STRUCTURAL ENGINEERING

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LONGMANS, GREEN AND CO.
55 FIFTH AVENUE, NEW YORK
221 EAST 20TH STREET, CHICAGO
TREMONT TEMPLE, BOSTON
210 VICTORIA STREET, TORONTO

STRUCTURAL ENGINEERING

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FOURTH EDITION, REVISED AND ENLARGED

LONGMANS, GREEN AND CO.

LONDON ♦ NEW YORK ♦ TORONTO

1928

Made in Great Britain

PREFACE.

During the last few years many excellent treatises have been published on Structural Engineering, the majority of which are principally devoted to a consideration of the subject from the purely theoretical point of view, whilst a few others treat the subject mainly with regard to the practical arrangement of structural details. The subject is one of so wide a range that it is manifestly impossible to attempt anything like a comprehensive survey of structures generally in a single volume. Between the theoretical computation of the loads and stresses in a structure and the evolution of a satisfactory practical design which shall have due regard to the exigencies of practical construction, there exists a gap which may only be bridged by considerable practical experience and knowledge of shop methods.

The Authors have endeavoured in the present volume to deal with the design of the more ordinary and commonly occurring structures from both points of view, and whilst necessarily stating the main outlines of theory involved, have extended the application of such theory to the practical design of a considerable variety of structures and structural details of everyday occurrence. The consideration of higher structures, such as rigid and two-hinged metal arches, suspension bridges, and structures of particular or unique character, has been intentionally omitted in order to make room for such examples as will be of interest to the majority of readers. It has been considered desirable to include a short summary of the properties of structural materials and weights of details in order that these may be readily available for reference in one volume, and the chapter on materials is not intended to supply other than a very brief compendium of the properties of materials. An extended acquaintance with these is very necessary to the structural designer, and such information in detail is readily accessible in treatises on materials.

Wherever possible, numerical data and arithmetical, in preference to analytical, methods have been adopted, and the use of mathematical formulæ has been avoided where not absolutely necessary. Although this treatment may occasionally result in slightly more protracted methods of calculation, the Authors are convinced, after many years of both practical and teaching experience, that it will render the subject matter more accessible to the greatest number of readers. The majority of practical designers have neither the time nor opportunity for acquiring an advanced knowledge of mathematics, and whilst not decrying its desirability, an acquaintance with higher mathematics is not necessary to the design of most ordinary structures. A thorough

understanding of the elastic beam theory *is* necessary, and this, it is hoped, has been stated in the fullest and simplest manner. The consideration of column strength has been treated a little more fully than is customary in works on Structural Engineering, and the Authors are particularly indebted to Mr. J. M. Moncrieff, M.Inst.C.E., for permission to make free use of the matter contained in his exhaustive investigation on this subject. Points relative to the design of bridges have been frequently alluded to and several bridge details illustrated; but as bridge design constitutes an extensive subject in itself, those portions relating to it in the present volume are only intended to be introductory. A brief section is devoted to tall building construction, and for more detailed information readers may be referred to the many excellent American treatises on this subject. It had been intended to include a short notice of simple reinforced concrete structures, but as these are already fully dealt with in specialized works and the subject is rapidly attaining such large proportions, such notice has been omitted through lack of space for adequate treatment. Masonry structures and types of engineering masonry, which are inseparably associated, have been given particular attention and several applied designs carefully worked out.

It is hoped the information contained in the work may prove of general utility to both draughtsmen and students, and although not exclusively compiled for the use of students, it will be found practically to cover the present syllabus of both the Ordinary and Honours Grade Examinations in Structural Engineering held by the City and Guilds of London Institute, whilst it will also be of valuable assistance to those preparing for the Associate Membership Examination of the Institution of Civil Engineers, and the B.Eng. degree of London University.

The Authors desire to acknowledge their indebtedness to Messrs. R. A. Skelton & Co. and Messrs. Mellowes & Co., Ltd., for information kindly supplied respecting broad-flanged beams and glazing; to Messrs. J. M. Moncrieff and E. Sandeman, MM.Inst.C.E., Professor T. Claxton Fidler, M.Inst.C.E., Mr. E. H. Stone, M.Am.Soc C.E.; to the Engineering Standards Committee; and to the *Minutes of Proceedings of the Inst. C.E.*, *the Transactions of the American Soc. C.E.*, *the Memoires of the French Soc. C.E.*, *The Engineer*, *Engineering*, *Annales des Ponts et Chaussées*, *La Revue Technique*, and other journals, frequent references to which have been annotated.

J. H.
W. H.

SHEFFIELD, *August*, 1911.

PREFACE TO SECOND EDITION.

THE first edition has been carefully revised, some of the illustrations re-drawn to a larger scale, and the table of equivalent distributed loads on railway bridges, on page 35, revised to meet the increased weights of present-day locomotives.

J. H.
W. H.

SHEFFIELD, *January*, 1914.

PREFACE TO THIRD EDITION.

The second edition has been revised and enlarged by the addition of new matter relating to beams, roofs, and stresses in lattice girders, including inclined girders. A new chapter has been added on simple influence lines, the principles of which have been explained by reference to several numerical examples which it is hoped will provide a useful introduction to the study of this important subject.

J. H.
W. H.

Sheffield, *February*, 1924.

PREFACE TO FOURTH EDITION

The third edition has been revised and new matter relating to cements and concrete has been added. References to British Standard Sections have been corrected to agree with the new standard sections. The Authors desire to acknowledge their indebtedness to Messrs. G. and T. Earle (1925), Ltd., Mr. J. E. Worsdale, B.Sc., the Cement Marketing Co., Ltd., the Lafarge Aluminous Cement Co., Ltd., and the Engineering Standards Committee for valuable information kindly supplied.

J. H.
W. H.

Sheffield,
May, 1928.

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CHAPTER I.

MATERIALS.

Stone.—The principal engineering structures for which stone is employed are masonry dams, piers, abutments and wing-walls for bridges, arches and retaining walls. The essential characteristics of stone for such works are durability, weight, and resistance to compression. In selecting suitable stone, the following general properties should receive consideration.

Durability.—Durability depends principally upon hardness, chemical composition and relative fineness of grain, and is very essential in masonry structures, since they are exposed to more or less severe atmospheric influences, or are permanently or intermittently submerged in water.

Weight.—The stability of structures such as dams, retaining walls, and lofty piers, is dependent on the weight of the material used, whilst in the case of ordinary walls and arches weight is not of such paramount importance.

Resistance to Compression.—All masonry structures being subject to compressive stress, it is necessary that the material should possess an adequate resistance to compression, especially in the case of massive and lofty structures Masonry is not employed under conditions where any considerable tensile stress will be developed. *Hardness* frequently decides the adoption or rejection of a stone which might otherwise be employed, on account of excessive expense in working.

Porosity.—Porosity is undesirable, since it contributes to rapid weathering and reduced resistance to compression. Very porous stones are especially liable to be affected by frost.

The following aids to judgment with regard to the above properties will be found useful. The best method of ascertaining the durability of a proposed stone is by an inspection of existing structures built of the same stone. Failing this, a careful examination should be made of the weather-resisting qualities of an exposed face of the quarry which has been undisturbed for a considerable length of time. The weight and resistance to compression are obtained by applying suitable tests, and figures relating to these will be found below. In all important works, special tests should be made of the actual stone employed. A suitable stone should exhibit on a clean fracture, a dense and finely grained structure free from earthy matter. The porosity of stone may be tested by immersing a dry sample in water and noting the volume of water absorbed, at the same time observing the effect on the clearness of the water, since this affords an indication of the

amount of soft earthy matter contained in the stone. The principal classes of stone employed in structural work are Granite, Limestone and Sandstone.

Granite.—Granite is the most durable stone employed in construction, but its hardness and expense of working restrict its use to very exposed structures and those subject to the heaviest stresses. Ordinary granite is composed of quartz, felspar and mica. Quartz is silicon oxide, and practically indestructible. Felspar is a mixture of silicates of alumina, soda, potash, or lime. It is much less durable than quartz. Its colour varying from grey to red gives the distinctive colour to the different varieties of granite. Mica is formed of silicates of alumina and other earths crystallized in thin laminae which readily split on weathering. It is the least durable constituent of the granite. Syenitic granites contain hornblende in addition to the above constituents. Hornblende is a silicate of lime and magnesia, is very hard and durable, and imparts a dark green colour to the granite. Syenitic granites are tougher and more compact than ordinary granite, but their scarcity makes the cost too great for general use. It is largely employed for ornamental purposes on account of the high polish it will take. An excess of iron in granite decreases its durability, and care should be exercised to select material without this impurity. The iron is easily detected when the granite is exposed to air, dark uneven patches of discoloration due to iron oxide forming on the surface. Its weight varies from 160 to 190 pounds per cubic foot, and the crushing load from 600 to 1200 tons per square foot. It absorbs a very small percentage of water.

Limestone.—Limestone is very varied in composition and physical characteristics, and includes all stones having carbonate of lime for the principal constituent. Some varieties have a crystalline structure, whilst others are composed entirely of shells and fossils cemented together. Marble is the hardest and most compact limestone, but is only used for decorative purposes. Other limestones make good building material, but care must be taken to select fine even-grained stone containing no sand-cracks or vents. It is easily worked, and forms a good evenly coloured surface. Its weight varies from 120 to 170 pounds per cubic foot, and the crushing resistance from 90 to 500 tons per square foot. It is less durable than granite or sandstone, and absorbs a relatively large percentage of water. It should always be laid on its natural bed when building, and it is preferable to allow it to season before using to free it from *quarry sap* or moisture it contains when quarried.

Sandstone.—As its name implies, sandstone has for its chief constituent sand, cemented together by various substances, and it is largely upon the nature of the cementing material that the quality of the stone depends, since the silica forming the sand is extremely durable. The best cementing material is probably silicic acid, but most sandstones have carbonate of lime, iron, alumina, or magnesia as part of the cementing medium. In selecting sandstone, a recent fracture, if examined through a lens, should be sharp and clean with grains of a uniform size, well cemented together. Its weight should be at least 130 pounds per cubic foot, and it should not absorb more than 5 per

cent. of its own weight of water when immersed for 24 hours. Sandstone is used for all the best ashlar work. The most important variety for heavy engineering works is termed *grit*, from the formation in which it occurs. It has a coarse-grained structure, is very strong, hard, and durable, and is obtainable in very large blocks. For lighter work *freestone* is used, being more easily worked.

TABLE 1.—PROPERTIES OF STONES.

	Weight per cubic foot.	Crushing weight per square foot.	Percentage absorption of its own weight.
Granites—	lbs.	tons.	per cent.
Peterhead	165	800 to 1200	0·25
Cornish	162	—	—
Guernsey	187	950	—
Killiney (Dublin)	171	690	—
Sandstones—			
Craigleith	140	860	3·6
Bramley Fall	132	240	3·7
Darley Top	139	520	3·4
Grinshill Freestone	122	210	7·8
Red Mansfield	143	600	4·5
White Mansfield	140	460	5·0
Limestones—			
Bath	128	97	7·5
Portland Whitbed	132	205	7·5

Bricks.—Bricks are manufactured from clay, sometimes mixed with other earths, by moulding the clay to the required size and shape and burning it in kilns. The quality of bricks depends upon the nature and proportions of the constituents of the clay, and the heat to which they are subjected in the kilns. A small quantity of lime, very finely disseminated through the clay, is advantageous, as it assists vitrification when burning; but if in lumps, on being burnt, it is converted into quicklime, which, on exposure to damp, sets up a slaking action with consequent expansion and liability to split the bricks. The proportion of iron contained in the clay influences the colour of the brick, which may vary between yellow, red, blue, or black. Bricks should be burnt until vitrification is just commencing. The characteristics of good bricks are, freedom from flaws, cracks, quicklime, and salt; they should be of uniform colour, shape, and dimensions, and when struck should give a clear ringing sound. The absorption of water is a good indication of the quality of the brick. No brick should absorb more than one-sixth its own weight of water.

A test often included in specifications is to place a saturated brick in a temperature of 20° F., and subject it to a load of 336 lbs. per square inch. If there are any signs of injury, such bricks are liable to be rapidly disintegrated by frost, and should be rejected.

There are many qualities of bricks, but the kinds most generally employed in first-class engineering structures, are Staffordshire blue bricks and the finer qualities of red bricks known as stock bricks. Blue brick has a glazed surface, making it almost impervious to water,

is very durable, and has a high compressive resistance. It is used for the face of works in contact with water, for piers and abutments of bridges, retaining walls, and tunnels. Stock bricks are usually employed for the interior portions of works. The crushing resistance of Staffordshire blue bricks varies between 0·9 and 2·8 tons per square inch, that of stock bricks about 1 ton per square inch. The compressive stress on bricks should never approach the crushing resistance, as the masonry, of which they form part, is, as a structure, much weaker than the individual bricks.

The weight and size of bricks vary according to the locality of manufacture, a size frequently adopted being 9 ins. × 4½ ins. × 3 ins., and weight about 9 lbs.

When used in curved structures such as small arches, bricks should always be tapered to the radius of the curve, to allow a uniform thickness of mortar in the joints, and ensure an even bearing.

Specification for Brickwork.—The bricks to be laid in the bond specified with joints not exceeding ¼ inch in thickness. Every course to be thoroughly flushed up with mortar, and the bricks well wetted before laying. The work to be substantial, with neat and workmanlike appearance on the face. Heading joints to be truly vertically over each other, and the horizontal joints perfectly straight and regular. Any cutting to be neatly done. Arches to be turned in half-brick rings with bond courses every five feet through the arch. No batts to be used, except where absolutely necessary. All joints to be neatly struck and drawn, and arch rings to be carefully kept in true curves. Special care to be taken in laying the different rings in arched work exceeding 9 inches in thickness, so that on easing the centering, the separate rings shall fully bear on each other throughout the whole length of the arch. If any lower ring settles away from an upper ring, so as to cause a ring joint to open, such joint shall not be pointed or stopped, excepting with permission of the engineer or the clerk-of-works, and any such defective rings are, if desired, to be entirely rebuilt. No bricks to be laid in frosty weather, and newly executed work to be adequately covered over at night. Circular work of less than three feet radius to be executed with special radius bricks.

Lime.—Lime is obtained by heating limestone to redness in kilns. This process, termed calcination, converts the limestone into quicklime, which on being slaked with water, either by sprinkling or immersion (the former is better), forms calcium hydrate. This substance, when mixed with sand and water, and left to dry, changes into a solid mass which is practically again limestone. The chemical action which takes place is as follows:—

$$\text{Limestone} = \underset{\text{(Carbonate of lime)}}{CaCO_3}$$

$$\text{by calcining} = \underset{\text{(Quicklime)}}{CaO} + \underset{\text{(Carbon dioxide)}}{CO_2}\rightarrow$$

on slaking, $$CaO + \underset{\text{(Water)}}{H_2O} = \underset{\text{(Calcium hydrate)}}{CaH_2O_2}$$

on setting, $$CaH_2O_2 + CO_2 = CaCO_3 + H_2O\rightarrow$$

Where the limestone is composed almost wholly of carbonate of lime, the resulting quicklime, called *fat* or air-lime, will only harden

in air, and unless used with a suitable sand, only the exposed portions will set in reasonable time. Limestones containing clay, when burnt form *hydraulic* lime, which has the property of setting more or less rapidly under water or in damp situations. Such limes are said to be feebly or strongly hydraulic according as the mortar of which they form the active ingredient, sets slowly or rapidly under water.

CLASSIFICATION OF HYDRAULIC LIMES.

Class.	Per cent. of clay.	Setting under water.
Feebly hydraulic .	5 to 12	Firm in 15 to 20 days. In 12 months hard as soap. Dissolves with great difficulty.
Ordinarily hydraulic	15 to 20	Resists pressure of finger in 6 to 8 days. In 12 months hard as soft stone.
Strongly hydraulic .	20 to 30	Firm in 20 hours. Hard in 2 to 4 days. In 6 months may be worked like a hard limestone.

In the preparation of mortar, sand is added to the lime to prevent excessive shrinkage and to save cost, but in no way affects the mixture chemically. Sharp, clean, gritty sand should be used. The proportions of lime and sand may be varied, but the following give good results for ordinary purposes: 2·4 parts of sand to 1 part pure slaked lime, and 1·8 parts of sand to 1 part of hydraulic lime.

Cement.—Cement is similar to the best hydraulic lime, but possesses stronger hydraulic properties. There are two classes of cements—natural and artificial. Natural cements are obtained by calcining naturally occurring limestones which are found to produce cement. Roman cement manufactured from nodules found in the London Clay is perhaps the best known natural cement. Having no great ultimate strength, it is unsuitable for use in heavy structures, and its use is confined to temporary work where quick setting is of great importance.

Portland Cement.—Portland Cement is the most widely used constructional material at the present time. It is the vital constituent of concrete which has largely superseded masonry and brickwork for heavy engineering works and all forms of construction capable of execution in reinforced concrete. The great bulk of Portland Cement is manufactured by calcining in rotary kilns an intimate mixture of suitable quantities of chalk and clay very finely ground and mixed with water to a milky consistency. The product of the fusion of this *slurry*, known as *clinker*, when finely ground, constitutes the Portland Cement of commerce. A brief outline of the various stages in the manufacture as practised in one of the most modern plants is here given.

Chalk is excavated on a large scale by steam diggers and railed to gyratory crushers capable of reducing 200 tons of stone per hour to 4-inch gauge. The crushed chalk passes to a rotary screen with 3-inch holes, the tailings from which are re-crushed by a jaw crusher. The crushed chalk is then elevated and charged into a reinforced concrete silo of 3000 tons capacity. The silo discharges into a set of combination

tube mills 32 feet long by 6 feet in diameter divided into three compartments, two of which contain forged steel balls and the third, flint pebbles. In these the chalk, mixed with water, is so finely ground as to leave a residue of not more than 10 per cent. on a 180 × 180 mesh sieve. The chalk slurry from the tube mills is next delivered into a circular reinforced concrete tank 66 ft. in diameter and 11 ft. deep, holding about 1500 tons of slurry and furnished with an electrically driven stirring mechanism of the sun and planet type. By this means the slurry is continuously agitated and the solid matter prevented from sinking to the bottom. The slurry tank discharges to a sump from which the slurry is lifted to an automatic measuring machine which passes forward exactly the desired quantity in any given time. From the measuring machine the slurry gravitates down an inclined shoot in an even film, to octagonal wash mills 20 ft. wide provided with rotary stirrers. At this stage the clay is added to the chalk slurry by tipping it down the same shoot as that down which the slurry is flowing. The clay is weighed previously to mixing with the chalk slurry. The combined chalk and clay slurry after mixing and stirring in the wash mills is strained through gratings to a sump, from which it is lifted to reinforced concrete triple mixers, each measuring 40 ft. long by 20 ft. wide by 10 ft. deep, and provided with three sets of rotary stirring arms. From these mixers samples are continually taken for analysis, and any adjustments in the proportion of the ingredients are made prior to the slurry passing forward to the slurry grinding tube mills. In the slurry grinding tube mills, each 20 ft. long by 5 ft. diameter, the slurry is so finely ground that, when dried and pulverized, 95 per cent. will pass through a standard sieve of 180 holes to the lineal inch. These tube mills discharge into a small tank with motor-driven stirring arms which forms the sump of the slurry pumps. The slurry pumps deliver the slurry to the feeding platform of the rotary kilns, and also into a set of storage silos of capacity sufficient to supply slurry to the kilns for from 4 to 5 days running in the event of casual stoppage of the supply of raw materials or breakdown of the grinding and mixing plant, since the kilns necessarily run continuously for long periods. In the storage silos, which are riveted steel tanks 50 ft. high and 23 ft. diameter, the slurry is kept agitated by blowing air at about 25 lbs. per square inch pressure upwards through the slurry for short periods at intervals.

The rotary kilns, 200 ft. to 250 ft. long and 9 ft. to 10 ft. in diameter, are of riveted steel plates lined with refractory brick. They are supported at an inclination of 1 in 20 by four cast-steel tyres running on rollers and driven through reduction gearing at the rate of about one-half to two revolutions per minute. The output of clinker from one such kiln averages between 7 and 8 tons per hour. The slurry from the pumps is delivered into a feed tank, from which it is measured by a scoop wheel into a trough which discharges into the feed spout leading into the upper end of the rotary kiln. A tell-tale on the firing platform indicates whether the feeding mechanism is operating satisfactorily or not. At the lower end of the rotary kiln pulverized coal, previously dried in revolving cylinders, is blown through the firing nozzle in the hood covering the end of the kiln and an intense flame plays axially into the kiln. The action inside the kiln is as follows :

As the slurry flows down the inclined rotating barrel, the water is first evaporated, and as it passes further down the kiln the calcium carbonate is decomposed by the more intense heat and its carbonic acid gas liberated. The lime is then free to combine with the silica and alumina of the clay, which chemical combination takes place in the burning or clinkering zone at a temperature of between 1300° and 1400° C., calcium silicates and aluminates being formed. The calcined product or *clinker*, in the form of greenish-black granules about the size of marbles, falls from the kiln into a rotating cylindrical cooler through which air is drawn. This air, heated by contact with the red-hot clinker, is subsequently passed into the kiln, where it contributes to the combustion of the coal.

The clinker falls on to a conveyor, which delivers it to silos constructed over the cement grinding mills. At this stage a small percentage of gypsum is added for regulating the setting time of the cement. The clinker is finally ground in combination tube mills similar to those already described, to a degree of fineness such that 5 per cent. only remains on a standard 180 × 180 mesh sieve. The ground cement is then conveyed to store, from which it is drawn to automatic weighing and bag-filling machines.

Bulking of Cement and Influence of Fine Grinding on Weight and Strength.—By bulking is meant the change in apparent volume due to increased fineness of grinding. A cubic foot of cement carefully measured is found to be of very variable weight dependent upon its degree of fineness. The finer the grinding the less is the weight per cubic foot. The increase in bulk of a given weight of cement with finer grinding is attributed to the cushioning action of air. The British Standard Specification for Portland Cement (1925) requires that the residue on a 180 × 180 sieve shall not exceed 10 per cent. after 15 minutes' sifting. Table 2 shows the influence of fine grinding on the weight per cubic foot. The specific gravity of cement varies between 3·05 and 3·15.

TABLE 2.—INFLUENCE OF FINE GRINDING ON WEIGHT OF CEMENT (J. E. WORSDALE).

Cement residue on 180 sieve	Actual weight per cubic foot, Measured dry	Weight per cubic foot based on uniform volume of paste.	Suggested agreed figure for weight per cubic foot.
per cent	lbs.	lbs	lbs
10	90	90·0	90
6	85·5	86·5	87
3	82·5	83·4	84
1	80 6	80·2	81

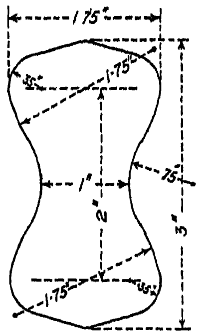
The bulking of cement has an important bearing on the quantity to be specified for making concrete, since if the cement be measured by volume, the actual weight employed will be very variable dependent upon the degree of fineness of grinding. For this reason it is preferable to specify the *weight* of cement to be mixed with stated *volumes* of

coarse and fine aggregate. Fineness of grinding also influences the strength of cement. If cement having a residue of 10 per cent. on a 180 × 180 sieve is considered to weigh 90 lbs. per cubic foot, then for every one per cent. reduction in residue the weight will be reduced approximately 1¼ per cent. On the other hand, for equal weights of cement made into concrete, one per cent. reduction in residue will increase the strength of the concrete by the amounts shown in Table 3.

TABLE 3.—INFLUENCE OF FINE GRINDING ON STRENGTH OF CONCRETE (J. E. WORSDALE).

Based on tests at age of	7 days.	28 days.	3 months.	1 year.
Percentage increase in concrete strength	2½	2	1¾	1½

Tests.—The tensile strength of cement is ascertained by making tests on briquettes of standard shape and dimensions. The B.S.S.,[1] 1925, requires the average tensile strength of six briquettes of the form shown in Fig. 1 to be not less than 600 lbs. per square inch, the tests being made seven days after gauging. The briquettes are to be kept in a damp atmosphere for 24 hours after gauging, then removed from the moulds and submerged in clean fresh water at a temperature of between 58 and 64° F. until the time for testing. All of the well-established brands of cement will be found to comply easily with the above specification, most being well above it.



Cross Section 1"×1"

FIG. 1.

The B.S.S. also requires that six briquettes made from a mixture of one part of cement to three parts by weight of standard (Leighton Buzzard) sand shall show the following average tensile strengths: Seven days after gauging, not less than 325 lbs. per square inch, and 28 days after gauging, not less than

$$\text{Breaking strength at 7 days} + \frac{10{,}000}{\text{Brkg. strength at 7 days}} \text{ lbs. per sq. in.}$$

Le Chatelier Test.—The soundness of cement is determined by the Le Chatelier test.[2] The apparatus, illustrated in Fig. 2, consists of a small split cylinder of spring brass, to which are attached two indicators. The cylinder is filled with cement gauged under the conditions of the B.S.S., and placed under water for 24 hours. It is then removed from the water and the distance between the ends of the indicators accurately

[1] Reproduced by permission of the Engineering Standards Committee, from Report No. 12, revised 1925.

[2] Reproduced from Report No. 12, revised 1925. British Standard Specification for Portland Cement. By permission of the Engineering Standards Committee.

measured. The mould is again immersed in cold water which is brought to boiling-point and kept boiling for six hours. After allowing the mould to cool the distance between the indicators is again measured. The expansion should not exceed 10 millimètres after 24 hours' previous aëration of the cement, nor 5 millimètres after 7 days' aëration.

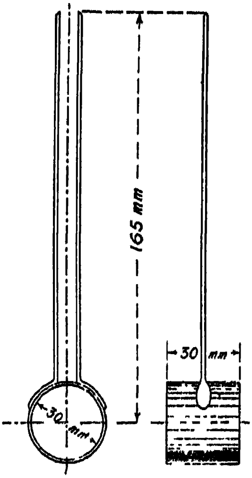


FIG. 2

Time of Setting.—The time of setting is determined by the aid of the Vicat needle apparatus. For the method of making the test the reader is referred to the B.S.S. The initial setting time of normal setting cement should not be less than 30 minutes and the final setting time not more than 10 hours. For quick-setting cement the initial setting time should not be less than 5 minutes and the final setting time not more than 30 minutes.

Rapid Hardening Cements.—The importance of speed in constructional concrete work has created a demand for Portland cements possessing quick-hardening properties. The advantages will be obvious. Concrete roads may be opened to traffic after two or three days, shuttering may be removed from reinforced concrete members in a few days, and concrete piles driven in from two to seven days after moulding instead of requiring curing for several weeks. In general, this class of cements possesses superior strength to ordinary Portland cement, and thus makes for economy in weight. Quick-hardening cements should not be confused with those quick-setting cements which have to be used with great speed and placed *in situ* within a few minutes, as, for instance, Roman cement. A considerable number of such cements are now on the market, some of the widest known being Ciment Fondu, Ferrocrete, and Tunnelite.

Ciment Fondu.—This cement, introduced in 1918, is often referred to as an aluminous cement. It will be seen from Table 5, its constitution differs considerably from that of normal Portland cement, in that it contains a much larger percentage of alumina and much less lime and silica, the resulting chemical action on hydrating producing an essential crystalline structure of aluminate of lime in place of silicate of lime, which structure matures and hardens in a much shorter time than a structure mainly siliceous. Ciment Fondu is not quick setting, its setting time being longer than that of Portland cement. It possesses greatly increased strength and develops considerable heat during setting, thus being advantageous for use in frosty weather. Since it contains no free lime it is more permanently retentive of colouring matter.

Ferrocrete.—Ferrocrete is a rapid-hardening cement having a final setting time a little less than that of ordinary Portland cement, and thus allows ample time for placing concrete in position before setting has appreciably progressed. In about four days concrete made with Ferrocrete attains approximately the same compressive strength as does

Portland cement concrete in 28 days. Also, in two days, Ferrocrete mixed with three parts of sand develops a considerably greater tensile strength than that required by the B.S.S. in 28 days, these results being well substantiated by independent testing authorities. It is obviously of great advantage in reducing the amount of shuttering required on extensive works, its superior strength permits of leaner mixes of concrete being used for equal strength with ordinary Portland cement concrete, precast units may be lifted and moved earlier, piles driven in a few days and concrete structures generally loaded much earlier.

Table 4 gives representative results of tensile tests on 3 to 1 sand and cement briquettes at various ages.

TABLE 4.—TENSILE TESTS OF SAND AND CEMENT BRIQUETTES.

Age.	Lbs. per square inch. 3 sand to 1 cement					
	1 day.	2 days	3 days.	5 days	7 days	28 days.
Portland cement . . .	220	340	480	520	[illegible]	622
Ferrocrete	312	516	583	611	688	713
Ditto	425	525	585	685	680	700
Ciment Fondu . . .	475	518	540	563	580	618

TABLE 5.—REPRESENTATIVE ANALYSES OF VARIOUS CEMENTS.

	Portland cement.	Ferrocrete.	Ciment fondu.	Roman cement
	per cent.	per cent.	per cent.	per cent
Insoluble residue	0·45	0·12	0·80	
Silica	21·17	20·58	7·80	25·18
Alumina	6·79	6·06	36·23	10·30
Iron oxide	2·65	2·82	8·17	7·41
Lime	64·22	64·56	41·97	44·54
Magnesia	1·10	1·25	0·47	2·92
Sulphuric anhydride . .	1·48	1·90	0·22	2·61
Sulphur (as sulphide) . . .	—		0·11	
Alkalies and Loss . . .	0 66	1·21	0·31	3·68
Loss on ignition . . .	1·48	1·50		
Titanium oxide . . .			3·92	
Other constituents				1·16

Concrete.—Concrete is formed by mixing together suitable proportions of broken stone or gravel, sand, cement and water. The mortar formed by the sand and cement is called the *matrix*. The larger material constitutes the *aggregate*. The sand and aggregate should be clean, and where great strength is required the aggregate should consist of hard material. The stone is better specified as *coarse* aggregate and the sand as *fine* aggregate. To secure the best results the stone should be well graded in size between specified limits so that the smaller pieces help to fill the relatively large voids which would otherwise obtain if the stone were of uniform size.

In Fig. 3 the whole of the voids in stone of sensibly uniform size is

filled with mortar. In Fig. 4, where stone of two screened sizes is used, more of the voids are occupied by solid material; and in Fig. 5, where the stone is well graded, the maximum density is obtained with the minimum quantity of cement. For concrete in mass the stone may be from 2½ inches downwards, but for reinforced concrete slabs and members generally, ¾ inch is usually specified as the maximum size of coarse aggregate to ensure thorough filling between the reinforcement. For massive concrete work, rubble or cyclopean concrete is used (see

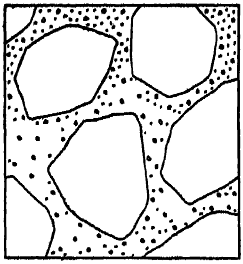


Fig. 3.

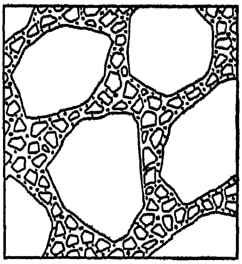


Fig. 4.

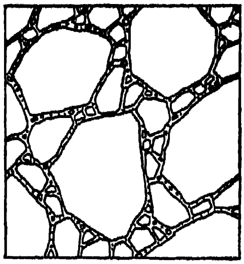


Fig. 5.

page 379). The sand should also be graded from ¼ inch down, the larger proportion passing a ⅛-inch mesh. Fine dust should be rigorously excluded from both stone and sand. It is a source of weakness, as it has no body to which the cement can adhere. The object of grading is to reduce the percentage of voids as far as possible, *as the purpose for which cement is added is to act as a binding material and not to fill the voids.*

Proportions for Concrete.—The careful proportioning of the constituents of concrete for a given purpose is a considerable subject in itself, but for most ordinary purposes the following method may be followed. The percentage voids of large and small aggregate should be known. For the large aggregate the percentage voids may be found by filling a tank of known capacity with the previously soaked aggregate and measuring the quantity of water required to fill up the tank. In estimating the voids in sand this method is not so satisfactory owing to the difficulty of releasing all the air between the closely packed grains. The specific gravity of the sand is preferably taken and the voids calculated therefrom.

The proportions are usually specified by volume, taking the cement volume as unity. Thus 1 : 2 : 4 indicates 1 part by volume of cement, 2 parts of fine aggregate, and 4 parts of coarse aggregate, and is known as 6 to 1 concrete. The voids of the large aggregate represent the *minimum* volume of mortar required to form a dense concrete if perfect mixing could be ensured and no wastage of cement occurred between mixing and laying, but the volume of mortar should exceed by about 20 per cent. the volume of the voids in the coarse aggregate to ensure thorough soundness. Since the percentage voids of most aggregates range from about 30 to 45 (see Table 6), this quantity of excess mortar will represent from 6 to 9 per cent. of the total volume of the aggregate, including its voids. The quality of the concrete may be varied by using different proportions of sand to cement, a larger proportion of

sand making a weaker but less expensive concrete. Table 6 gives the approximate percentage of voids to be expected in various aggregates, but where doubt exists these should be actually measured.

TABLE 6.—PERCENTAGE VOIDS IN AGGREGATES.

Material.	Size.	Per cent. of voids.
Granite chippings . .	¾ in.	47·3
Crushed sandstone . .	¼ in.	41·4
Sandstone	2 in.	42·5
Limestone	3 in. to 1 in.	41·0
Sandstone	1 in. to ¼ in.	39·3
Limestone	1½ in. to ¾ in.	38·5
Broken brick . .	1½ in.	38·5
Thames ballast . . .	1 in. to $\frac{1}{16}$ in.	37·5
Trent sand	⅛ in. down.	37·3
Round pebbles . . .	3½ in. to 1½ in.	37·0
Thames ballast . . .	1½ in. to ⅜ in.	35·3
Gravel	⅜ in. to ⅛ in.	34·8
Sand	⅛ in. down.	30·0
Gravel	1 in. to ¼ in.	27·3
Mixed gravel and sand.	—	23·2

EXAMPLE 1.—*To find the relative quantities of dry materials to be used of coarse aggregate having 45 per cent. voids and fine aggregate or sand having 30 per cent. voids, the mortar to be 1 of cement to 2 of sand and to be 20 per cent. in excess of the voids of the large aggregate.*

Let x = volume of cement; $2x$ = volume of sand.

$$\text{Volume of sand voids} = \frac{30}{100} \times 2x = \frac{3x}{5}$$

$$\text{Excess of cement over sand voids} = x - \frac{3x}{5} = \frac{2x}{5}$$

$$\text{Volume of mortar} = 2x + \frac{2x}{5} = \frac{12x}{5}$$

Hence, volume of voids of large aggregate should $= \dfrac{100}{120} \times \dfrac{12x}{5} = 2x$,

and volume of large aggregate $= \dfrac{100}{45} \times$ its voids $= \dfrac{100}{45} \times 2x = 4{\cdot}44x$.

The required proportions by volume are therefore $1 : 2 : 4\frac{4}{9}$.

The volume of concrete resulting will be the volume of the large aggregate including voids + 20 per cent. of its voids $= 4{\cdot}44x + \frac{20}{100}$ of $2x = 4{\cdot}84x$.

The volume of ingredients before mixing $= x + 2x + 4{\cdot}44x = 7{\cdot}44x$.

Hence for 100 cub. ft. of concrete *in situ* $\dfrac{7{\cdot}44}{4{\cdot}84} \times 100 = 153{\cdot}72$ cub. ft. of dry ingredients would be required, assuming no waste in mixing and transporting. A slightly richer mix would be desirable if watertightness was a special consideration.

While it is still a general practice to specify the proportions by volume, it should be remembered that a cubic foot of cement may contain a very variable weight of cement according as it is loosely

measured or well shaken down. The degree of fineness also affects the weight as shown in Table 2. For these reasons it is preferable to specify the *weight* of cement to be used with stated volumes of aggregate. Mr. J. E. Worsdale has recently proposed a new specification for the proportions of concrete as follows: "The proportions of coarse and fine aggregates shall be 5 cub. ft. of coarse aggregate and 2½ cub. ft. of fine aggregate to 1 cwt. sack of cement." These proportions are based on a cement having a fineness which just complies with the British Standard Specification, but for cements having fineness given below, the quantities of aggregates may be increased according to the following schedule:—

Cement residue on 180 × 180 sieve.	Increased proportions of aggregate per cwt. of cement.	
	Coarse	Fine.
per cent.		
6	0·17	0·08
3	0·35	0·17
1	0·55	0·27

Such a specification ensures the proper quantity of cement being employed. Taking one cubic foot of cement equal to 90 lbs., one cwt. would be the equivalent of practically 1¼ cub. ft., and a specified mix of 1 cwt. of cement to 2½ cub. ft. of fine and 5 cub. ft. of coarse aggregate is equivalent to ⅘ cwt., or one cubic foot of cement actually weighing 90 lbs. to 2 cub. ft. of fine and 4 cub. ft. of coarse aggregate. Specified in this way, the proportions in Example 1, above, would be—

1 cub. ft. × 1¼ = 1 cwt. of cement,
2 „ × 1¼ = 2½ cub. ft. of fine aggregate,
and 4·44 „ × 1¼ = 5·55 cub. ft. of coarse aggregate.

Slump Test for Consistency.—The consistency of concrete is readily ascertained by the Slump Test. A sample from the mix is filled in three layers, with a pricker, into a truncated conical metal mould 12 in. high, 8 ins. base and 4 ins. top diameter. The mould is then lifted and the slump or settlement of the concrete measured. A slump of ½ in. to 1 in. represents concrete of "Normal Consistency." Slumps of ½ in. to 1 in., 3 in. to 4 in. and 5 in. to 6 in. represent consistencies suitable for machine-finished roads, heavy reinforced sections and thin vertical reinforced sections respectively.

Compressive Tests for Concrete.—Compression tests, which are more representative of the strength of concrete as generally used, are usually made on 6-in. cubes for small aggregate up to 1-in. and 9-in. or 12-in. cubes for larger aggregates. The cubes should be accurately shaped and cast in metal moulds. They are removed from the moulds in 24 hours and kept immersed in water at 60° F., or they may be exposed to the weather under damp cloths, at the site where the concrete is to be used, until ready for testing. The following table gives typical results of compressive tests on concrete made with different cements at various ages:—

COMPRESSIVE STRENGTH OF CONCRETE. 6-IN. CUBES.

Age.	Lbs. per square inch.				
	1 day.	2 days.	7 days.	28 days.	1 year.
Portland Cement.					
1 : 1½ : 3 . . .		1850	3940	5050	7729
1 : 2 : 4		1550	3190	4490	6173
1 : 2½ : 5 . . .		1170	2860	3400	5582
1 : 3 : 6		840	1900	2540	
Ferrocrete					
1 : 1½ : 3 . .		3470	6790	8240	
1 : 2 : 4		3000	6020	7350	
1 : 2½ : 5 . . .		2290	4500	5230	
1 : 3 : 6		1350	3400	4080	
Ciment Fondu					
1 : 2 : 4	8000		8810		
1 : 5·14 . . .		6258	7642	8100	11,200

Asphalte.—Asphalte is used to a great extent in engineering works for damp-proof courses, and layers in masonry and metal bridges, roads, roofs, etc. It is a combination of bitumen and calcareous matter, naturally or artificially combined. The natural asphaltes are usually found as limestones saturated with 8 to 12 per cent. of bitumen. In preparing such asphalte for use, the rock is ground to a powder, mixed with sand or grit and heated with mineral tar. (Coal-tar should not be used, being brittle, easily crushed, and readily softened under heat.) The mastic, as the mixture is then called, is laid *in situ* as a thick liquid. If the natural rock contains a large percentage of bitumen it may be laid, after grinding and heating, as a powder, in which case it must be thoroughly rammed, whilst hot, so as to form one solid mass.

The proportion of grit in the mastic will vary according to the purpose for which the asphalte is to be used.

For roofs, lining of tanks, etc. . . 2 of grit to 18 of asphalte.
,, flooring, footways, etc. . . 2 ,, 16 ,,

Timber.—The uses of timber in engineering structures may be classified as follows:—

1. For marine works.
2. ,, exposed structures other than marine works.
3. ,, parts of structures under cover.
4. ,, paving.

For the first three classes strength and durability are essential, and for the last hardness is the chief quality required. All timber structures situated in sea-water are subject to the attacks of sea-worms, which bore into most varieties of timber, and reduce or entirely destroy its strength. For such situations the most suitable timbers are greenheart or jarrah, as these timbers contain an oil which renders them to some extent immune from the attacks of sea-worms. For the second class the timber is usually preserved from weathering by creosoting or other means, which is unnecessary for the third class.

Selection of Timber.—The selection of timber should be entrusted only to a thoroughly experienced person, as the quality can only be judged after much personal experience. For all the main members of structures the heartwood only should be used, the outer portions, or sapwood, being inferior in strength and durability. The annual rings should be regular, close, and narrow. Darkness of colour is generally a good indication of strength. When freshly cut, the timber should have a sweet smell; a disagreeable smell usually betokens decay. The surface should be firm and bright when planed, and when sawn the teeth of the saw should not be clogged. Knots should not be large, numerous, or loose.

Classification of Timber according to Size.—Timber is converted from the logs or balks to commercial sizes by different methods of sawing, depending upon the uses to which the timber is to be put.

The usual sizes of timber employed in engineering construction are the following:—

Whole or square timber	9 in. × 9 in. to 18 in. × 18 in.
Half timber	9 in. × $4\frac{1}{2}$ in. to 18 in. × 9 in.
Planks	11 in. to 18 in. wide by 3 in. to 6 in. thick
Deals	9 in. wide by 2 in. to 4 in. thick
Battens	$4\frac{1}{2}$ in. to 7 in. wide by $\frac{3}{4}$ in. to 3 in. thick.

Varieties of Timber.—For engineering purposes the following are the most generally used timbers:—

Pine, Baltic, sometimes known as red or yellow fir, consists of alternate hard and soft rings. It has a strong resinous odour, is easily worked, but is not nearly so durable as some of the harder timbers. The best varieties are those in which the annual rings do not exceed $\frac{1}{10}$ in. in thickness, and contain little resinous matter. In seasoning the maximum shrinkage is $\frac{1}{40}$th part of its original width.

Sizes obtainable - Balks 10 to 16 in. square, 18 to 45 ft. long.
Deals 2 to 5 in. thick, 9 in. wide, 18 to 50 ft. long

Uses.—Decking, temporary work, or in positions where there is little or no stress.

American Red or Yellow Pine, is clean, free from defects, and easily worked, but is weaker and less durable than Baltic pine.

Sizes.—Balks 10 to 18 in. square, 16 to 50 ft. long
Deals 2 to 5 in. thick, 9 in. wide, 16 to 50 ft. long.

Uses. - Similar to Baltic pine.

Pitch Pine, obtained from the southern states of America, is a timber very largely employed in engineering works on account of its strength and durability. It is very hard and heavy, and contains a large proportion of sapwood full of resin. It is liable to cupshakes, will not take paint, and is very hard to work.

Sizes.—Balks 10 to 18 in. square, 20 ft. to nearly 80 ft. long.
Deals 3 to 5 in. thick, 10 to 15 in. wide, 20 to 45 ft. long.

Uses.—For the heaviest timber structures, where strength and durability are essential. The long lengths in which it is obtainable make it very suitable for piles.

Oak, English, is the most durable of northern latitude timber. It

is very strong, hard, tough, and elastic. Contains gallic acid, which increases its durability, but corrodes any iron penetrating the timber.

Uses.—It is too expensive for use in general engineering works, and is only employed where extreme strength and durability are required.

Oak, American, is similar in many respects to English, but is inferior in strength and durability. It is sound, hard, tough, elastic, and shrinks little in seasoning.

Sizes.—Balks 12 to 28 in. square, 25 to 40 ft. long.

Uses.—Similar to English oak.

Greenheart is probably the strongest timber in use. It is dark green to black in colour, and obtainable only from the northern part of South America. It has a fine straight grain, is very hard and heavy, has an enormous crushing resistance, and contains an oil which resists, to a great extent, the attacks of sea-worms. It is very apt to split and splinter, and care must be taken when working it.

Sizes.—It is imported rough in logs 12 to 24 in. square, and up to 70 ft. in length.

Uses.—For piles, dock gates, jetties, and all marine structures.

Beech is black, brown, or white in colour, is light, hard, compact, not hard to work, and is durable if kept constantly either wet or dry, but if alternately wet and dry it quickly rots.

Use.—For piles and sleepers.

Elm possesses great strength and toughness. It has a close fibrous grain, warps, and is difficult to work. It should be used when freshly cut, and kept continually under water.

Uses.—For piles and fenders.

TABLE 7.—WEIGHT AND STRENGTH OF TIMBER.

Timber (seasoned)	Weight per cubic foot	Resistance to crushing in direction of grain in tons per square inch		Breaking load at centre for square beam 1" × 1" × 1 ft. span
	lbs.			lbs.
Pine, Dantzig	36	3·1		400 to 700
„ American Red	34	2·1		390 to 570
„ „ Yellow	30	1·8		470
„ Pitch	42 to 48	3·0		350 to 500
Oak, English	48 to 60	2·9	4·5	500 „ 750
„ American	54	3·1		500 „ 700
Greenheart	60 to 70	5·8	6·8	900 „ 1500
Beech	43 to 53	3·4	4·2	550 „ 700
Elm	36	2·6	4·6	350 „ 450
Teak	41 to 55	2·3	5·4	600 „ 700
Jarrah	60 to 63	3·2		500 „ 650

Teak has a dark brown colour, is light, but very strong and stiff. It contains a resinous oil, which makes it very durable, and able to resist the attacks of ants. It is easily worked, but splinters.

Sizes.—Balks 12 to 15 in. square, 23 to 40 ft. long.

Uses.—Used to a great extent in shipbuilding for backing armour,

as the oil it contains does not corrode iron. It is also used for decking and structures liable to the attacks of ants.

Jarrah.—A red Australian timber, with a close wavy grain. It is very durable, and contains an acid which partially resists sea-worms and ants. It is full of resin, brittle, liable to cupshakes, and shrinks and warps in the sun.

Sizes.—Balks 11 to 24 in. square, 20 to 40 ft. long.

Uses.—Similar to greenheart. It is also used to a large extent for wood pavement.

Where two values are given in the above table for the crushing resistance, the first is for timber moderately dry, and the second for thoroughly seasoned timber. Timber when wet does not possess nearly the same strength as when thoroughly dried.

Shearing Strength.—The shearing resistance of timber is very variable, and few reliable experiments have been made to ascertain such resistance. Rankine says the shearing strength along the grain is practically the same as the tensile strength across the grain. The following are approximate shearing values along the grain :—

Oak, elm, ash, birch	over 1000	lbs.	per sq. in.
Sycamore, Cuban pine	„ 600	„	„
Norway pine, white pine, spruce .	„ 400	„	„
English oak trenails, across the grain	„ 4000	„	„

Decay of Timber.—*Dry rot* is caused by the confinement of gases, produced by warmth and stagnant air, around timber The fungus thus produced feeds upon the wood, and reduces it to a powder. The rot will spread to any wood in the vicinity. Unseasoned timber is more liable to dry rot than seasoned. Thorough seasoning, ventilation and protection from dampness are the best precautions against dry rot. *Wet rot* is the decomposition of wood by chemical action through being kept in a wet state. It is not infectious excepting through actual contact. Thorough seasoning and preserving by painting, creosoting, etc., will prevent wet rot.

Destruction of Timber.—All marine timber structures are liable to be destroyed by sea-worms, chief amongst which is the Teredo. By employing greenheart or jarrah the action of the sea-worms is to some extent prevented. Other precautions adopted are, covering, after tarring, of all timber below high-water level, with sheet zinc or copper, or studding the whole surface with scupper nails. Where timber is liable to be attacked by white ants, teak, jarrah, or other ant-resisting timber should be selected for use.

Seasoning.—The object of seasoning is to expel any sap there may be remaining in the timber. *Natural* seasoning is performed by stacking balks in layers, under cover, and allowing a free circulation of air to pass around each balk. This process requires a considerable time, taking between three and twenty-six months, according to the description of the timber and the sizes under treatment. *Water* seasoning reduces the time occupied. By this method the timber is placed under water, preferably in a stream, for a fortnight, after which it is stacked under cover and allowed to thoroughly dry. Whilst under

water the sap is diluted and carried away, thus reducing the time required to season when stacked.

Dessicating is seasoning the timber by enclosing it in a chamber and circulating hot air around it. This method reduces the time required for seasoning, but care must be taken that the heat is not sufficient to cause the timber to split. This method of seasoning is unsuitable for large balks.

Seasoning reduces the weight of timber by 20 per cent. to 33 per cent.

Preservation of Timber.—The most effective means of preserving timber is, after thoroughly seasoning, to fill up the pores so as to exclude all moisture from penetrating below the surface. Many processes for accomplishing this have been adopted, but the most successful is that of creosoting.

Creosoting.—By this process the timber is thoroughly impregnated with dead oil of coal tar. Two methods of accomplishing this are in use. The older method consists of placing the timber, after seasoning, in an airtight cylinder, exhausting the air and then admitting the creosoting oil, at a temperature of 120° F. and 170 lbs. per square inch pressure. After maintaining the pressure for a short time, the oil is removed from the cylinder and the timber allowed to dry. By this treatment up to 20 lbs. of creosote per cubic foot of timber can be injected.

A second method, known as the Rueping process, has latterly been extensively adopted in America. After placing the seasoned timber in the cylinder, the air pressure is increased to 75 lbs. per square inch, and whilst at that pressure the cylinder is filled with creosote. The pressure is then gradually increased to 150 lbs. per square inch, and allowed to remain at such pressure for 15 minutes, after which the pressure is reduced to that of the atmosphere and the creosote drained from the cylinder. The air in the cells of the timber, having been compressed to 150 lbs. per square inch, expands and forces any surplus creosote from the cells. To remove any oil from around the outside of the pores a vacuum of 22 inches is created. The total operation requires about 4½ hours. By this treatment about 5 lbs. of creosote per cubic foot of timber is sufficient to preserve the timber against decay in ordinary situations.

The amount of creosote required per cubic foot of timber will vary according to the nature of the timber and the proportion of sapwood in the piece.

Boucherie's Process consists of forcing copper sulphate along the fibres until the timber is thoroughly impregnated, and is very successful in preventing dry rot; but if the timber be exposed to the action of water the salt is dissolved and carried away. It has not the power to repel the attacks of white ants or sea-worms.

Kyan's Process.—Bichloride of mercury is, by this process, injected into the pores of the timber. To some extent it prevents destruction by ants and sea-worms, and is effective against dry rot, but is now seldom employed.

Metals.—The metals of greatest use in construction are cast iron, wrought iron, and steel. All being derived from iron ore, iron forms

the chief constituent of each. The iron in the ore is in chemical combination with other materials, which are partially removed by *smelting*. The iron after smelting is known as *pig iron*. Cast iron is manufactured directly from suitable pig iron by remelting and running the molten metal into moulds. Wrought iron is made from pig iron by processes termed refining, puddling, and rolling. There are a number of methods of converting iron into steel, the chief being the Bessemer, Siemens, and Siemens-Martin processes. The description of the processes of manufacture cannot be entered into here, but may be found in works on metallurgy.

The chief difference in the chemical composition of the three metals is the proportion of carbon contained by each. The following table gives typical percentages of carbon and other impurities in the metals.

TABLE 8.—ANALYSES OF IRON AND STEEL.

	Cast iron.	Wrought iron	Steel.
	Per cent	Per cent.	Per cent
Carbon	2·0 to 6·0	0 to 0·25	0·15 to 1·8
Silicon	0·2 to 2·0	0·032	0 015
Phosphorus	0 038	0 004	0·041
Manganese	0·013	trace	0 683
Sulphur	0 014	0 114	0·035

The carbon in cast iron exists partly in the state of a mechanical mixture, when it is visible as black specks in the mass and gives the iron a dark grey colour, and partly as an element in the chemical compound. Free carbon makes the metal softer and more adaptable for casting. When chemically combined, carbon imparts strength and brittleness to the iron.

The chief differences of physical properties are shown in the following table.

TABLE 9.—PHYSICAL PROPERTIES OF IRON AND STEEL.

Cast iron	Wrought iron	Steel
Hard	Soft	Medium to hard.
Brittle	- -	—
Fusible.	- -	Fusible when containing a high percentage of carbon.
	Malleable	Malleable.
	Ductile.	Ductile.
	Forgeable	Highly elastic
	Tenacious.	Temperable.
	Weldable	Weldable.

Cast Iron.—The use of cast iron for constructional purposes has diminished to a very great extent in recent years owing to the improved

methods of manufacturing wrought iron and steel. Its use is now restricted to columns, bed-plates, cylinders, and similar members that are subject to purely compressive stress.

Ultimate strength of cast iron—

Tension	7 to 11	tons per square inch.	
Compression . . .	35 to 60	,,	,,
Shear	8 to 13	,,	,,

The transverse strength is often specified in preference to the compressive and tensional strengths. Test pieces, cast on the main casting and afterwards removed, are tested by supporting them as beams and applying central loads. A usual specification for such tests is as follows: Test pieces to be 2 inches deep, 1 inch wide and 3 feet 6 inches long, and placed on supports 3 feet apart. The central breaking load to be not less than 28 cwts., and the deflection to be at least $\frac{5}{16}$ inch.

The very marked difference between the strength in tension and compression makes it unsuitable for girders, and although formerly used for such, the practice is now discontinued. The cost of production and the high compressive resistance of cast iron render it an economical material for use in compression members subject to steady dead loads, but it is liable to fracture under sudden severe shocks. Grey iron, in which there is a large percentage of free carbon, should be used for casting. Columns should always be cast in a vertical position to ensure a uniform density of metal, and to allow air bubbles and scoriæ to rise to the head and be removed. In casting hollow columns the core is more easily adjusted and kept in position by the above method than if the column were cast in a horizontal position. Castings should be clean, sound, and free from cinders, air holes, and blisters. Wavy surfaces on castings indicate unequal shrinkage and want of uniformity in the texture of the iron. Filled-up flaws can be detected by tapping on the faces; a dull sound is given out by the filling.

The uncertainty of obtaining perfectly sound castings necessitates a high margin of safety being adopted.

Wrought Iron.—Wrought iron has a fibrous structure, and, compared with cast iron, is soft and ductile. Its quality depends, in a great measure, upon the rolling process it has undergone. The best quality is produced by repeating the process of piling, welding, and rolling some three or four times.

Strength.—To ascertain the strength of wrought iron, samples should be cut from each rolling and tested in tension, noting the ultimate stress per square inch, the elastic limit, the percentage elongation, and the percentage reduction of area at fracture. The usual requirements for strength are given in the following table:

TABLE 10.—TENSILE STRENGTH OF WROUGHT IRON.

	Ultimate tensile stress per square inch.	Elongation per cent.
	tons	per cent.
Round or square bars .	23 to 24	20 to 50
Flat bars	22 „ 23	25 „ 45
Angle iron	22 „ 23	25 „ 45
T or H iron	20 „ 22	10 „ 45
Plate { grain lengthways .	18 „ 22	10 „ 20
Plate { „ crossways .	17 „ 19	5 „ 12
Sheet { grain lengthways	20 „ 22	10 „ 20
Sheet { „ crossways .	18 „ 19	5 „ 12

The percentage elongation is a measure of the ductility of the material. It is usual to specify bending tests in addition to the tensile tests, and Table 11 shows the bending tests required by the Admiralty for two qualities of wrought-iron plates.

TABLE 11.—BENDING TESTS FOR WROUGHT IRON.

Plates	Angle through which plates must bend without cracking.				
	Hot	Cold			
	Thickness				
	1 inch thick and under	1 inch	¾ inch	½ inch.	¼ inch
	degrees	degrees	degrees	degrees	degrees
B.B. grain lengthways	125	15	25	35	70
B B grain crossways .	90	5	10	15	30
B grain lengthways	90	10	20	30	65
B. grain crossways .	60	—	5	10	20

Rivets should bend double when cold without showing any signs of fracture. Angles, tees, and channels should bend, whilst hot, to the following shapes without fracture, Fig. 6.

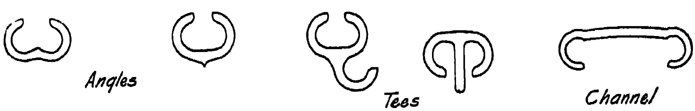


FIG. 6.

Market Forms.—*Sheet iron*, so called when the thickness does not

exceed No. 4, B.W.G. (0·239 inch), is little used in engineering construction.

Corrugated sheets are made by passing the sheets between grooved rollers, after which they are usually galvanized, *i.e.* coated with zinc, to preserve them from rusting. The widths of the corrugations or flutes are made 3, 4, or 5 inches. The width of sheet is specified in terms of the number and width of the corrugations; thus a 10/3 inch sheet would cover a net width of 2 feet 6 inches when laid, the actual width of the sheet being a little more to allow for side lap. The usual sizes employed range from 18 B.W.G. to 22 B.W.G. in thickness. The maximum ordinary length of sheet is 10 feet.

TABLE 12.—DIMENSIONS OF CORRUGATED SHEETS.

Thickness B.W.G.	Thickness in inches.	Widths of sheets
No. 16	0·065	5/5″, 6/5″
„ 17	0·058	5/5″, 6/5″
„ 18	0·049	5/5″, 6/5″, 6/4″, 7/4″,
„ 19	0·042	8/3″, 10/3″
„ 20	0·035	6/4″, 7/4″, 8/3″, 10/3″
„ 21	0·032	
„ 22	0·028	
„ 23	0·025	8/3″, 10/3″
„ 24	0·022	
„ 26	0·018	

Plates.—The ordinary sizes are as follows: Thickness, ¼ inch to 1 inch; width, 1 foot to 4 feet; length, up to 15 feet. The superficial area of a sheet must not exceed 30 square feet, nor the weight be more than 4 cwts. Larger sheets are obtainable, but are charged extra. Plates of less width than 12 inches are known as *flats*. Flats can be readily obtained up to 25 feet in length.

Round and square bars are rolled in sizes from 3 inches to 6 inches in diameter or side.

Wrought iron L, T, and H sections are obtainable, but are now very little used in engineering structures. The sizes are similar to those specified for mild steel.

Steel.—For all important structural works steel is the metal almost exclusively used. The two chief varieties for such work are cast and mild steel. The former is used for all important castings, and although possessing many of the characteristics of cast iron, it is much superior in strength, uniformity of texture, and as a material from which sound castings may be produced.

Mild Steel.—For structural purposes the sizes of rolled sections, strength and methods of testing have been standardized by the British Engineering Standards Committee and published in specification form, which is generally adopted for structural works.

Strength.—

TABLE 13.—TENSILE STRENGTH OF STEEL.

	Ultimate tensile strength per square inch.	Elastic limit.	Elongation.
	tons	tons.	
Mild steel . . .	28 to 32	17 to 18	20 per cent. in 8 in.
Cast steel (annealed)	30 to 40	15 to 17	10 to 20 per cent. in 10 in.
Rivet steel . . .	26 to 30	15 to 17	25 per cent.

Bending tests for mild steel. Test pieces, not less than $1\frac{1}{2}$ inches wide, should withstand without facture, being doubled over until the internal radius is not greater than $1\frac{1}{2}$ times the thickness of the test piece. Rivet shanks should withstand without fracture, being bent over, when cold, until the two parts of the shank touch.

Rolled Sections.—*Flats.*—Sections obtainable :—

Width, $\frac{3}{4}$ inch to 12 inches ; thickness, $\frac{1}{8}$ inch to 2 inches.

Maximum ordinary length, 40 feet.

Plates are rolled to maximum width, length, or area. The following table shows the maximum dimensions for a few thicknesses of plates.

Thickness	Length	Width	Area
inches	feet	inches.	sq ft.
$\frac{1}{8}$	14	48	48
$\frac{1}{4}$	26	72	100
$\frac{3}{8}$	36	81	200
$\frac{1}{2}$	44	81	200
$\frac{3}{4}$	44	81	220
1	44	81	175
$1\frac{1}{4}$	44	81	135
$1\frac{1}{2}$	44	81	115

As the width multiplied by the length must not exceed the maximum area, plates cannot possess both the maximum length and maximum width

Chequered Plates.—The maximum dimensions for chequered plates will be found in the following table.

Thickness on plain	Length	Width	Area
inches	feet	inches	sq ft
$\frac{3}{16}$	20	52	52
$\frac{1}{4}$	25	54	54
$\frac{3}{8}$	25	55	64
$\frac{1}{2}$	25	55	64
$\frac{5}{8}$	25	55	60
$\frac{3}{4}$	20	54	50
$\frac{7}{8}$	20	52	43
1	20	52	38

Buckled plates are made in sizes 3 feet to 5 feet square, the camber varying between 2 and 3 inches. The thicknesses are ¼ inch to ⅜ inch, rising by $\frac{1}{16}$ths.

Stamped Steel Troughing.—The sizes of troughing will be found in Table 15. The maximum dimensions of troughing stamped from single plates are ⅝ inch thick, 18 inches deep, and 36 feet long.

Round bars may be obtained up to 6 inches in diameter, the maximum length up to 1⅜ inches diameter being 60 feet. The maximum length rapidly decreases for larger sections, 15 feet being the maximum for 3 inches diameter and over. The sections of *angles, tees, joists, channels, zeds, and rails* have been standardized, and the full lists, published by the Engineering Standards Committee, should be consulted. The maximum ordinary length for all the above sections is 40 feet. Some of the standard sections are used to a greater extent than others, and consequently such sections are rolled more frequently, and are easier to obtain at short notice. Rolling lists specifying these sections are to be obtained from the makers. The weight per foot length cannot be exactly adhered to by the rollers, and they reserve the right to supply material within the limits of 2½ per cent. under or over the specified weights. Lengths beyond the ordinary maximum may be obtained on payment of special "extras." Extras are also charged for any workmanship, such as cutting to dead lengths, bevelling, etc.

The following references to the publications of the British Engineering Standards Association, 28, Victoria Street, London, S.W.1, will be found useful. The number is the official number of the specification or report, and the year is the date of the latest issue to 1928.

No. 1—1920. Lists of Rolled Sections for Structural Purposes.
„ 6—1921. Dimensions and Properties of Rolled Steel Sections for Structural Purposes.
„ 9—1922. Specification and Sections for Bull Head Railway Rails.
„ 12—1925. Specification for Portland Cement.
„ 15—1912. Specification for Structural Steel for Bridges and general Building Construction.
„ 63—1913. Specification for Sizes of Broken Stone and Chippings.
„ 76—1916. Tars, Pitches, Bitumens, and Asphalts.
„ 144—1921. Specification for Creosote for the Preservation of Timber.
„ 153. Specification for Girder Bridges.
Parts 1 and 2—1922. Materials and Workmanship.
„ 3, 4, and 5—1923. Loads and Stresses, Details of Construction and Erection.
Appendix I.—1925. Tables of Unit Loadings for Railway and Highway Bridges.

CHAPTER II.

LOADS AND WORKING STRESSES.

Dead and Live Load.—*General Considerations.*—The load on a structure may be divided broadly into two classes—the *dead load*, and what is usually termed *live load*. The dead load comprises the weight of the structure itself, which is constantly imposed. The live load would perhaps better be defined as incidental or intermittent load. It comprises all load which is applied to and removed from the structure at intervals. The live load may be of very varied character. In the case of bridges it consists of the weight of rolling traffic, such as trains or road vehicles, together with pedestrian traffic and wind pressure. On roofs the principal live load is that due to wind pressure; on crane girders, the weight of the traveller, together with the load lifted increased by accelerating force; on columns, wind pressure combined with intermittent loads in cases where the columns support girders subject to live loads. It will be noted that all structures in the open are subject to wind pressure. Whilst the character of the dead load is that of an unchangeable static load, that of the live load is very varied on different structures. Thus the live load imposed on a short bridge girder during the passage of a train at high speed differs greatly from the very gradually applied pressure as the tide rises against the gates of a dock entrance, or the still more slowly increasing pressure behind a dam as the reservoir gradually fills with water. Both these latter are, however, examples of live load, although their effect on the structure is much less intense than in the former case.

The dead load may be conveniently divided into two portions, one comprising the weight of the main girders or frames of the structure, and the other including the accessory parts of the structure necessary for giving it the desired utility. The weight of the main girders of a bridge, the principals of a roof, and the main columns and girders of a framed building, are examples of the first subdivision, whilst the weight of the flooring, permanent way or pavement, roof covering, etc., represents the second subdivision. In commencing to design any structure, it is necessary to make as careful an estimate as possible of the dead weight of the structure itself, and it is obvious the exact weight of a structure is indeterminable until the structure has been completely designed. The *nominal amount* of live load for which a structure is to be designed is generally fairly accurately known, although owing to the varied character of different live loads, their effect as regards the stress brought into action is often a debatable point. Of the dead load, the weight of the accessory parts of a structure is very closely estimable,

but the weight of such portions as main girders, roof principals, etc., may only be approximately estimated, and the closeness of agreement between such estimated weight and the final weight of the structure as designed, will determine whether the design is suitable or otherwise. Frequently a re-calculation of the structure becomes necessary, owing to the final weight considerably exceeding the estimated weight. This matter constitutes one of the principal difficulties in design, and the difficulty increases with the magnitude of structure and lack of precedent. In a small structure exposed to the action of a considerable live load, a very close estimate of the dead weight of the structure is of relatively little importance, since by far the greater proportion of the stress will be caused by the live load. In a very large structure, the bulk of the stress will be caused by the dead load, and it is important to estimate more accurately the probable weight of such a structure before commencing its design. Unfortunately, the probable weight of large structures is always more difficult to forecast, since fewer precedents exist for purposes of comparison, and much greater judgment and experience are essential, on account of the greater complexity of the structure.

Innumerable formulæ have been devised, which aim at giving the probable weight of main girders, roof principals, piers, etc., but such formulæ can only furnish a general guide to be supplemented by careful judgment, since the assumptions on which they are based are seldom realized in the particular structure under consideration. Many such formulæ are framed on records of the weight of existing structures similar to the type under consideration, and where ample precedent exists the formulæ will naturally be more reliable. Formulæ, however, are less reliable for the larger and more uncommon types of structures, and it will be realized from the above remarks that considerable experience and judgment are absolutely necessary to undertaking the design of large and important structures.

Dead Load.—Table 14 gives the weight of the various kinds of masonry and materials employed in structural work.

TABLE 14.—WEIGHT OF VARIOUS MATERIALS.

Material.	Weight in lbs. per cubic foot
Masonry.	
Granite ashlar masonry in cement mortar	165
Freestone ashlar " "	145
Limestone rubble " "	154
Freestone rubble " "	122 to 138
Blue brickwork " "	147
Red brickwork " "	122
Concrete.	
Rubble concrete in masonry dams	140 to 162
Coke breeze concrete (1 to 6)	95
Ballast concrete	140
Clinker concrete (1, 2, 4)	112
Cement concrete (1 to 5)	130
Granolithic concrete (1 to 2)	138
Reinforced concrete, including average reinforcement	150

TABLE 14.—WEIGHT OF VARIOUS MATERIALS—*continued.*

Materials.	Weight in lbs. per cubic foot.
Ballast (Dry).	
Limestone, 35 per cent. voids	110
" 40 " "	102
" 45 " "	93
Sandstone, 35 " "	94
" 40 " "	86
" 45 " "	79
Broken slag $2\frac{1}{2}$ inches, containing about 35 per cent. voids	95 to 100
Gravel	90
Miscellaneous.	
Slag, solid	150
Sand, damp	118
" dry	90
York paving flags	154
Granite paving (Penmaenmawr)	172
Asphalte	150
Timber.	
Elm	36
Red pine and spruce fir	30 to 44
American yellow pine	30
Larch	31 to 35
Oak (English)	48 to 60
" (American)	54
Teak	41 to 55
Greenheart	60 to 70
Pitch pine	42 to 48
Jarrah (wood pavement)	60 to 63
Iron and Steel.	
Cast iron	448
Wrought iron	480
Mild steel	490
Cast steel	492
Glass.	
Flint	187
Plate and sheet	169
Fresh water	62·5
Sea water	64·125

Dead Load on Bridges.—In estimating the dead load on railway bridges, the permanent way may be conveniently bulked as so much per foot run. The following detail weights may be taken as representative for the standard gauge of 4 feet $8\frac{1}{2}$ inches.

Sleepers, 9 ft. × 10 in. × 5 in. at 40 lbs. per cub. ft. = 125 lbs. each. Chairs, 52 lbs. Rails, 86 or 100 lbs. per yard. Fishplates, 32 lbs. per pair. Fishplate bolts, 5 lbs. per set of four. Spikes, $\frac{3}{4}$ lb. each.

Weight of 30 feet of single track—

	lbs.
20 yards of rail at 86 lbs.	1720
11 sleepers (pine) at 125 lbs.	1375
22 chairs at 52 lbs.	1144
22 keys at 50 lbs. per cub. ft.	68
2 pairs fishplates at 32 lbs.	64
2 sets fishplate bolts at 5 lbs.	10
22 sets spikes and trenails at 4 lbs.	88
	4469

Say 2 tons per 30 feet of single track, or $\frac{1}{15}$ ton per foot run, say 150 lbs. per foot run. With 100-lb. rails and suitably heavier chairs and fastenings = 166 lbs. per ft. run.

Rails.—*Railway rails* weighing 85, 86, and 100 lbs. per yard are in general use for main-line traffic, usually in 30 or 36 feet lengths.

Tramway Rails.—The following are in use. South London, 102 lbs. per yard. Newcastle, 101. Leeds, 100. Birkenhead, 100. For purposes of preliminary estimate, 105 lbs. per yard may be taken.

Ballast and Pavement.—These weights are stated in Table 14. The mean width of ballast may be taken as 12 feet for a single track, 23 feet for double track, and 45 feet for quadruple track, for standard gauge. The average depth of ballast used is 18 to 19 inches, the lower 9 inches, called pavement, consisting of larger stones roughly hand packed, leaving 9 to 10 inches of gravel or broken stone ballast above to the upper level of sleepers. On the Great Central Co.'s main line, the quantity laid per mile of double track is 7500 cubic yards. The lower layer of pavement is not laid on bridges.

Flooring.—The flooring of railway bridges consists of flat or buckled plates on cross-girders and rail-bearers, or one of the many types of troughing. For highway bridges, troughing levelled up with concrete, or jack arches turned between cross girders, are most suitable. Table 15 gives the weight of various floor details.

TABLE 15.—WEIGHT OF BRIDGE FLOOR DETAILS.

Detail.	Weight lbs. per sq. ft.
Flat Steel Plates.	
Per thickness of $\frac{1}{8}$ inch	5.1
Buckled Plates. Rise 2 to 3 inches.	
$\frac{1}{4}$ inch thick	10.3
$\frac{3}{8}$ " "	15.4
$\frac{1}{2}$ " "	20.5
Troughing. Lap-jointed as Fig. 7, including rivets.	

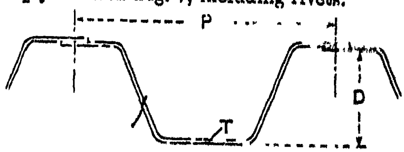


FIG. 7.

TABLE 15.—WEIGHT OF BRIDGE FLOOR DETAILS

Detail.	Weight. Lbs. per sq. ft
1 foot 8 inch pitch, 6 inches deep, 5/16 inch thick	19·71
1 ,, 8 ,, ,, 6 ,, ,, 3/8 ,, ,,	23·45
2 ,, 0 ,, ,, 7½ ,, ,, 3/8 ,, ,,	23·56
2 ,, 0 ,, ,, 7½ ,, ,, 1/2 ,, ,,	31·15
2 ,, 6 ,, ,, 10 ,, ,, 3/8 ,, ,,	23·69
2 ,, 6 ,, ,, 10 ,, ,, 1/2 ,, ,,	31·39
2 ,, 8 ,, ,, 12 ,, ,, 3/8 ,, ,,	24·61
2 ,, 8 ,, ,, 12 ,, ,, 1/2 ,, ,,	32·68
2 ,, 10 ,, ,, 14 ,, ,, 3/8 ,, ,,	25·30
2 ,, 10 ,, ,, 14 ,, ,, 1/2 ,, ,,	33·61

Butt-jointed as Fig. 8

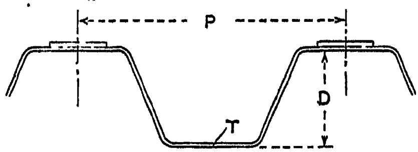


FIG. 8.

Detail.	Weight. Lbs. per sq. ft
2 feet 0 inch pitch, 7½ inches deep, 3/8 inch thick	26·08
2 ,, 0 ,, ,, 7½ ,, ,, 1/2 ,, ,,	34·27
2 ,, 6 ,, ,, 10 ,, ,, 3/8 ,, ,,	25·58
2 ,, 6 ,, ,, 10 ,, ,, 1/2 ,, ,,	33 70
2 ,, 8 ,, ,, 12 ,, ,, 3/8 ,, ,,	26·49
2 ,, 8 ,, ,, 12 ,, ,, 1/2 ,, ,,	34 94
2 ,, 10 ,, ,, 14 ,, ,, 3/8 ,, ,,	27 07
2 ,, 10 ,, ,, 14 ,, ,, 1/2 ,, ,,	35 73
2 ,, 6 ,, ,, 14 ,, ,, 3/8 ,, ,,	29·03
2 ,, 6 ,, ,, 14 ,, ,, 1/2 ,, ,,	38 29
2 ,, 8 ,, ,, 15 ,, ,, 1/2 ,, ,,	37 97
2 ,, 8 ,, ,, 15 ,, ,, 5/8 ,, ,,	47 18

Butt-jointed with reinforced bottom flange as Fig. 9.

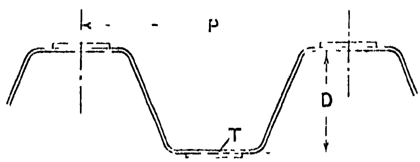
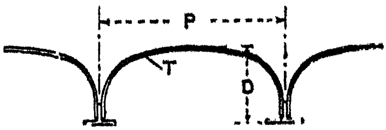


FIG. 9.

Detail.	Weight. Lbs. per sq. ft
2 feet 0 inch pitch, 7½ inches deep, 3/8 inch plate, 1/2 inch straps	34 00
2 ,, 6 ,, ,, 10 ,, ,, ,, ,,	34 63
2 ,, 8 ,, ,, 12 ,, ,, ,, ,,	34 98
2 ,, 10 ,, ,, 14 ,, ,, ,, ,,	35·06
3 ,, 0 ,, ,, 16 ,, ,, ,, ,,	35·18
3 ,, 0 ,, ,, 16 ,, ,, 1/2 inch plate, 5/8 inch straps	45·47
4 ,, 6 ,, ,, 18 ,, ,, with double straps top and bottom, 1/2 inch thick	55·48
,, ,, ,, ,, with double straps top and bottom, 5/8 inch thick	68·61

TABLE 15.—WEIGHT OF BRIDGE FLOOR DETAILS—*continued.*

Detail.	Weight Lbs. per sq. ft.
Arched troughing as Fig. 10.	
 FIG. 10.	
1 foot 9 inch pitch, 9 inches deep, ¼ inch thick	20·31
2 „ 0 „ „ 10 „ „ ¼ „ „	20·00
2 „ 8 „ „ 12 „ „ $\frac{5}{16}$ „ „	25·67
2 „ 6 „ „ 12 „ „ ⅜ „ „	29·53
2 „ 6 „ „ 15 „ „ $\frac{5}{16}$ „ „	24·48
2 „ 6 „ „ 15 „ „ ⅜ „ „	33·00

Weight of Cross-girders.—In railway bridges with framed floors, consisting of cross-girders, rail-bearers, and plate flooring, the most economical spacing of cross-girders is from 7 ft. to 8 ft. The growing practice of building locomotives with small wheels close together, tends to a closer spacing in the near future. With these spacings, cross-girders for double-track railway bridges are usually 26 ft. to 27 ft. long, and 2 ft. 3 in. to 2 ft. 6 in. deep. Adopting these proportions, the weight of one cross-girder may be taken as 2·25 tons. For single track bridges with cross-girders 14 ft. to 15 ft. long, and 15 in. to 18 in. deep, the weight of one cross-girder may be taken as 0·5 ton. Cross-girders between main braced girders with wide bays will be much heavier, and an independent estimate is necessary. Cross girders for highway bridges will be appreciably lighter for the same spans, but owing to the variable character of the traffic and width of highway bridges, a preliminary estimate is necessary in each case.

Rail-bearers.—Rail-bearers 7 ft. to 8 ft. long may be estimated at 700 lbs. to 900 lbs. each.

Jack arching is usually 9 inches to 13½ inches thick of brickwork, and its weight is readily estimated.

Weight of Main Girders.—Plate Girders. The probable weight of main plate girders, provided the depth is about one-tenth the span, may be fairly closely obtained from the following formulæ. Whenever reliable information may be obtained as to the weight of well-designed existing girders of the same span, similarly loaded, such records are preferable to results calculated from formulæ.

W = Total distributed or equivalent distributed load carried by one girder, exclusive of its own weight. L = Span in feet.

Probable weight of plate girders. Depth about $\frac{1}{10}$ span.

$$\text{From 20 ft. to 40 ft. span} = \frac{W \times L}{580} \text{ tons.}$$

$$\text{,, 40 ft. ,, 60 ft. ,,} = \frac{W \times L}{560} \text{ ,,}$$

$$\text{,, 60 ft. ,, 100 ft. ,,} = \frac{W \times L}{530} \text{ ,,}$$

Plate girders are seldom employed beyond 100 ft. span.
Probable weight of lattice girders. Depth about $\frac{1}{8}$ span.

$$\text{From 20 ft. to 50 ft. span} = \frac{W \times L}{600} \text{ tons.}$$

$$\text{,, 50 ft. ,, 90 ft. ,,} = \frac{W \times L}{590} \text{ ,,}$$

$$\text{,, 90 ft. ,, 120 ft. ,,} = \frac{W \times L}{530} \text{ ,,}$$

The weights of lattice girders of larger span should be carefully estimated in detail, having regard to the probable stresses coming upon the various members. The weight of main lattice girders is more influenced by variations in arrangement of detail than is that of plate girders.

Weight of Rolled Steel Section Bars.—Table 16 gives the weight per foot run of the most commonly used rolled sections in mild steel. *Full* lists of these are published in most section books.

TABLE 16.—WEIGHT OF ROLLED STEEL SECTION BARS.

Equal Angles

Size (in × in × in)	Weight lbs per foot
$6 \times 6 \times \frac{3}{4}$	28 70
$5 \times 5 \times \frac{5}{8}$	19·92
$4\frac{1}{2} \times 4\frac{1}{2} \times \frac{1}{2}$	14·46
$4 \times 4 \times \frac{1}{2}$	12 75
$3\frac{1}{2} \times 3\frac{1}{2} \times \frac{1}{2}$	11 05
$3 \times 3 \times \frac{3}{8}$	7 18
$2\frac{1}{2} \times 2\frac{1}{2} \times \frac{3}{8}$	5 89
$2\frac{1}{4} \times 2\frac{1}{4} \times \frac{5}{16}$	4·45

Unequal Angles

Size (in × in × in)	Weight lbs per foot
$6 \times 4 \times \frac{5}{8}$	19 92
$6 \times 4 \times \frac{1}{2}$	16·15
$6 \times 3\frac{1}{2} \times \frac{1}{2}$	15 31
$5 \times 3 \times \frac{1}{2}$	12 75
$4 \times 3 \times \frac{1}{2}$	11 05

TEES.

Size (in × in × in)	Weight lbs per foot
$6 \times 4 \times \frac{1}{2}$	16·22
$6 \times 3 \times \frac{1}{2}$	14·53
$5 \times 4 \times \frac{1}{2}$	14·51
$5 \times 3 \times \frac{1}{2}$	12·79

CHANNELS

Size (in × in)	Weight lbs per foot
17×4	44·34
15×4	36 37
12×4	31 33
$12 \times 3\frac{1}{2}$	25·25
$10 \times 3\frac{1}{2}$	24·46
$9 \times 3\frac{1}{2}$	22·27
$8 \times 3\frac{1}{2}$	20·21
$7 \times 3\frac{1}{2}$	18 28
7×3	14·22
$6 \times 3\frac{1}{2}$	16·48
6×3	12·41

The weights of several sizes of rolled joists are given on pages 143–146, together with other properties of those sections.

TABLE 17.—WEIGHT OF BOLTS, NUTS, AND RIVET-HEADS.

Diameter.	Weight in lbs. of			
	1 inch length of bolt.	One hexagonal nut and bolt-head.	One square nut and bolt head.	100 rivet-heads.
in.				
$\frac{3}{8}$	0·0313	0·057	0·071	
$\frac{1}{2}$	0·0556	0·135	0·169	4·17
$\frac{5}{8}$	0·0869	0·261	0·330	8·15
$\frac{11}{16}$	—	—	—	10·85
$\frac{3}{4}$	0·1252	0·450	0·570	14·08
$\frac{13}{16}$	—	—		17·30
$\frac{7}{8}$	0·1703	0·720	0·90	22·30
$\frac{15}{16}$	—			27·50
1	0·2225	1·07	1·35	33·38
$1\frac{1}{8}$	—		—	47·53
$1\frac{1}{4}$	0·3476	2·09	2·63	65·19
$1\frac{1}{2}$	0·5060	3·61	4·55	
$1\frac{3}{4}$	0·6815	5·70	7·20	
2	0·8901	8·56	10·80	

In estimating the weight of riveted work, an allowance of from 2 per cent. to 5 per cent. of the weight of the structure is usually made, depending on the class of work, to cover the weight of rivet-heads. Such allowance is usually excessive, and is really intended to cover the wastage of drilling and punching the holes.

Dead Load on Roofs.—*Principals.* The probable weight of roof principals of ordinary V-types may be fairly estimated from the following formulæ :—

W = Weight of one principal in lbs. L = Span in feet. D = Distance apart of principals in feet.

For roofs with heavy covering, $W = \frac{3}{4} DL \left(1 + \frac{L}{10}\right)$
„ „ medium „ $W = \frac{1}{2} DL \left(1 + \frac{L}{12}\right)$
„ „ light „ $W = \frac{11}{16} DL \left(1 + \frac{L}{12}\right)$

Purlins and Common Rafters consist of angle, zed, or joist section if of steel, or rectangular sections if timber. The weights of the former are given above. The weight of timber members may be estimated on a basis of 35 lbs. to 40 lbs. per cubic foot.

Roof Coverings.—*Boarding* 1 inch thick, 3 lbs. to $3\frac{1}{2}$ lbs. per square foot.

Slating.—The usual sizes and weights of slates used for roofing are given in Table 18, for slates laid with 3-inch lap and nailed near the centre. A *square* of slating is 100 square feet, and a nominal 1000 of slates contains 1200.

TABLE 18.—SIZES AND WEIGHT OF SLATES.

	Size.	Area covered by 1200 slates.	Weight per sq. foot of roof surface.
	in. in.	squares.	lbs.
Doubles	13 × 6	2·5	8·25
Ladies	16 × 8	4·75	8·0
Countesses	20 × 10	7·0	8·0
Duchesses	24 × 12	10·0	8·5

Glazing.—The weight of glass such as used for roof covering is 14 ozs. per square foot for each $\frac{1}{16}$ inch in thickness. The sheets are usually 2 feet wide and $\frac{1}{4}$ inch thick, and weigh $3\frac{1}{2}$ lbs. per square foot.

Glazing Bars.—A large number of patent bars are in use. The following table of the weights of Messrs. Mellowes and Co.'s "Eclipse" glazing bars will afford a guide in estimating the weight of this detail.

TABLE 19.—WEIGHT OF GLAZING BARS.

"Eclipse" bar	Bearing, centres of purlins.	Weight in lbs. per foot run
	ft in	
No 7	6 0	2 796
,, 8	7 6	3·202
,, 9	8 6	3 578
,, 9A	9 3	3·906
,, 10	10 0	4 906

The standard spacing of these bars is $24\frac{1}{2}$ inches.

Corrugated Sheeting.—These sheets are rolled of such sizes that the side lap is about 1 inch, whilst the end lap is generally made 6 inches. Allowing for these laps, Table 20 gives the weight per square foot for various gauges.

TABLE 20.—WEIGHT OF CORRUGATED SHEETING

Thickness B W G	18	20	22	24	26
Weight in lbs. per square foot, including laps . .	2 78	2·20	1 80	1 49	1 12

Galvanized Fittings for Corrugated Sheets.—The weight of fittings per 100 square feet of roof surface may be taken as 12·6 lbs. if the sheets are attached to steel purlins by hook-bolts, and 7·6 lbs. if screwed to timber purlins.

Galvanized Ridging and Louvre Blades.—Ridging weighs from 1 lb. to 8 lbs. per foot run for girths of from 10 inches to 36 inches, and thicknesses of from No. 26 to No. 16 B.W.G.; Louvre Blades, from 2·5 to 5 lbs. per foot run for each blade of 11 inches girth, and thicknesses of from $\frac{1}{16}$ inch to $\frac{1}{8}$ inch.

Stamped Steel Gutters usually vary from $\frac{3}{16}$ inch to $\frac{1}{4}$ inch thick, and weigh from 9 lbs. to 25 lbs. per foot run according to section.

Lead for flashing and covering flats usually weighs from 5 to 7 lbs. per square foot, and allowing for laps, rolls, and nails, may be taken at 5·8 to 8·5 lbs. per square foot.

Zinc.—The thickness of zinc sheeting follows the zinc gauge, ranging from Nos. 9 to 16, and corresponding closely with Nos. 27, 25, etc., to 19 of the B.W.G. respectively. Nos. 15 and 16 are generally used for roofing. The sheets are 7 feet to 8 feet long by 2 feet 8 inches to 3 feet wide. Allowing for laps, the weight is $1\frac{1}{2}$ to $1\frac{3}{4}$ lbs. per square foot.

Asphalte.—The thickness and weight of asphalte laid on floors and roof flats is given in Table 21.

TABLE 21.—WEIGHT OF ASPHALTE ON FLOORS AND ROOFS.

	Thickness.	Wt. lbs. per sq. ft.
	Inch.	
Roof flats and bridge floors	$\frac{3}{4}$	$9\frac{3}{8}$
Goods warehouse floors	$1\frac{1}{4}$	$15\frac{5}{8}$
Railway platforms	1	$12\frac{1}{2}$
Waterproofing backs of arches	$\frac{1}{2}$	$6\frac{1}{4}$

In laying asphalte and waterproof coating, care should be taken to give the finished surface a good fall to ensure proper drainage.

Snow.—The snow load accumulates slowly, and may be treated as dead load. 5 lbs. per square foot of covered area is usually assumed in England.

Dead Load on Floors.—Floors being of very varied construction, no general weight per square foot may be cited. The type of floor being decided, its weight per square foot may be readily estimated in detail by reference to the preceding tables.

Live Load.—*Live Load on Floors.* The live load on floors of different classes may be assumed as equivalent to the following dead loads :—

For dwelling-houses, hotels, and hospitals	60 to 70 lbs. per square foot.
For schools, assembly halls, offices, and retail shops	110 to 130 lbs. „ „
For warehouse buildings	200 to 250 lbs. „ „

Machine shop floors are examples where especially heavy loading may occur. Usually the lighter classes of machine tools only are placed on upper floors, heavier machinery requiring special foundations being necessarily placed in the basement. The type of machines to be employed will determine the weight for which the floor is to be designed. The load on such floors is less uniformly distributed than on ordinary floors, and solid and rigid types should be adopted to ensure the best distribution of the locally concentrated loads to the girders beneath.

Live Load on Bridges.—The intensity of the live load on railway bridges is governed by the type of locomotives in use. These actually impose heavy concentrated rolling loads upon the various members, such loads depending on the weight of engine and tender and spacing

of axles. Two methods of treating the rolling load are in general use. 1. The *concentrated load method*, in which the bending moments and shearing forces due to the actual concentrated loads at known spacings are considered. 2. The *equivalent distributed load method*, in which the system of actual concentrated loads is replaced by a distributed load of uniform intensity which will create the same maximum bending moment at any point on the span, as that caused by any position of the concentrated axle loads. The former method is generally adopted in American, and the latter in English practice.

Equivalent Distributed Load on Main Girders.—Fig. 11 shows the loads and axle spacings of two of the heaviest locomotives at present in

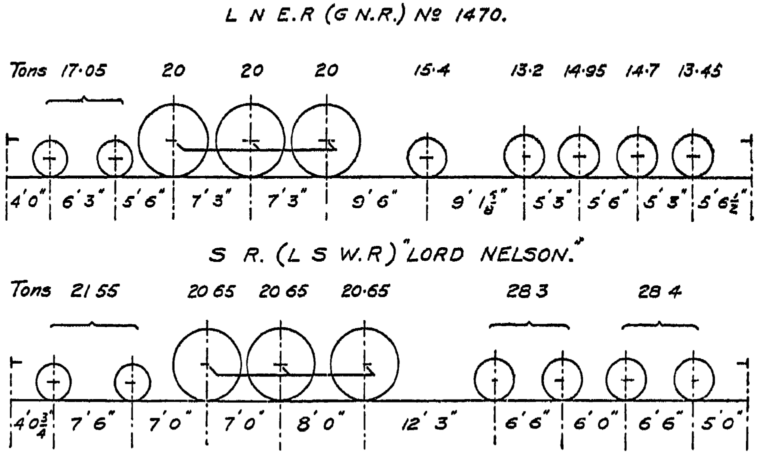


FIG 11.

use, in this country. In Table 22 are given equivalent uniformly distributed loads for present-day British locomotives (1928), for spans ranging from 10 to 200 feet.

TABLE 22.—EQUIVALENT DISTRIBUTED LOADS ON RAILWAY BRIDGES.

Equivalent distributed load in tons per foot run for single line of way

Span in feet	For bending	For shear.	Span in feet	For bending	For shear
10	4·40	5·80	90	2 11	2·38
15	3·60	5 06	100	2 08	2 33
20	3·25	4·50	110	2 06	2·29
25	3·04	4·08	120	2 04	2 27
30	2·90	3 73	130	2 03	2·24
35	2·75	3·46	140	2 02	2 21
40	2·62	3·25	150	2 01	2 19
45	2·54	3 09	160	2 00	2·16
50	2 46	2·94	170	1·99	2·14
60	2 30	2·70	180	1·97	2·12
70	2·20	2 54	190	1·96	2·10
80	2·14	2·45	200	1·95	2·09

In Table 22, the loads in the columns headed "For bending" are to be used for determining the sectional areas of flanges, and those in the columns headed "For shear" are to be used for determining sectional areas of webs of plate girders or web members of braced girders.

For larger spans, the live load is generally taken as that due to the heaviest type of mineral or freight train, preceded by two of the heaviest type of locomotives. The weight of a train made up of high capacity wagons, or wagons loaded with boilers, girders, machinery, etc., falls not far short of the weight of a train of engines. An equivalent load of 1·5 tons per foot run for a single line of way probably covers the effect of such a train for spans exceeding 200 feet. Spans up to about 210 feet would be practically covered by three large engines and tenders.

Live Load on Cross-Girders.—Cross-girders, however closely spaced, have each in turn to carry the heaviest axle load, which is usually that on the driving axle, or 19 tons per axle in present English practice. As the distance between the two most heavily loaded axles is usually from 5 to 8 feet, any closer spacing of the cross-girders is wasteful. Cross-girders between lattice main girders are usually at wider spacing than 8 feet, and in such cases the maximum loads will be equal to the maximum reactions due to the rail-bearers or stringers when these are most heavily loaded.

Live Load on Troughing.—With continuous trough flooring, it is impossible for a single trough to carry the full load of the heaviest axle, since the continuity of the floor causes the load to be distributed over several trough sections to right and left of the load. The extent of this distribution depends on the relative rigidity of rails and troughing, but the maximum load coming upon any one trough will not exceed one-half the actual axle load and may be as low as one third

Live Load on Rail-Bearers.—On short rail-bearers from 5 to 8 feet span, the maximum live load will be the load on the heaviest axle when at the centre of span. Longer rail-bearers will accommodate two, three, or more axles simultaneously, and the worst position of such axle loads for creating bending moment will determine the maximum loading.

Live Load on Highway Bridges.—This, whilst not so intense as on railway bridges, is very variable. Table 23 gives particulars of special loads on highway bridges. For ordinary wheeled traffic over bridges exempt from the special loads in Table 23, an equivalent distributed load of 130 lbs. per square foot of roadway should cover the requirements. For pedestrian traffic on footpaths, 120 lbs. per square foot is a sufficient allowance.[1]

Wind Load.—The question as to the magnitude of wind pressure and its mode of action on structures is one on which great uncertainty still exists. The following facts, however, appear to be satisfactorily established.

1. That exceptionally high intensities of pressure only prevail over local and relatively small areas, and that the greater the area exposed to the wind, the less will be the average pressure on such area. From

[1] For further information on rolling loads, see *Mins. Proceedings Inst. C.E.*, vol. clviii p. 328; vol. cxli. p. 2; vol. clviii. p. 380; vol. clxi. p. 261; vol. ccii. p. 368.

TABLE 23.—TRAFFIC LOADS ON HIGHWAY BRIDGES.

Vehicles.	Length over all	Width over all	Length of wheel base	Weight on rectangle enclosing vehicle.	Number of axles.	Maximum weight per wheel	Total weight loaded.
	ft in	ft in	ft in.	lbs per sq ft		tons	tons.
Tramway car, double deck, single truck	27 5	7 0	6 0	137	2	3	$11\frac{3}{4}$
,, ,, bogie trucks	33 6	7 1	13 6	139	4	$2\frac{3}{4}$	$14\frac{3}{4}$
,, ,, ,,	35 0	7 0	16 0	159	4	$3\frac{1}{4}$	$17\frac{1}{2}$
,, single deck ,,	35 0	7 0	17 0	138	4	3	15
,, (vestibule combination car with single truck)	30 10	6 9	6 0	103	2	$2\frac{1}{2}$	$9\frac{1}{2}$
20 ton Road roller	20 3	8 3	11 $7\frac{1}{2}$	263	2	$6\frac{3}{4}$	20
15 ,, ,,	19 7	7 3	11 6	235	2	5	15
Traction engines I.	—	7 2	10 2	—	2	7	19
,, II	—	9 7	15 0	—	2	8	22
,, III	18 4	7 10	10 7	176	2	—	11
Boiler wagon I	variable	—	13 0	—	2	12	48
,, II	,,	—	22 0	—	2	$9\frac{1}{2}$	38
Stone wagon	—	6 8	9 0	—	2	4	16
Steam lorry	—	—	11 6	—	2	$4\frac{1}{4}$	$10\frac{1}{2}$
11 ton Petrol commercial lorry	24 0	7 9	13 7	133	2	4	11
12 ton Foden steam lorry	26 0	7 6	15 3	138	2	4	12
14 ton Steam tractor.	18 2	6 10	10 0	254	2	5	14
16 ton ,,	18 0	7 0	10 0	265	2	$5\frac{1}{2}$	16
13 ton Petrol tractor	21 8	8 9	12 0	154	2	5	13
11 ton Caterpillar, with treads 6′ 6″ × 2′ 0″. $4\frac{1}{2}$ tons on each caterpillar tread	23 6	8 9	15 $8\frac{1}{2}$	120	3	—	11
14 ton Caterpillar, with treads 6′ 6″ × 2′ 0″. $4\frac{1}{2}$ tons on each caterpillar tread	21 0	8 9	16 $8\frac{1}{2}$	171	3	—	14

extended records of wind pressure obtained at the site of the Forth Bridge, the average pressure upon a board 20 feet by 15 feet was generally only *two-thirds* of the pressure upon a small wind-gauge of $1\frac{1}{2}$ square feet area, both in moderate and high winds.

2. The total or effective pressure against thin flat plates is considerably greater than the pressure calculated by multiplying the exposed windward area by the wind pressure per square foot acting on the windward face. This is due to the creation of a partial vacuum or negative pressure behind the plate, caused by the suction effect of the eddies set up by the resistance of the sharp edges to the free passage of the wind current. Thus, taking the effective wind pressure on a thin square plate as 100, the total pressure upon a cube presenting the same area to the wind is represented by 80, and on a prism having a length double that of the cube, by 72. The suction effect, therefore, rapidly diminishes as the length of the obstacle in the direction of the wind current increases. Roughly speaking, from $\frac{1}{3}$ to $\frac{1}{4}$ of the total effective pressure on an exposed area of thin plating is due to the negative pressure on the leeward side, and $\frac{2}{3}$ to $\frac{3}{4}$ to the positive pressure on the windward side. The exact ratio in any particular case varies, however, with the shape of the exposed surface.[1] This has a direct bearing on the effective pressure on the windward side of a V roof. The sharper the angle at the ridge, the greater will be the negative pressure on the leeward side, with consequent greater *effective* pressure than on roofs of flatter slope.

3. The extent to which one plate surface will shelter another similar one placed parallel with, and to leeward of the first, is actually very small, unless the two surfaces are relatively close together. Thus the windward girder of a bridge or the windward face of a lattice pier offers little protection to the leeward girder or face. In the case of a bridge with main *plate* girders and a continuous floor, the pressure against the leeward girder will no doubt be less than on the windward girder, but such reduction is less than is frequently assumed. Experiments on square and rectangular plates show that the maximum shielding effect occurs when the plates are separated by a distance of about $1\frac{1}{2}$ times their least cross dimension, and that the shielding effect of long rectangular plates is considerably less than that of circular plates.

4. Results deduced from the overturning of railway rolling stock show the *mean* side pressure to have varied from 26 to 31 lbs. per square foot, and, excepting in the case of structures in abnormally exposed situations, it is unlikely that *any considerable area* is ever exposed to a much higher *positive* mean pressure than 35 lbs. per square foot. Much higher pressures have been recorded by wind-gauges in which the pressure is registered by the deflection of a spring, but such pressures may be, and probably are, due to the dynamic effect of a sudden gust of wind blowing for a very short period. At the same time it should be remembered that a wind pressure of 30 lbs. per square foot applied suddenly will deflect the spring of a wind-gauge to the same extent as a steadily applied pressure of 60 lbs. per square foot, provided the sudden gust last long enough to produce a complete vibration of the spring. In a similar manner, a bridge or tall building will

[1] *Mins. Proceedings Inst. C.E.*, vol. clvi. p. 78.

possess a time period of vibration depending upon its mass, which will, however, be much longer than that of a light spring. If, then, a sudden gust of wind prevail long enough to deflect a bridge or building laterally through the full extent of its vibration, the *stress* produced will be that due to the higher pressure as recorded by the spring-loaded indicator of the wind-gauge.

5. The pressure of wind increases with the height above the ground, so that the upper portions of lofty buildings, chimneys, piers, and bridge girders are exposed to higher wind pressures than the lower portions, which are further often sheltered by neighbouring buildings.

6. The pressure on convex surfaces is much less, and on concave surfaces greater than on flat surfaces presenting the same projected area to the direction of the wind current. Thus, on the area of a circular chimney as seen in elevation, the *effective* wind pressure is only about half the *actual* pressure which would act on a thin flat surface of similar outline and area.

Intensity of Wind-Pressure on Structures.—In the case of bridges, it is generally assumed that no train will be traversing a bridge when the wind pressure exceeds 30 to 35 lbs. per square foot, whilst a maximum pressure of 45 to 50 lbs. may act on the structure when unloaded. The area of the bridge exposed to the higher pressure will be from once to about three times the area as seen in elevation, depending on the type of construction. Thus a tubular girder with continuous roof and floor will expose the elevational area only, a plate girder bridge from once to twice that area, the degree of shelter afforded by the leeward girder depending largely on the width of the bridge. A large bridge with double lattice girders may expose an effective area equal to three times the elevational area, or even more if the main girders be very broad. The lower pressure of 30 lbs. per square foot will act on the effective area of the windward girder, and on the train considered as a continuous screen, the upper edge being 12 feet 6 inches and the lower edge 2 feet 6 inches above the rails. In the latter case the train will largely shelter the leeward girder, and the wind pressure will only act to any considerable extent on that portion of it (if any) projecting above the train. The wind pressure on a moving train is, of course, to be considered as a rolling load, since it travels with the train.

The suction effect previously referred to is most active in the case of lattice-work structures which present a series of thin plate surfaces to the wind. On tall buildings which have considerable depth in the direction of the wind, the maximum *effective* pressure will be from $\frac{2}{3}$ to $\frac{3}{4}$ of that assumed for open-work structures, or about 35 to 38 lbs. per square foot, whilst the larger area makes it still less probable that the *mean* pressure will actually reach this figure The usual allowance for tall buildings and roof structures will be stated subsequently.

Further, it is obvious that judgment must be exercised in adopting a suitable maximum wind pressure for a given structure. The lower portions of many buildings, low roofs, etc., are often almost completely sheltered by neighbouring and higher buildings, whilst for others in lofty and more exposed situations the effect of the maximum wind pressure must necessarily be considered.

WORKING STRESSES AND SECTIONAL AREA OF MEMBERS.

General Considerations.—After making a careful estimate of the load to be borne by a structure, the stresses in the various members due to such load are next calculated. The methods of deducing the stresses in the various members of structures due to specified loads are considered in subsequent chapters. The sectional area of each member then depends primarily on the magnitude of the stress occurring in it, having due regard to a suitable disposition of the material in the section according as the member is subject to tension, compression, bending, etc., or to more than one of these actions combined. In arranging the disposition of material in the cross-section, further regard must be paid to convenience of making joints and connections with neighbouring members and deduction of area occupied by rivet-holes in the case of tension members. These features are fully considered later, and at present attention is directed to the determination of the actual sectional area.

The *working stress* in any member is understood to be the *actual* maximum stress in tons per square inch created in the member under the most unfavourable conditions of loading. The principal difficulty lies in determining at all accurately the *actual maximum stress* which takes place in any member. In cases where the stress is entirely due to a perfectly dead load, no doubt exists as to the value of the maximum stress, and a relatively high working stress may safely be employed in proportioning the sectional area. Examples of purely dead load stresses in actual practice are, however, rare, and the maximum stresses in the members of practical structures are generally due to the combined effect of both dead and live loading. That portion of the stress due to the dead load may be very closely estimated, whilst that caused by the live load is in many cases a very indefinite quantity, depending on the variable character and mode of application of the live load, examples of which have already been mentioned. It is a well-established fact that the actual momentary stress due to a suddenly applied live load may be double that due to the same amount of static or dead load. But between a suddenly applied and a very gradually applied live load, there exists a series of infinite gradations, each of which must be considered on its own particular merits. The additional stress caused in such cases, over and above that due to the *nominal* amount of the live load, is commonly referred to as *dynamic stress* or stress due to *impact*, and there is little doubt that the time period of application of the live load is the principal factor in determining the magnitude of the dynamic stress created.

Further, it appears from the results of numerous experiments that a great number of repetitions of fluctuating stress in a member exercises a more deteriorating effect on the material than a static stress of equal maximum intensity. These facts obviously have a specially important bearing on the design of members of bridges which are most subject to frequent and rapidly applied fluctuations of load. It is evident, therefore, in fixing the working stress, that regard should be paid to the *range of stress*, or difference between maximum and

minimum stress in a member as well as the time period of application of the live load. The character of live loads and degree of suddenness of their application are so varied that a proper estimate of the probable dynamic effect, and therefore the actual maximum stress caused, becomes a more or less complex matter. Many rules have been proposed for making allowance for the above-mentioned influences in designing the sectional area of members, and some of the principal of these will now be stated. It may be mentioned, however, that the present state of exact knowledge on this question is far from complete, whilst useful investigation is much retarded by the difficulty of making reliable experiments.

Factor of Safety.—Formerly the working stress was deduced by dividing the ultimate or breaking strength of the material by 4, 5, 6, etc., according to the reduction considered suitable for the character of the load, and in cases where a fairly reliable estimate of the dynamic effect of the live load is possible, this method used by persons possessing sound judgment and experience may produce satisfactory results. Its abuse lies in its indiscriminate application to all or any cases of live loading. It is necessary that the working stress be kept safely within the minimum elastic limit of the material in order to avoid permanent strain or set. For average mild steel, such as employed in general structural work, the ultimate strength varies from 27 to 32 tons per square inch, and the elastic limit from about 14 tons per square inch upwards, the latter being a more variable quantity than the ultimate strength. The actual working stress should therefore not exceed 9, or at most 10 tons per square inch, in order to leave a reasonably safe margin as regards the elastic limit. These figures correspond to a factor of safety of 3 with respect to the ultimate strength.

For purely dead loads, that is where the *actual* maximum stress is very closely estimable, a working stress of 9 tons per square inch for mild steel may be safely adopted for tension members, and 8 tons per square inch for compression members, excepting where the length of the latter necessitates a reduction in accordance with the liability to fail by buckling.

From the results of experiments on members subjected to fluctuating stresses the following general deductions are made.

1. That a factor of safety of $4\frac{1}{2}$ should be employed when the stress fluctuates between the maximum value and zero.

2. That a factor of safety of 9 should be employed when the stress fluctuates between a certain amount of compression and a similar amount of tension. This case would be met by trebling the nominal live load stress and using a factor of safety of 3, as for a dead load.

Case 2 probably represents the extreme effect of a variable load, and between it and the case of a purely dead load there are numberless others in which the ratio of dead to live load stress may have any value. Taking these deductions as a basis, therefore, the factor of safety for members subject to maximum and minimum stresses of the *same* kind will vary between 3 and 4·5, and for members subject to maximum and minimum stresses of *opposite* kinds the factor of safety will vary between 4·5 and 9. The value chosen in any particular case

will depend principally on the extent to which the live or dead load stress preponderates.

Most of the formulæ devised for determining the sectional area or working stress of a member, or what is practically the same thing – the factor of safety to be employed, provide in effect, a sliding factor of safety dependent upon the relative values of the nominal or apparent maximum and minimum stresses, whilst some take into account the degree of suddenness of application of the live load.

Wöhler's Experiments.—The following results were deduced from a number of experiments made by Herr Wohler with a view to ascertaining the effect on the strength of materials when subjected to known alternations of stress repeated until fracture ensued. The material was found to break under a stress considerably less than that which it would withstand when the load was applied steadily under normal conditions of testing. The apparent loss of ultimate strength also varied according to the difference between the maximum and minimum stresses applied.[1] Broadly stated the actual results were as follows :—If t = breaking stress due to a steadily applied load with no variation (usually called the static breaking stress), then under a large number of applications of load varying between zero and u, the material eventually broke under the load u when the apparent stress caused was equal to $\frac{t}{2}$, or the strength of the material was apparently reduced by one-half. Under the repeated application of a load varying between $+v$ and $-v$, that is between a certain compression and the same amount of tension, the material eventually broke under the load v when the apparent stress caused was equal to $\frac{t}{3}$, corresponding with an apparent reduction in strength of two-thirds.

Launhardt-Weyrauch Formula. This formula was devised to express the reduced ultimate strength of a member when subject to alternating nominal or apparent maximum and minimum stresses, as indicated by the results of Wohler's experiments.

Let Max. S and Min. S = the higher and lower apparent stresses to which the member is subjected ; t = static breaking stress of the material in tons per square inch ; P = the reduced breaking stress in tons per square inch due to the fluctuating load. Then

$$P = \frac{2}{3}t\left(1 + \frac{1}{2}\cdot\frac{\text{Min. S}}{\text{Max. S}}\right)$$

To apply this to purposes of design, the suitable *working* stress p tons per square inch is obtained by dividing the breaking stress P by a factor of safety of 3, whence –

$$p = \frac{2}{9}t\left(1 + \frac{1}{2}\cdot\frac{\text{Min. S}}{\text{Max. S}}\right)$$

Taking t for mild steel = 27 tons, and for wrought iron = 21 tons per square inch—

[1] For a detailed discussion of the results of these experiments, see *A Practical Treatise on Bridge Construction*, by T. Claxton Fidler.

$$p = 6\left(1 + \tfrac{1}{2} \cdot \frac{\text{Min. S}}{\text{Max. S}}\right) \text{ for mild steel}$$

and
$$p = 4{\cdot}66\left(1 + \tfrac{1}{2} \cdot \frac{\text{Min. S}}{\text{Max. S}}\right) \text{ for wrought iron.}$$

If Max. S and Min. S be of opposite kinds, Min. S will be denoted as a *negative* stress, and the formula becomes—

$$p = 6\left(1 - \tfrac{1}{2} \cdot \frac{\text{Min. S}}{\text{Max. S}}\right)$$

EXAMPLE 2.—*The maximum and minimum stresses in a mild steel member are respectively 47 and 22 tons of tension. Determine the net sectional area.*

Working stress $= 6(1 + \frac{1}{2} \cdot \frac{22}{47}) = 7{\cdot}4$ tons per square inch, and net sectional area $= \frac{47}{7{\cdot}4} = 6{\cdot}35$ square inches.

EXAMPLE 3.—*The maximum and minimum stresses being respectively 47 tons tension and 10 tons compression, to determine the net sectional area.*

Here the minimum stress of 10 tons is negative. Working stress $= 6(1 - \frac{1}{2} \cdot \frac{10}{47}) = 5{\cdot}35$ tons per square inch, and net sectional area $= \frac{47}{5{\cdot}35} = 8{\cdot}8$ square inches.

Relatively few members are actually allowed to suffer reversals of stress, a counter-brace being usually provided to take up the reversed stress, when the principal member is then subject to a stress fluctuating between Max. S and zero. The working stress then equals $6(1 + 0) = 6$ tons per square inch, and the factor of safety $= \frac{27}{6} = 4{\cdot}5$.

Where Max. S = Min. S, but is of opposite kind, the working stress $= 6(1 - \frac{1}{2}) = 3$ tons per square inch, and the factor of safety $= \frac{27}{3} = 9$, as previously mentioned.

The apparently reduced breaking strength of the material when subjected to alternating stresses is considered by some authorities to be due to a deteriorated condition of the material to which the term *fatigue* has been applied, and the results of the Wohler tests are generally ascribed to some such condition of the material. It will be noticed that the Launhardt-Weyrauch formula, being based on the results of Wohler's tests, makes allowance for the so-called *fatigue* of the material, but does not allow for the *dynamic* or *impact* effect of the load, since the suddenness of application of the load does not enter into the results deduced from the tests.

Modified Launhardt Formula.—The following modified form of this formula is in use in America :—

$$p = 3{\cdot}34\left(1 + \frac{\text{Min. S}}{\text{Max. S}}\right) \text{ for wrought iron}$$

the lower resulting value for p being assumed to cover the effects of both "fatigue" and impact.

Whether it is necessary to make a separate allowance for the effects

of "fatigue" and dynamic stress is a much-debated question. Prof. T. Claxton Fidler advances cogent reasons for supposing that the effects of "fatigue" and dynamic action are one and the same.[1]

Fidler's Dynamic Formula.—The following formula has been proposed by Prof. T. Claxton Fidler for determining the sectional area of members. It makes allowance for the dynamic effect of the fluctuating load in a manner stated briefly as follows. If a member be subject to an initial stress of, say, 2 tons per square inch due to the dead load, the *sudden* application of a further load such as would create an additional 3 tons of stress if applied very gradually, may cause a momentary or dynamic increase of stress of $2 \times 3 = 6$ tons per square inch, thus creating an actual maximum stress of $2 + 6 = 8$ tons per square inch. Similarly, the sudden application of a stress equal to one-half the breaking strength of the material may readily account for the failure of a bar by the creation of a dynamic stress equal to the breaking stress. This is, broadly speaking, the line of argument by which the results of Wohler's tests may be explained by reference to the dynamic theory of stress, and, assuming this action to take place, the material does not undergo any *actual* reduction of breaking strength, but fails at its normal static breaking stress, simply because that stress is momentarily created by the suddenness of application of the load.

If Max. S and Min. S represent as before, the maximum and minimum stresses to which the member is subjected, the dynamic increment of stress $= w =$ Max. S $-$ Min. S.

Hence the actual maximum stress to be provided for in designing the sectional area = nominal maximum stress + increment = Max. S $+ w$; and for mild steel tension members—

$$\text{Net sectional area} = \frac{\text{Max. S} + w}{9} \text{ square inches}$$

For compressive members not liable to buckle—

$$\text{Gross sectional area} = \frac{\text{Max. S} + w}{7} \text{ square inches.}$$

For members subject to alternating stresses of opposite kinds, Min. S will be negative, and $w =$ Max. S $- (-$ Min. S$) =$ Max. S + Min. S.

For wrought-iron tension members—

$$\text{Net sectional area} = \frac{\text{Max. S} + w}{6{\cdot}66}$$

and for wrought-iron compression members

$$\text{Gross sectional area} = \frac{\text{Max. S} + w}{5{\cdot}33}$$

Prof. Fidler recommends that in all cases where the increment w is applied instantaneously, or practically so, its full value = Max. S $-$ Min. S, is to be added to the nominal Max. S. This will be the case in

[1] *Bridge Construction*, T. C. Fidler, pp. 248 *et seq.*

web members of girders, cross-girders and longitudinals, whilst in the case of the flanges of main girders exceeding 100 feet span, *i.e.* in which an appreciable period of time elapses between minimum and maximum loading, w is to be taken

$$= \frac{\text{Max. S} - \text{Min. S}}{2}$$

This is, of course, a purely arbitrary allowance, but at the same time reasonable. The formula used in this manner becomes very elastic, and may obviously be applied generally, by varying w in accordance with the time period of loading, but such modification must depend entirely on the judgment of the designer.

EXAMPLE 4.—*Required the sectional area for a mild steel member in which the stress alternates rapidly from* 50 *tons of tension to* 18 *tons of tension.*

$$w = 50 - 18 = 32 \text{ tons}$$

and $$\text{sectional area} = \frac{50 + 32}{9} = 9{\cdot}1 \text{ square inches.}$$

EXAMPLE 5.—*The maximum stress in a mild steel strut is* 57 *tons of compression and the minimum stress* 12 *tons of tension. Required the sectional area exclusive of consideration of buckling tendency.*

Here, $w = 57 + 12 = 69$ tons, and if the change from minimum to maximum loading take place instantaneously, the actual maximum stress to be designed for $= 57 + 69 = 126$ tons, whence—

$$\text{sectional area} = \tfrac{126}{7} = 18 \text{ square inches.}$$

Stone's "Range" Formula.[1]—The following formula has been proposed by Mr. E. H. Stone, M.Inst.C.E. Briefly, this formula takes into account, separately, what is defined as the *immediate effect* and the *cumulative effect* of the moving load. Quoting from the author's original paper, the immediate effect is "observable every time a train traverses a bridge, due to the sudden and violent manner in which the load is applied—irregularities in the track, peculiarities of the engine, or other causes." The cumulative effect is "an effect produced in course of time by repeated loading and unloading." The object of the inquiry was to establish a co-efficient to be applied to the nominal moving load, which should express its total effect on the members of the structure in terms of effective or equivalent fixed load.

1. The "immediate" effect of the moving load, as compared with that due to the same amount of static or fixed load, is deduced from experiments, information regarding which is given in the paper above referred to.

2. The "cumulative" effect of the moving load, as compared with that due to the same amount of fixed load, is deduced from Wohler's experiments. The "immediate" and "cumulative" effects are then combined in a rational manner to give the total effect of the moving load, as compared with that of an equivalent amount of fixed load. The results for mild steel are given in Table 24.

[1] *Transactions Am. Soc. C.E.*, vol. 41, pp. 467 to 553.

TABLE 24.—WORKING STRESSES FOR MILD STEEL BY RANGE FORMULA.—Safe Working Stress = 9 − (5 × R′).

Nominal load.			Co-efficient to obtain equivalent of moving load in terms of fixed load.	Effective load.			Working results. Factor of safety 3. Permissible stress, Tons per sq. in.	
Total nominal load.	Composition of nominal load. Fixed load.	Moving load.		Total effective load.	Composition of effective load. Fixed load.	Moving load.	Due to nominal load.	Due to effective load.
tons.	tons.	tons.		tons.	tons.	tons.		
100	0	100	2·2500	225·0000	0	225·0000	4·00	9·00
100	2·5	97·5	2·1478	211·9105	2·5	209·4105	4·25	9·00
100	5	95	2·0585	200·5575	5	195·5575	4·49	9·00
100	10	90	1·9091	181·8190	10	171·8190	4·95	9·00
100	15	85	1·7889	167·0565	15	152·0565	5·39	9·00
100	20	80	1·6897	155·1760	20	135·1760	5·80	9·00
100	25	75	1·6061	145·4575	25	120·4575	6·19	9·00
100	30	70	1·5344	137·4080	30	107·4080	6·55	9·00
100	33·3	66·6	1·4918	132·7866	33·3	99·4533	6·78	9·00
100	35	65	1·4719	130·6735	35	95·6735	6·89	9·00
100	40	60	1·4167	125·0020	40	85·0020	7·20	9·00
100	45	55	1·3678	120·2015	45	75·2015	7·49	9·00
100	50	50	1·3226	116·1300	50	66·1300	7·75	9·00
100	55	45	1·2817	112·6765	55	57·6765	7·99	9·00
100	60	40	1·2439	109·7560	60	49·7560	8·20	9·00
100	65	35	1·2086	107·3010	65	42·3010	8·39	9·00
100	66·6	33·3	1·1974	106·5800	66·6	39·9133	8·44	9·00
100	70	30	1·1754	105·2620	70	35·2620	8·55	9·00
100	75	25	1·1439	103·5975	75	28·5975	8·69	9·00
100	80	20	1·1136	102·2720	80	22·2720	8·80	9·00
100	85	15	1·0844	101·2660	85	16·2660	8·89	9·00
100	90	10	1·0559	100·5590	90	10·5590	8·95	9·00
100	95	5	1·0278	100·1390	95	5·1390	8·99	9·00
100	100	0	1·0000	100·0000	100	0·0000	9·00	9·00

"It is found by experiment, as might have been expected, that the co-efficient representing the extra effect of the moving load on a girder (or on a member of a bridge truss) is a variable quantity depending on the relative amount of fixed load and moving load in the total load, being greatest where the proportion of moving load in the total load is comparatively high."

The first column of Table 24 represents the total *nominal* load or stress in the member, due to both fixed and moving load, expressed as 100 per cent. The second and third columns give the percentage of nominal stress due to fixed and moving load respectively. The fourth column contains the deduced co-efficients by which the nominal stress due to moving load is to be multiplied in order to give effective or equivalent fixed load stress. The values in the fifth column are obtained by multiplying the percentages in column 3 by the co-efficients in column 4, and adding the percentages in column 2, and express the equivalent stress due to the total nominal load in terms of fixed or dead load stress. Columns 6 and 7 show the composition of this equivalent stress, that is, the proportion of it due to the effect of the

fixed load, and that due to the effect of the moving load. Since the values in column 5 represent equivalent fixed load stress, a uniform working stress of 9 tons per square inch may be adopted in every instance, and the sectional area may be computed on the total effective load at 9 tons per square inch, or on the total nominal load at some reduced number of tons per square inch. The reduced working stress to be adopted in any particular case

$$= 9 \text{ tons} \times \frac{\text{total nominal load}}{\text{total effective load}}$$

Thus, for a member having a total nominal stress of 100 tons, 30 tons being due to dead load, and 70 tons to moving load, the equivalent dead load stress from column 5 = 137·408, and sectional area $= \frac{137\cdot4}{9} = 15\cdot27$ square inches. If, however, it is desired to calculate the sectional area with respect to the total nominal stress of 100 tons, the reduced working stress to be employed $= \frac{100}{137\cdot4}$ of $9 = 6\cdot55$ tons per square inch, which is found in column 8, and sectional area $= \frac{100}{6\cdot55} = 15\cdot27$ square inches as before. It may be noted that for the same example, the sectional area by the Launhardt formula would be 14·5 square inches, and by Claxton Fidler's formula, 18·9 square inches.

The other values in column 8 are deduced similarly, and the table exhibits at a glance the unit working stress to be adopted for a given nominal total stress, of which the percentage composition is known. A formula giving the working stress is readily obtained by plotting the curve showing the relation between the working stresses of column 8 and the corresponding varying percentages of fixed and moving load constituting the total nominal load. This curve being slightly modified to allow for a reasonable amount of shock and jarring on lighter members subject to considerable variation of stress, the formula eventually deduced is as follows—

Let R = proportionate range of stress

$$= \frac{\text{Stress due to moving load}}{\text{Fixed load stress} + \text{moving load stress}} = \frac{\text{Range of stress}}{\text{Total stress}}$$

Then safe working stress per square inch

$= 9 - 5R^2$ for mild steel

and $= 7 - 4R^2$ for wrought iron.

EXAMPLE 6.—*A mild steel member is subject to a tensile stress of* 57 *tons, due to moving and fixed load combined, and* 18 *tons of tension, due to fixed load alone. Required the sectional area by the Range formula.*

Stress due to moving load = 57 − 18 = 39 tons. Hence, ratio $R = \frac{39}{57} = 0\cdot684$.

Working stress $= 9 - 5 \times (0\cdot684)^2 = 6\cdot66$ tons per square inch,

and net sectional area $= \frac{\text{total nominal load}}{\text{working stress}} = \frac{57}{6{\cdot}66} = 8{\cdot}55$ square inches.

The working stress may also be taken from Table 24, thus—

$$\text{Percentage of fixed load stress} = \frac{18}{57} \times 100 = 31{\cdot}6$$

From Table 24—

Working stress for 30 per cent. fixed load = 6·55
" " $33\frac{1}{3}$ " " 6·78
∴ " " 31·6 " " is practically the mean of these two values, or 6·66 tons per square inch.

EXAMPLE 7.—*A mild steel member is subject to a fixed load stress of 40 tons of tension, and alternations of 30 tons of tension and 30 tons of compression caused by a moving load. To deduce the sectional area.*

Here maximum tensile stress = 40 + 30 = 70 tons. When the moving load stress of 30 tons *compression* comes into operation, 30 of the 40 tons tension due to the fixed load is neutralized, leaving a "residual fixed load stress" of 10 tons of tension. The range of stress is therefore 60 tons, whilst the "total stress" = stress due to moving load + residual fixed load stress at the instant the maximum tensile stress of 70 tons is in operation, = 60 + 10 = 70 tons. It is to be noticed that 60 of the 70 tons of maximum stress is really due to the action of the moving load, and that *for the moment*, only 10 of the 40 tons tension originally due to the fixed load is in operation *as fixed load stress.*

Hence, $R = \frac{60}{70} = 0{\cdot}86$

and working stress $= 9 - 5 \times (0{\cdot}86)^2 = 5{\cdot}3$ tons per square inch

whence sectional area $= \frac{70}{5{\cdot}3} = 13{\cdot}2$ square inches.

The above treatment of cases of fluctuating stresses is recommended by Mr. Stone as being the most reasonable, and employed in this manner, the "range" formula becomes applicable to all cases of loading.

EXAMPLE 8.—*A mild steel member is subject to a fixed load stress of 20 tons compression, and a moving load stress which alternates between 80 tons of compression and 40 tons of tension.*

In this case the maximum compression = 20 + 80 = 100 tons. At the instant the 40 tons of tension is created by the action of the moving load, the whole 20 tons compression originally due to the fixed load, is more than neutralized, and the effect due to the weight of the structure itself here constitutes moving load effect, since it is applied and removed each time the moving load comes into action. The "residual fixed load stress" is therefore zero, and "total stress" = range of stress + residual fixed load stress = 120 + 0 = 120 tons.

Hence the ratio $R = \frac{\text{range of stress}}{\text{total stress}}$ becomes $= 1$, for all cases where actual reversal of stress takes place.

$\therefore$ Working stress $= 9 - 5 \times 1^2 = 4$ tons per square inch, and sectional area $= \frac{120}{4} = 30$ square inches.

The three formulæ cited are representative of those in general use, which in one form or another may be relegated to one of the above types. It should be noticed that in *every* case a factor of safety of 3 is uniformly employed on the "equivalent dead load" stress, whilst on the "nominal maximum stress" a variable factor of safety results from using one or other of the formulæ.

Impact Formulæ.—In the case of members of railway bridge girders, the probable additional stresses caused by the rolling load over and above those arrived at by calculating the stresses due to the various *static* positions of the load, are usually computed by the aid of one of the many suggested Impact formulæ. The principal causes of impact stress are unbalanced driving wheels of locomotives, uneven track, wheel tyres worn out of truth, pitching and rolling of engines, rapidity of application of load and deflection of cross beams and longitudinals. The term "impact" is understood to include all effects of the rolling load which create stresses in excess of the computed static stresses.

It is manifestly impossible to express by a formula the combined effect of so many complex factors, and the present state of exact knowledge of this question is far from complete. With track and rolling stock in good condition, the chief causes of impact are unbalanced rotating parts and speed of load. At best, since an accurate estimate is impossible, it is desirable to make an allowance known to be on the safe side, and most of the impact formulæ in general use probably accomplish this end.

Pencoyd Formula, proposed by Mr. C. C. Schneider, Past President of the Am Soc. C. E.

$$I = \frac{300}{L + 300} \times S$$

where S = stress due to rolling load considered at rest in the position which causes the maximum stress in the member under consideration; L = length in feet of that portion of the span which the load has traversed to reach that position from the point where it first began to produce stress in the member; I = the stress to be added to S to allow for impact. The fraction $\frac{300}{L + 300}$ is known as the "impact coefficient." This formula, although widely used, probably gives results slightly too low for short spans up to about 70 feet on which impact is most severe, and too great for large spans exceeding about 200 feet.

American Railway Engineering Association Formula.—This formula has been suggested following the results of observations on forty-six spans up to 500 feet at speeds varying from 10 to 70 miles per hour.

$$I = \frac{1}{1 + \frac{L^2}{20000}} \times S$$

Prof. Turneaure suggests a modification of this formula by employing the constant 30,000 instead of 20,000.

Waddell's Formula.—Mr. J. A. L. Waddell, the eminent American bridge engineer, has proposed the following formula—

$$I = \frac{165}{nL + 150} \times S$$

where I, L, and S have the same significance as above and n = the number of tracks carried by the bridge. For highway bridges, this formula is modified as follows—

$$I = \frac{100}{nL + 200} \times S$$

in which n = total width of roadway and sidewalks divided by 20.

The following are the conclusions arrived at and published by the Committee of the American Railway Engineering Association in connection with the observations above referred to.[1]

"(1) With track in good condition the chief cause of impact was found to be the unbalanced drivers of the locomotive. Such inequalities of track as existed on the structures tested were of little influence on impact on girder flanges and main truss members of spans exceeding 60 to 75 feet in length.

"(2) When the rate of rotation of the locomotive drivers corresponds to the rate of vibration of the loaded structure, cumulative vibration is caused, which is the principal factor in producing impact in long spans. The speed of the train which produces this cumulative vibration is called the 'critical speed.' A speed in excess of the critical speed, as well as a speed below the critical speed, will cause vibrations of less amplitude than those caused at or near the critical speed.

"(3) The longer the span length the slower is the critical speed; and therefore the maximum impact on long spans will occur at slower speeds than on short spans.

"(4) For short spans, such that the critical speed is not reached by the moving train, the impact percentage tends to be constant so far as the effect of the counter-balance is concerned, but the effect of rough track and wheels becomes of greater importance for such spans.

"(5) The impact as determined by extensometer measurements on flanges and chord members of trusses is somewhat greater than the percentages determined from measurements of deflection, but both values follow the same general law.

"(6) The maximum impact on web members (excepting hip verticals) occurs under the same conditions which cause maximum impact on chord members, and the percentages of impact for the two classes of members are practically the same.

"(7) The impact on stringers is about the same as on plate-girder

[1] American Railway Engineering Association, *Bulletin* No. 125, 1910.

spans of the same length, and the impact on floor beams and hip verticals is about the same as on plate girders of a span length equal to two panels.

"(8) The maximum impact percentage as determined by these tests is closely given by the formula

$$I = \frac{100}{1 + \frac{L^2}{20000}}$$

in which I = impact percentage, and L = span length in feet.

"(9) The effect of differences of design was most noticeable with respect to differences in the bridge floors. An elastic floor, such as furnished by long ties (sleepers) supported on widely spaced stringers, or a ballasted floor, gave smoother curves than were obtained with more rigid floors. The results clearly indicated a cushioning effect with respect to impact due to open joints, rough wheels, and similar causes. This cushioning effect was noticed on stringers, floor beams, hip verticals, and short span girders.

"(10) The effect of design upon impact percentage for main truss members was not sufficiently marked to enable conclusions to be drawn. The impact percentage here considered refers to variations in the axial stresses in the members, and does not relate to vibrations of members themselves.

"(11) The impact due to the rapid application of a load, assuming smooth track and balanced loads, is found to be, from both theoretical and experimental grounds, of no practical importance.

"(12) The impact caused by balanced compound and electric locomotives was very small, and the vibrations caused under the loads were not cumulative.

"(13) The effect of rough and flat wheels was distinctly noticeable on floor beams, but not on truss members. Large impact was, however, caused in several cases by heavily loaded freight cars moving at high speeds."

For further useful information on this subject, reference may be made to a paper "On Impact Coefficients for Railway Girders" and the accompanying discussion thereon, by C. W. Anderson, M.Inst.C.E., published in volume cc. of the *Minutes of Proceedings of the Inst. C.E.*

CHAPTER III

BENDING MOMENT AND SHEARING FORCE.

Bending Moment.—Suppose the beam in Fig. 12 to be fixed at A and loaded with 2 tons at the free end E, then the load multiplied by its distance from A is called the *moment* of the load about A. Since the effect of this moment is to cause bending of the beam, the moment is further called the *bending moment* at A. Its value = 2 tons × 12 feet = 24 foot-tons.

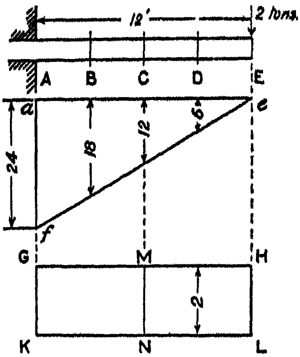


FIG. 12.

Since the bending moment equals load × distance, the bending moment at B distant 9 feet from E = 2 × 9 = 18 foot-tons. Similarly at C, the bending moment = 2 × 6 = 12 foot-tons, at D = 2 × 3 = 6 foot tons, and at E = 2 × 0 = 0. Plotting these values below a horizontal line, *ae*, the points so obtained lie on the straight line *ef*, which is such that the vertical depths between it and *ae* represent the bending moments at corresponding points along the beam. The figure *aef* is called the *bending moment diagram* for the beam. In this case, having obtained point *f*, the diagram might have been completed by joining *f* to *e*, without calculating the bending moment for any intermediate points. It will be seen that *ae* represents the length of the beam to any convenient scale, which however has no connection with the scale to which the bending moments are plotted. For instance, if *ae* be made 3 inches and *af* 2 inches, the scale for distance along the beam would be 4 *feet* to 1 inch, and for the bending moments, 12 *foot tons* to 1 inch. The practical value of a bending-moment diagram is to enable the bending moment to be scaled off for *any* point along the beam, the diagram being usually readily drawn after calculating the moment at a *few* points only.

Shearing Force.—Apart from the bending action, the load has another effect on the beam as indicated at A, Fig. 13. This is the tendency to shear the beam vertically at any section such as A or B. Such an effect is due to vertical *shearing force*. In this case it is equal in amount to 2 tons at every vertical section along the beam. The shearing force is represented diagrammatically below the bending

moment diagram in Fig. 12, where GH again represents the span and GK is made equal to 2 tons to scale. The depth of the rectangle GHLK is a measure of the shearing force at any section of the beam.

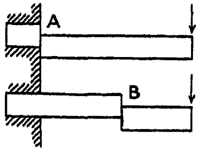


FIG. 13.

Relation between Bending Moment and Shearing Force.—The following relation always exists between the bending moment and shearing force at any section of a beam. The area of the shearing-force diagram, between the free end of the beam and any vertical section, equals the bending moment at that section. Thus in Fig. 12 the area of the rectangle GHLK, between the free end H and the vertical section GK = GH × GK = 12 feet × 2 tons = 24 foot-tons, which was the value obtained for the bending moment *af*. Again, considering section C, the area of the shearing-force diagram between H and M is the rectangle MHLN, which = 6 feet × 2 tons = 12 foot-tons, the bending moment previously obtained for section C.

A beam such as the above, having one end fixed and the other free, is called a *cantilever*. The bending moment in this case, if expressed in general terms = Wl foot-tons, where W = the load and l = span.

Cantilever carrying any number of Concentrated Loads.—Suppose the cantilever in Fig. 14 to be 30 feet long and to carry loads as indicated. The bending moment at D is nothing. At C the bending moment = 5 tons × 10 feet = 50 foot-tons, which is plotted on the bending-moment diagram at cc'. At B there is the combined bending moment, due to 5 tons acting at a leverage of 22 feet, and 6 tons at a leverage of 12 feet. Hence—

Bending moment at B = 5 × 22 + 6 × 12 = 182 foot-tons, which is plotted at bb'.

The bending moment at A = 5 × 30 + 6 × 20 + 8 × 8 = 334 foot-tons, which is plotted at aa'. The diagram is completed by joining points a', b', c', d by straight lines. Note that the inclination of the boundary line $a'b'c'd$ changes under each point of application of the loads. If the loads were very numerous and close together, the broken line $a'b'c'd$ would approximate to a curve.

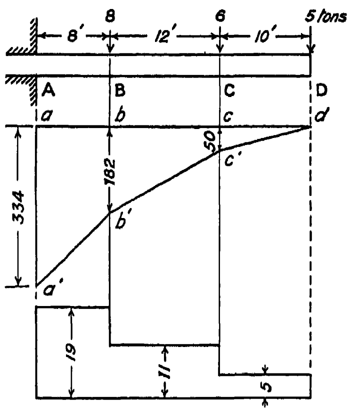


FIG. 14.

The shearing force between D and C is 5 tons. Between C and B it is augmented by the additional load of 6 tons, giving 5 + 6 = 11 tons, and between B and A this 11 tons is further augmented by the load of 8 tons, giving 5 + 6 + 8 = 19 tons. The construction of the shearing-force diagram is indicated in the lower figure.

Cantilever carrying a Uniformly Distributed Load.—A distributed load is any load which is applied continuously along the whole length of a beam. In practice the majority of distributed loads to be considered

are *uniformly* distributed loads; that is, each foot length of the beam carries an equal amount of load, such, for example, as a beam supporting a wall of uniform height. The dead weight of the beam itself, if of uniform section, is also a uniform load.

Suppose the cantilever in Fig. 15 to be 16 feet long, and to carry a load of 2 tons per foot run. The bending moment at A will be the same as if the whole load were concentrated at its centre of gravity. The total load = 16 × 2 = 32 tons, and its centre of gravity is situated at the middle of its length; that is, 8 feet from A. The bending moment at A therefore = 32 × 8 = 256 foot-tons. At the section B, the portion of the load creating bending moment is that distributed over the 12 feet length BE, having its centre of gravity distant 6 feet from B.

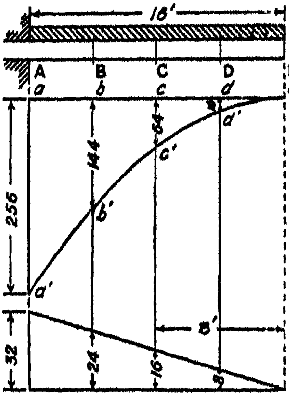


FIG. 15.

∴ bending moment at B
= (12 × 2) tons × 6 feet
= 144 foot-tons.

Similarly, at section C, the bending moment = (8 × 2) tons × 4 feet = 64 foot-tons, and at section D the bending moment = (4 × 2) tons × 2 feet = 16 foot-tons. At E the moment is 0. These values are plotted as before at *aa'*, *bb'*, etc.

If, instead of considering only four sections of the beam, a very great number were taken and the resulting bending moments plotted, the points so obtained would be found to lie on an even curve passing through *a'b'c'd'e*, which in this case is a semi-parabola tangent to the horizontal line *ae* at *e*. Knowing this to be so, it is only necessary to calculate the bending moment at A and to fit in the curve by the geometrical contraction shown in Fig. 16. Set out *aa'* = 256 ft.-tons to scale and *ae* = 16 ft. to scale. Divide *ae* into any number of equal parts and *aa'* into the *same* number of equal parts. Draw verticals through *a*, *b*, *c*, and *d*, and join *e* to points 1, 2, 3 on *aa'*. The intersections of *e*1 with *bb'*, *e*2 with *cc'*, and *e*3 with *dd'*, give points *b'*, *c'*, *d'*, through which a free curve is drawn. More frequent points on the curve may be obtained by dividing *ae* and *aa'* into a greater number of parts.

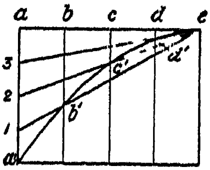


FIG. 16.

The shearing force is nothing at the free end E, and increases by 2 tons for each foot length of the cantilever from E to A, so that at D it is 4 × 2 = 8 tons, at C, 8 × 2 = 16 tons, at B, 24 tons, and at A, 32 tons. The S.F. diagram is therefore the triangle in the lower figure. Note the relation between the two diagrams. The area of the S.F. diagram between the free end G and centre of beam = $\frac{16 \times 8}{2}$ = 64

ft.-tons, or the value of the B.M. obtained for section C. Expressed in general terms, the B.M. at the fixed end $= \frac{wl^2}{2}$, where w = load per foot run and l = span in feet.

Beam supported at both Ends and carrying a Concentrated Load at the Centre.—Suppose the beam in Fig. 17 to be 50 feet span, and loaded at the centre with 8 tons. Obviously half the load will be carried by each support, giving rise to equal upward reactions of 4 tons. If the beam were inverted it would be similar to a balanced cantilever supported at the centre and having each end loaded with 4 tons. The B.M. at the centre therefore = 4 tons × 25 ft. = 100 ft.-tons, or ½ load × ½ span. In this case the beam bends with its concave side uppermost, whereas in the cantilevers previously considered the bending took place with the concave side downwards. It is customary to distinguish between these two kinds of bending action by designating the bending in Fig. 18, A as positive, and Fig. 18, B as negative. Diagrams of positive bending moments are plotted *above* the horizontal line representing the span, and negative moments *below*.

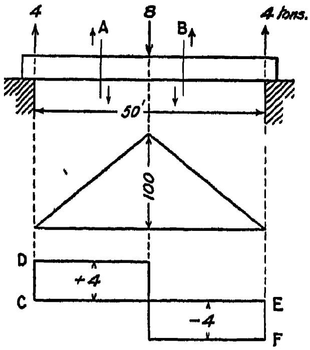


FIG. 17.

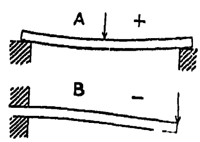


FIG. 18.

The shearing force from the left-hand abutment to the centre of the beam is 4 tons acting *upwards on the left* of any section such as A, Fig. 17, and *downwards on the right* of such section. At the centre of the beam the upward shear of 4 tons is more than neutralized by the downward acting load of 8 tons, so that the shear now acts *downwards on the left* of any section B, to the right of the centre and *upwards on the right* of such section. As the relative direction of the shear reverses at the centre, it is customary to designate the shearing force from the left abutment to the centre of the beam as *positive*, and from the centre to the right abutment as *negative*. From the centre to the right abutment the vertical shear of 4 tons acts downwards on the left of every section, until *at* the abutment it is neutralized by the upward reaction of 4 tons and becomes reduced to zero. This is shown on the S F diagram by plotting CD = 4 tons *above* the horizontal line CE, and EF = 4 tons *below* CE, and completing the diagram as shown. It should be noted that, immediately under the load (in this case at the centre of the beam), the diagram exhibits a positive shear of 4 tons and a negative shear of 4 tons, so the actual resultant shear is + 4 − 4 = 0. This zero shear only exists at the central vertical section of the beam, and, according to the diagram, it *suddenly* increases to the full value of 4 tons on sections *immediately* to the right or left of the centre. This state of shear only exists on the assumption that the load is applied at a single point on the beam, a condition impossible of realization in

actual practice, since all loads must have an appreciable area of bearing on the beams supporting them. As this point frequently gives rise to some difficulty in correctly interpreting the meaning of shear force diagrams, a reference to the following practical example will render it more clear. Suppose the load of 8 tons in Fig. 19 to be applied by a rolled joist having flanges 8 inches wide. The bearing area on the beam AB is now 8 inches wide instead of being a single point, and the "concentrated" load of 8 tons is actually a distributed load of 1 ton per inch run over the central 8 inches of the length of AB. From abutment A to the left-hand edge of the lower flange of the joist, the shear is + 4 tons as before. At 1 inch in from the edge of the flange, a downward load of 1 ton has been applied, reducing the shear to + 3 tons. At 2 inches in, the shear is further reduced by another ton, and equals + 2 tons; at 3 inches it is reduced to + 1 ton, and at the centre to zero. At 5 inches from the left hand edge of flange, 5 tons of downward acting load having been applied, the shear is + 4 − 5 = − 1 ton; at 6 inches it is + 4 − 6 = − 2 tons, etc.

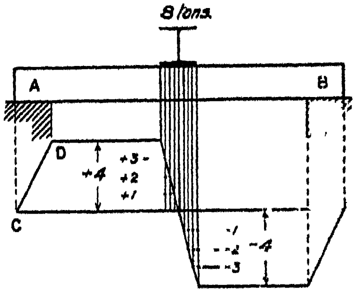


FIG. 19.

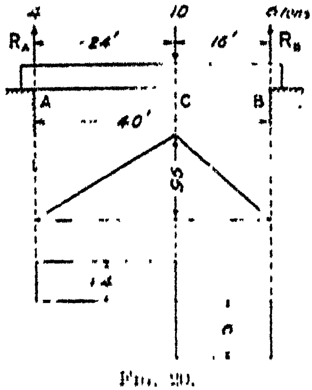


FIG. 20.

Thus the shear is really *gradually* decreased from + 4 tons at the left-hand edge of bearing to − 4 at the right hand edge, and cannot change abruptly as implied by the vertical steps on S.F. diagrams for concentrated loads as usually drawn. The same is true for the bearing surfaces on the supports, the shear increasing gradually from C to D over the horizontal bearing of 8".

Beam supported at both Ends and carrying a Concentrated Load at any Point.—Suppose the beam AB in Fig. 20 to be 40 ft. span, and to carry a concentrated load of 10 tons at 24 feet from A. The reaction at B (usually denoted R_B) is found by taking moments about the opposite end A of the beam. Thus—

$$R_B \times 40' = 10 \text{ tons} \times 24'$$
$$\therefore R_B = 10 \times \tfrac{24}{40} = 6 \text{ tons.}$$

The reaction R_A at A must be the difference between the total load and R_B, or 10 − 6 = 4 tons. The B.M. at C = 6 tons × 16 ft. = 96 ft.-tons, which enables the B.M. diagram to be plotted. The B.M. at C

may also be obtained by working from the reaction at A. Thus 4 tons $\times$ 24 ft. = 96 ft.-tons as before. The S.F. is + 4 tons from A to C, and − 6 tons from C to B, and the diagram is plotted as shown. The reactions at A and B are always inversely proportional to the segments into which the span is divided by the load point C. Thus $\frac{16}{40}$ of 10 tons gives R_A, the reaction at the *opposite* end to the 16 ft. segment, and $\frac{24}{40}$ of 10 tons gives R_B. By remembering this rule, the reactions may readily be written down from a simple inspection of the position of the load. **Note.**—The areas of the positive and negative portions of the S.F. diagram for *any* beam are always equal, since each is a measure of the B.M. at the same point.

Beam supported at both Ends and carrying any number of Concentrated Loads.—Suppose the beam in Fig. 21 to be 80 ft. span and to carry loads as indicated.

Reaction $R_A = \frac{16}{80}$ of 10 tons $+ \frac{50}{80}$ of 16 tons $+ \frac{60}{80}$ of 6 tons $= 2 + 10 + 4\frac{1}{2} = 16\frac{1}{2}$ tons. $R_B = 6 + 16 + 10 - 16\frac{1}{2} = 15\frac{1}{2}$ tons.

B.M. at $E = R_B \times 16$ ft. $= 15\frac{1}{2} \times 16 = 248$ ft.-tons.

Considering section D, the reaction R_B tends to bend the length DB upwards, whilst the intermediate load of 10 tons acting at a leverage of 50 ft. − 16 ft. = 34 ft., tends to hold down the length DB, and the B.M. at D will $= R_B \times 50$ ft. − 10 tons $\times$ 34 ft. $= 15\frac{1}{2} \times 50 - 340 = 435$ ft.-tons.

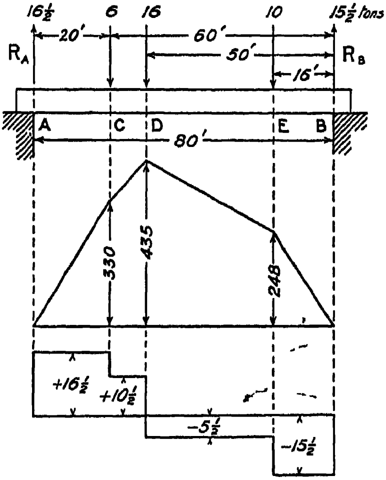


Fig. 21.

The Bending Moment at C is most readily obtained from $R_A \times 20$ ft. $= 16\frac{1}{2} \times 20 = 330$ ft.-tons. The same result will be obtained if calculated from the opposite end B of the beam. Thus B.M. at $C = R_B \times 60 - 10 \times 44 - 16 \times 10 = 330$ ft.-tons as before. Plotting 330, 435 and 248 ft.-tons beneath C, D and E respectively, the B.M. diagram is obtained.

For the S.F. diagram, the shear from A to C $= +16\frac{1}{2}$ tons. At C it is reduced by 6 tons, giving $+ 16\frac{1}{2} - 6 = +10\frac{1}{2}$ tons, the shear from C to D. From D to E, the S.F. $= + 10\frac{1}{2} - 16 = - 5\frac{1}{2}$ tons, and from E to B, $-5\frac{1}{2} - 10 = -15\frac{1}{2}$ tons, which checks with the reaction of $15\frac{1}{2}$ tons at B.

Beam supported at both Ends and carrying a uniformly distributed Load over the Whole Span.—Suppose the beam AB in Fig. 22 to be 60 ft. span, and to carry a load of $1\frac{1}{2}$ tons per foot run. R_A and R_B each equal half the total load $= \frac{1}{2} \times 60 \times 1\frac{1}{2} = 45$ tons. The *upward* B.M. at C = 45 tons $\times$ 30 ft. = 1350 ft.-tons. The *downward* B.M. at C is due to the load of 45 tons, extending over CB, which may be supposed concentrated at its centre of gravity distant 15 ft. from C.

$$\therefore \text{B.M. at } C = 45 \times 30 - 45 \times 15 = 675 \text{ ft.-tons.}$$

If the B.M. be calculated for several intermediate points between B and C in a similar manner to that adopted for the cantilever in Fig. 15, the plotted values will be found to range themselves on a parabola passing through *abc*. Hence make *cm* = 675 ft. tons, and draw in each half of the curve by the geometrical method previously given. (It may be mentioned here that all B.M. diagrams for uniformly distributed loads are bounded by parabolic curves, and that diagrams for concentrated loads are bounded by straight lines.) The shearing force at each abutment is equal to the reaction of 45 tons, and decreases uniformly to zero at the middle of the span. Expressed in general terms the B.M. at the centre $= \frac{wl^2}{8}$, w and l having the same significance as before.

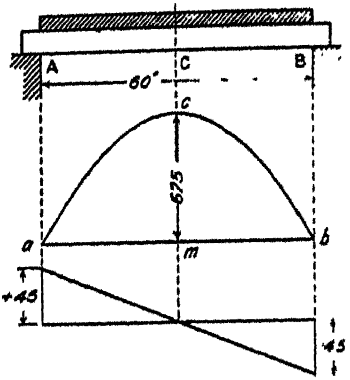


FIG. 22.

Beam supported at both Ends, and carrying a uniformly distributed Load over a Portion of the Span.—Suppose the beam in Fig. 23 to

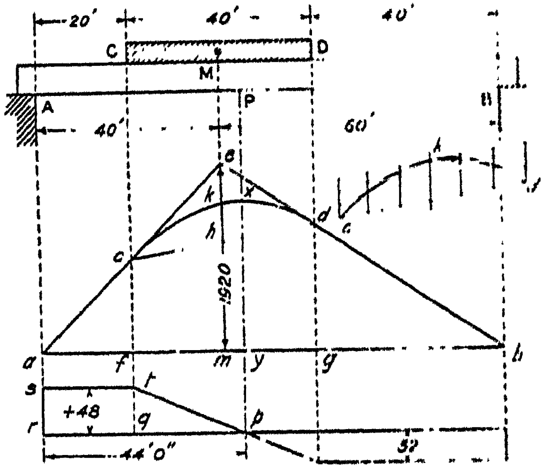


FIG. 23.

be 100 ft. span, and to carry a load of 2 tons per foot run over a length of 40 ft. as indicated. The reactions at A and B are the same as if the *total* load were concentrated at the centre of gravity of the distributed load on CD. Total load = 40 × 2 = 80 tons. ∴ $R_A = \frac{60}{100}$ of 80 = 48 tons, and R_B = 32 tons. The B.M. at M, supposing the load to be concentrated at M, would be 32 tons × 60 ft. = 1920 ft.-tons, and triangle *aeb*, obtained by making

me = 1920 ft.-tons, would be the B.M. diagram. Project C and D to *c* and *d* respectively. Join *c* to *d*, cutting *me* in *h*. Bisect *eh* in *k*. *cf* and *dg* are the bending moments at C and D, whether the load be all concentrated at M, or distributed between C and D. Since the load is actually distributed over CD and not concentrated at M, as above supposed, the boundary of the required diagram from *c* to *d* will be a parabolic curve passing through *ckd*. The curve is drawn by the same method as in Fig. 16, but is oblique, instead of rectangular, as shown in the inset. The boundary of the diagram for the whole span is then *ackdb*. This may be verified by calculating the moments at C, M, and D. Thus—

B.M. at C = 48 × 20 = 960 ft.-tons.
„ D = 32 × 40 = 1280 „
„ M = 32 × 60 − (20 ft. × 2 tons) × 10 = 1520 ft.-tons.

These values will be found to agree with the scaled moments *cf*, *dg*, and *hm*. The shearing force from A to C equals the reaction at A, 48 tons. From C towards D it diminishes at the rate of 2 tons per foot, so that at 24 ft. from C the S.F. is zero. It further diminishes to −32 tons at D, and remains constant from D to B. The maximum B.M. occurs at P, 44 ft. from A, and vertically above *p*, where the S.F. diagram cuts the horizontal base line, and is given by the area of the shear force diagram between *either* end of the beam and point *p*.

$$\therefore \text{Max. B.M.} = \text{area rect. } qrst + \text{area triangle } pqt$$
$$= 48 \times 20 + \tfrac{1}{2} \times 48 \times 24 = 1536 \text{ ft.-tons,}$$

which should correspond with the maximum scaled moment *xy*.

Note.—The following is a special case of the above when the load extends from one abutment partly over the span, Fig. 24.

Taking the same span with a load of 2 tons per ft. run, extending 70 ft. from A, the same construction applies. $R_A = \frac{65}{100}$ of (70 × 2) = 91 tons, and R_B = 140 − 91 = 49 tons.

B.M. at M, supposing the load concentrated = 49 × 65 = 3185 ft.-tons. Make *me* = 3185 to scale. Join *ae* and *be*. Project D to *d*. Join *ad*, bisect *eh* in *k* and draw the parabola *akd*.

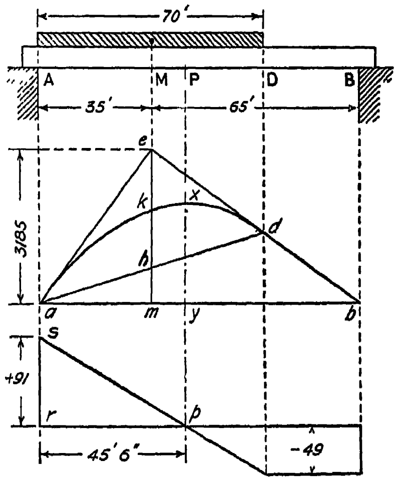


Fig. 24.

The shearing force is + 91 tons at A, diminishing to zero at P, distant 45·5 ft. from A, and further diminishing to − 49 tons at D, then remaining constant from D to B. The maximum B.M. occurs at P, and is given by area

of triangle $prs = \frac{1}{2} \times 91 \times 45{\cdot}5 = 2070\frac{1}{4}$ ft.-tons, which should correspond with the scaled moment xy.

The bending moment on girders carrying concentrated loads consists of two portions that due to the dead weight of the girder, usually considered as a distributed load, and that due to the concentrated loads. In drawing the diagram of moments, it is convenient first to construct separate diagrams for the distributed and concentrated loads, and then to combine them in order to obtain the total B.M. diagram. The following example illustrates this method. Suppose the girder in Fig. 25 weighs 0·25 ton per foot-run, and carries concentrated loads as indicated.

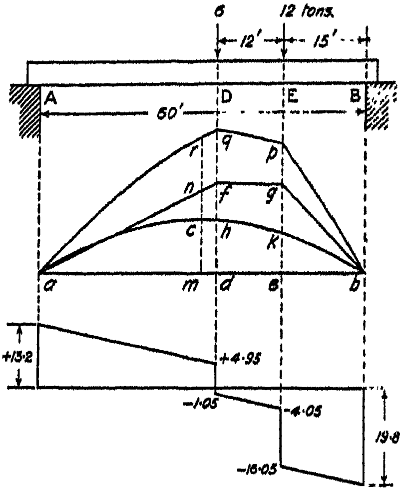


Fig. 25.

The B.M. at the middle of the span due to the weight of the girder

$$= \frac{wl^2}{8} = \frac{0{\cdot}25 \times 60 \times 60}{8} = 112{\cdot}5 \text{ ft.-tons.}$$

Make $mc = 112{\cdot}5$ ft.-tons to scale, and draw the parabola acb. The reaction at A due to the concentrated loads at D and E $= \frac{27}{60} \times 6 + \frac{15}{60} \times 12 = 5{\cdot}7$ tons. $R_B = 18 - 5{\cdot}7 = 12{\cdot}3$ tons. B.M. at D due to the concentrated loads alone $= 5{\cdot}7 \times 33 = 188{\cdot}1$ ft.-tons. B.M. at E $= 12{\cdot}3 \times 15 = 181{\cdot}5$ ft.-tons. Make $df = 188{\cdot}1$, and $eg = 181{\cdot}5$ ft.-tons to the same scale as used for mc. Join $afgb$.

The total B.M. at E $= ek + eg = ep$.
" " D $= dh + df = dq$.
" " M $= mc + mn = mr$.

Other points between a and r and b and p may be found similarly, and will lie on the curved boundary lines aq, qp, and pb, which complete the total B.M. diagram.

For the S.F. diagram, the total reaction at A $= 5{\cdot}7$ tons $+ \frac{1}{2}(60 \times \frac{1}{4}) = 13{\cdot}2$ tons, which equals the S.F. at A. From A to D the S.F. diminishes at the rate of 0·25 ton per foot, and therefore immediately to the left of D equals $+13{\cdot}2 - (33 \times 0{\cdot}25) = +4{\cdot}95$ tons. The load at D further reduces it by 6 tons, so that immediately to the right of D the S.F. $= +4{\cdot}95 - 6{\cdot}0 = -1{\cdot}05$ tons. From D to E it is reduced by a 12-ft. length of the distributed load of $\frac{1}{4}$ ton per ft., and immediately to left of E the S.F. $= -1{\cdot}05 - (12 \times 0{\cdot}25) = -4{\cdot}05$ tons. Immediately to right of E the S.F. $= -4{\cdot}05 - 12{\cdot}0 = -16{\cdot}05$ tons, which is still further reduced between E and B by

$(15 \times 0 \cdot 25) = 3 \cdot 75$ tons, giving $-16 \cdot 05 - 3 \cdot 75 = -19 \cdot 8$ tons, which checks with the reaction at B.

Bending Moment and Shear Force Diagrams for Balanced Cantilevers.—In applying the cantilever principle to bridges, one or other of the two arrangements in Fig. 26 is adopted.

Fig. 26, A, shows the elevation of a cantilever bridge in which a single cantilever girder, GH, rests on two piers at P and Q, whilst the outer ends GP and HQ of the girder project beyond the piers. The two remaining openings KG and HL are each bridged by an independent girder, one end of which rests on an abutment and the other on one of the projecting arms of the cantilever. The condition of loading in this case is as shown in the lower diagram, where the ends *g* and *h* of the projecting arms each carry a *concentrated* load equal to one-half

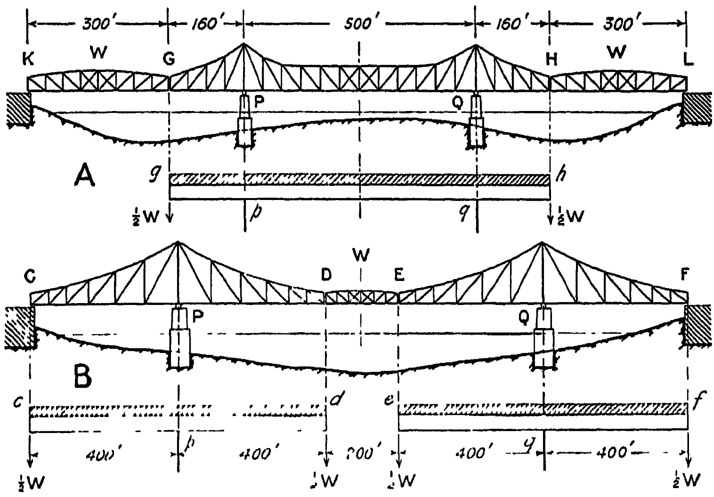


FIG 26

that on the independent girders, whilst the cantilever girder GH further supports its own weight together with the live load placed upon it.

In Fig 26, B, two cantilever girders CD and EF are employed, each supported at the centre by piers P and Q The central opening DE is bridged over by an independent girder resting on the cantilever arms at D and E. The tendency of the weight of this central girder to overbalance the cantilevers is counteracted by suitable balance weights or "kentledge" applied at the ends C and F. The condition of loading of the cantilevers CD and EF is then as shown in the lower diagram, where the arms *d* and *e* each carry one-half of the weight W of the central girder DE, and the opposite arms *c* and *f* each carry a similar amount of balance weight.

The method of drawing the B.M. and S.F. diagrams for the cantilever in case A is as follows. In this example the dead load alone will be considered. Suppose the bridge to have the dimensions indicated in Fig. 26, A, and the dead load to be 2 tons per foot run.

The weight of one of the detached spans KG or HL is 300 × 2 = 600 tons. One half this weight is concentrated on each end G and H of the cantilever, whilst the whole length of 820 ft. is further loaded with 2 tons per foot run.

In Fig. 27 the B.M. at P and Q due to the concentrated loads alone = 300 × 160 = 48,000 ft.-tons. Set off *pr* and *qs* each = 48,000 ft.-tons to scale. The B.M. at P and Q due to the distributed load on the overhanging arms GP and QH = (160 × 2) × 80 ft. = 25,600 ft.-tons. Set off *pa* and *qb* each = 25,600 ft. tons to the same scale as *pr* and *qs*. Since both these moments are acting simultaneously, the two diagrams *gpa* and *gpr* require combining in the same manner as in Fig. 25, by adding together their vertical depths at several points. The

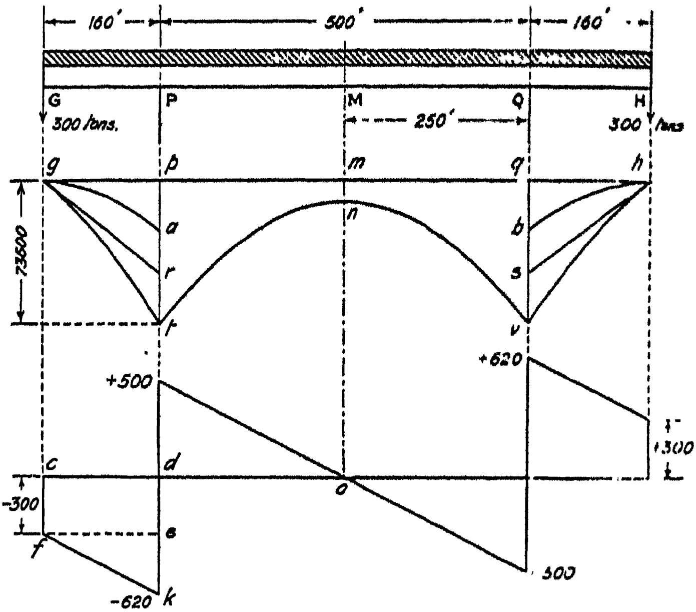


FIG. 27.

curve *gt* is the result, and the curve *hv* will be similar. The total moment at P or Q = −48,000 −25,600 = −73,600 ft.-tons = *pt*. The *total* load on the girder GH = 300 + 300 + (820 × 2) = 2240 tons. Since this load is symmetrically disposed, one-half or 1120 tons will be borne by each of the piers P and Q. Considering the right-hand half MH, the upward reaction at Q = 1120 tons. To obtain the B.M. at M the middle of the span, take moments about M. The downward acting moments = 300 tons × 410 ft. = 123,000 ft.-tons due to the concentrated load at H + (410 × 2) tons × 205 ft. (the distance of the c.g. of the distributed load on MH, from M) = 291,100 ft.-tons. The upward acting moment = 1120 tons × 250 ft. = 280,000 ft.-tons. The downward acting moment being the greater, the resultant moment at M = − 291,100 + 280,000 = − 11,100 ft.-tons. Set off *mn* =

11,100 ft.-tons, below the horizontal (since negative), and draw a parabola through points t, n, and v. The complete B.M. diagram for the cantilever is then the figure $gtnvh$, the moments being measured vertically below gh.

The S.F. immediately to the right of G is $-$ 300 tons. From G to P a further downward load of (160 × 2) tons is to be subtracted, giving $-300 - 320 = -620$ tons immediately to the left of pier P. At P an upward force of + 1120 tons is applied, giving $-620 + 1120 = +500$ tons immediately to the right of P. Between P and M this is gradually diminished by the downward acting load of (250 × 2) tons, giving $+500 - 500 = 0$ at M. Similarly the shearing force falls to -500 tons immediately to the left of Q, becomes $-500 + 1120$

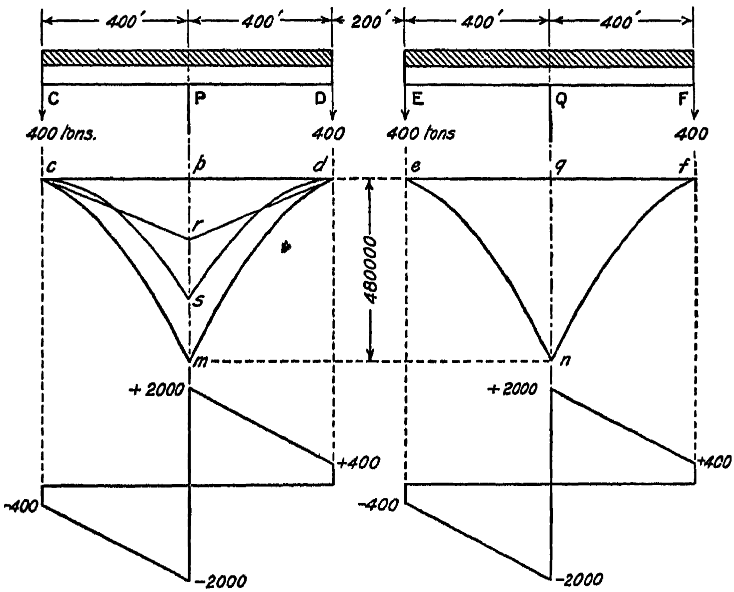


FIG. 28

$= +620$ tons to the right of Q, and falls to $+620 - 320 = +300$ tons immediately to the left of H, finally becoming $= +300 - 300 = 0$, at H, the point of application of the concentrated load of 300 tons. Note that the area $cdkf$ of the shear-force diagram between the free end of the girder and the section over the pier P, equals the B.M. at P.

$$\text{Thus area } cdef = 300 \times 160 = 48{,}000 \text{ ft.-tons,}$$
$$\text{area } efk = 320 \times \frac{160}{2} = 25{,}600 \quad \text{,,}$$

giving area $cdkf = 73{,}600$ ft.-tons, the value previously deduced for the B.M. at P.

Considering next the cantilevers in Fig. 26, B, their outline is reproduced in Fig. 28. Suppose the bridge to carry a dead load of

4 tons per foot run. The weight of the detached central girder = 200 × 4 = 800 tons. One-half this weight, or 400 tons, is applied as a concentrated load at D and E, and will require a balancing weight or downward pull applied by means of anchor ties at each of the points C and F.

The B.M. at P due to this concentrated load of 400 tons = 400 × 400′ = 160,000 ft.-tons. Set off *pr* = 160,000 ft.-tons and join *cr* and *dr*. The distributed load between C and P = 400 × 4 = 1600 tons, and its moment about P considering it concentrated at its *c.g.*, 200 ft. from P = 1600 × 200 = 320,000 ft.-tons. Set off *ps* = 320,000 ft.-tons, and draw the semi-parabolas *cs* and *ds*. Combining the two diagrams as before, the resulting curves are *cn* and *dn*. The maximum B.M. at P = *pr* + *ps* = 160,000 + 320,000 = 480,000 ft.-tons of *negative* moment. A similar diagram *efn* will, of course, obtain for the cantilever EF. For the shearing force, the total load on one pier = 400 (balance weight) + (800 × 4) distributed weight + 400 (half-weight of detached span) = 4000 tons = the upward reaction at P. Below C, set off − 400 tons. Between C and P a further downward acting load of (400 × 4) tons gives a total negative shear of − 400 − 1600 = − 2000 tons. The upward reaction of + 4000 tons converts this to + 2000 tons, which again diminishes to + 400 tons at D.

Cantilever Girder with Unsymmetrical Load. - Taking the dimensions of the cantilever bridge in Fig. 28, A, suppose the right-hand half of the bridge to carry an additional uniformly distributed load of 2 tons per foot run. This would correspond fairly closely with the case of two trains extending from the middle point M to the right hand abutment. The left-hand detached span KG weighs 600 tons as before, so that the end G of the arm PG carries a concentrated load of 300 tons. The right-hand detached span HL, together with its additional load of 2 tons per ft. run, now weighs 1200 tons, so that the end H of the arm QH carries a concentrated load of 600 tons. The cantilever girder GH also carries a load of 2 tons per foot run from G to M, and a load of 4 tons per foot run from M to H. These loads are indicated in Fig. 29. The loading being unsymmetrical, the reactions at P and Q will no longer be equal. To obtain the reaction at Q, take moments about the point P.

$$R_q \times 500' + 300 \times 160' + 320 \times 80' = 500 \times 125' + 600 \times 660' + 1640 \times 455',$$

from which R_q = 2262·2 tons. The total load on the *cantilever* = 300 + 600 + (410 × 2) + (410 × 4) = 3360 tons.

$$\therefore \text{Reaction } R_p = 3360 - 2262{\cdot}2 = 1097{\cdot}8 \text{ tons.}$$

The B.M. at P due to the concentrated load of 300 tons at G = 300 × 160′ = 48,000 ft.-tons. Set off *pr* = 48,000 ft.-tons and join *gr*. The B.M. at P due to the distributed load of 2 tons per foot extending over the arm GP = 320 × 80′ = 25,600 ft.-tons. Set off *pa* = 25,600 ft.-tons and draw the semi-parabola *ga*. Combining *ga* and *gr* the resultant curve *gt* is obtained, the negative moment at the pier P being = *pa* + *pr* = *pt*, or 25,600 + 48,000 = 73,600 ft.-tons. This portion of the diagram is identical with that in Fig. 27, since the loading on the arm PG has not been altered.

Proceeding similarly for the arm QH, the B.M. at Q due to the concentrated load of 600 tons at H $= 600 \times 160' = 96{,}000$ ft.-tons. This is set off to scale at *qs* and *hs* joined. The B.M. at Q due to the distributed load of 4 tons per foot run extending from Q to H $= 640 \times 80' = 51{,}200$ ft.-tons. This is set off to scale at *qb*, and the semi-parabola drawn from *b* to *h*. Combining *hb* and *hs*, the curve *hv* is obtained, the negative moment at the pier Q being $= qb + qs = qv$, or $96{,}000 + 51{,}200 = 147{,}200$ ft.-tons. The moments between P and Q will be represented by a parabolic curve from *t* to *v*.

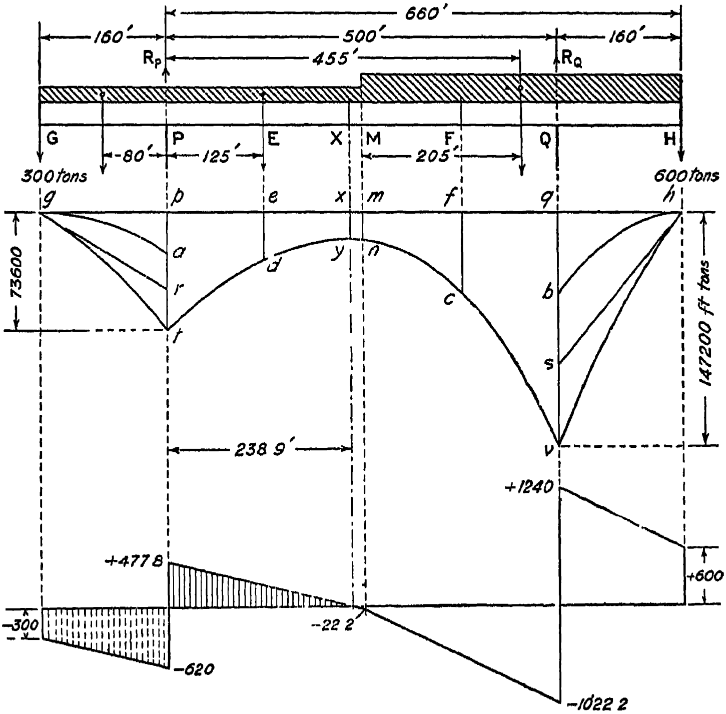


FIG. 29

If the position of the point *n* on this curve, midway between P and Q, be determined, the parabola may be drawn geometrically through points *t*, *n*, and *v*. The B.M. at M, calculating from the left-hand side, $= -(300 \times 410' + 820 \times 205')$ acting downwards $+ 1097{\cdot}8 \times 250'$ (the moment of R_P acting upwards) $= -16{,}650$ ft.-tons *negative* moment. Set off $mn = 16{,}650$ ft.-tons and draw the parabola *tnv*. The complete diagram of moments is then bounded by *gtnvh*. Note that if the moment at M had been positive, *mn* would have been set off *above* the horizontal *gh*, and the parabola *tnv* would cut *gh* and project above it.

The shearing force at G $= -300$ tons
immediately to left of P $= -300 - (160 \times 2) = -620$ tons
„ right of P $= -620 + 1097{\cdot}8 = +477{\cdot}8$ „
at M $= +477{\cdot}8 - (250 \times 2) = -22{\cdot}2$ „
„ left of Q $-22{\cdot}2 - (250 \times 4) = -1022{\cdot}2$ „
„ right of Q $-1022{\cdot}2 + 2262{\cdot}2 = +1240$ „
and at H $= +1240 - (160 \times 4) = +600$ „

which agrees with the concentrated load at the end of the arm QH. The maximum negative moments occur at the piers where the shear force diagram crosses the horizontal line. The *minimum* negative moment between P and Q occurs at X, vertically over the point where the shear force diagram crosses the horizontal a little to the left of M, and equals xy to scale. Its value may also be obtained from the area of the shear force diagram between G and X. Thus PX scaled or calculated from the S.F. diagram $= 238{\cdot}9$ feet. Then

$$\text{Negative area from G to P (dotted shading)} = -\frac{300 + 620}{2} \times 160$$
$$= -73{,}600 \text{ ft.-tons.}$$
$$\text{Positive area from P to X (full shading)} = +\tfrac{1}{2} \times 477{\cdot}8 \times 238{\cdot}9$$
$$= +57{,}073{\cdot}2 \text{ ft.-tons.}$$
$$\therefore \text{Minimum negative moment at X} = -73{,}600 + 57{,}073{\cdot}2$$
$$= -16{,}526{\cdot}8 \text{ ft.-tons.}$$

Bending Moment and Shearing Force Diagrams for Rolling Loads.—The following simple cases of rolling loads will illustrate the methods of drawing the B.M. and S.F. diagrams for concentrated loads which, rolling over a span, successively occupy a number of different positions upon it. Such cases occur when a locomotive crosses a bridge, or a crane traveller moves from one position to another on the crane girders. The concentrated loads are the weights supported by the various axles and wheels. The intervals between the loads remain constant, dependent upon the design of the locomotive or crane, etc. The treatment of rolling loads will be best understood by considering first the case of a single concentrated load rolling over a span.

Beam supported at both Ends and carrying a Single Concentrated Rolling Load.—Suppose the beam in Fig. 30 to be 60 feet span, and the load 6 tons. Divide the span into, say, six equal intervals of 10 feet, by lines I, II, III, IV, V. These indicate five successive positions of the load, and by calculating the B.M. for each position, five values will be obtained, which being plotted and connected by a curve, will give a diagram showing the B.M. for all intermediate positions of the load. Thus

Load at I, $R_A = 5$ tons, and B.M. at I $= 5 \times 10 = 50$ ft.-tons.

This is plotted to scale at $1 - 1'$ on the B.M. diagram, and $a1'b$ is the B.M. diagram for the load standing at position I on the beam. Load at II, $R_A = 4$ tons, and B.M. at II $= 4 \times 20 = 80$ ft.-tons. Plot $2 - 2' = 80$ ft.-tons. Load at III, $R_A = 3$ tons, and B.M. at III $= 3 \times 30 = 90$ ft.-tons, plotted at $3 - 3'$. Load at IV, $R_A = 2$ tons.

B.M. at IV $= 2 \times 40 = 80$ ft.-tons, plotted at $4 - 4'$. Load at V, $R_A = 1$ ton. B.M. at V $= 1 \times 50 = 50$ ft.-tons, plotted at $5 - 5'$.

The curve connecting these five points and the ends of the span includes *all* the possible B.M. diagrams for the load occupying *any* position on the span, and its height above the base line *ab* at any point therefore gives the B.M. at the instant the rolling load passes that particular point. The curve is a parabola, so that it is only necessary to calculate the B.M. $3 - 3'$ when the load is at the centre, and then draw in a parabola through the points $a3'b$. The *triangular* diagram

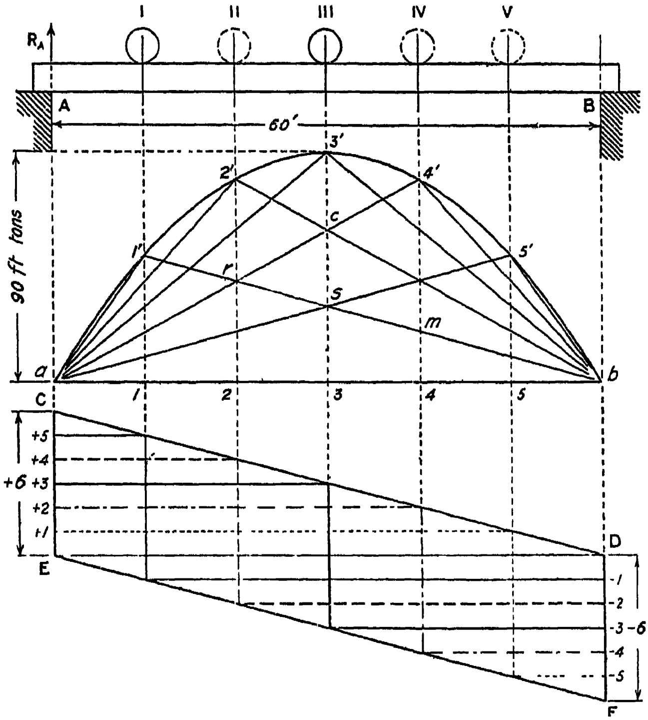


FIG 30

$a3'b$ is obviously the same as that for a load at rest on the centre of a beam, as in Fig. 17, and the height $3 - 3'$ of the parabola in this case is therefore equal to $\frac{Wl}{4}$. The parabola here is not to be confused with the parabola showing the B.M. for a *distributed* load *at rest.* Here, the parabola is the enclosing curve of a large number of individual triangular diagrams, and has no connexion whatever with the diagram for a distributed load.

The shearing force diagram is shown in the lower figure. With the load immediately to the right of point A, the whole 6 tons is carried by the abutment A, giving a shear of $+ 6$ tons at A, and nothing

at B. With the load at I, the shear at A equals the reaction of 5 tons, and at B it equals -1 ton, the diagram for this position being indicated by the thin full line. With the load at II, the shear at A $= +4$ tons, and at B $= -2$ tons, indicated by the long-dotted line. At position III, the shear at A $= +3$ tons, and at B, -3 tons, the diagram being shown by the heavy full line. With the load at IV the shear diagram is as shown by the chain-dotted line, and at V by the short-dotted line. The lines CD and EF, enclosing all the possible shear diagrams as the load changes its position, give the maximum positive and negative shearing forces for every position of the load. The shearing forces are scaled off *above* or *below* the horizontal base line ED according as the shear to *left* or *right* of the load is required, and *not* by scaling the distance between CD and EF, which is of course everywhere equal to 6 tons.

The following distinction between B.M. and S.F. diagrams for stationary and rolling loads should be carefully noted. For a stationary load the diagrams indicate the B.M. and S.F. existing simultaneously at every point along the beam due to the particular position of the load, which position is permanent. The rolling load diagrams are in every case enveloping diagrams, such that the particular B.M. or S.F. diagram consequent upon any specified position of the rolling load may be readily obtained from them by projection. Thus in Fig. 30, the parabola $a1'2'3'4'5'b$ does not mean that moments $= 1 - 1'$, $2 - 2'$, $3 - 3'$, etc., exist simultaneously, but that when the load is in position I, the bending moment at section I $= 1 - 1'$, at section II $= r2$, at section III $= s3$, and at section IV $= m4$. Similarly when the load is in position IV, the B.M. at section IV $= 4 - 4'$, but at section II the B.M. $= r2$, *not* $2 - 2'$. With load at V, the B.M. at section V $= 5 - 5'$, and at section III $= s3$. With regard to the shearing force, when the load is in position IV, the shearing force diagram is as shown by the chain-dotted line, obtained by projecting point IV on to the lines CD and EF, and ruling in the horizontals through $+2$ and -4. Thus for this position of the load the shear from A to IV is constant and $= +2$ tons, whilst from IV to B it is constant and $= -4$ tons.

In order to ascertain the B.M. at *any* point on a beam due to a concentrated rolling load at any *other* point, the following simple method obtains. Suppose, in Fig. 30, the load to be at position IV, and the B.M. is required at II and III. Project point IV on to the enveloping parabola, and join $4'$ to a, cutting verticals through II and and III in r and s. The vertical heights $2r$ and $3s$ give the required moments at sections II and III respectively. The application of this method will be necessary in constructing the diagrams for two or more rolling loads.

Beam supported at both Ends and carrying Two Unequal Concentrated Rolling Loads at a Constant Distance apart. Suppose the beam in Fig. 31 to be 50 feet span, the loads 5 tons and 10 tons separated by a distance of 10 feet, and that they roll over the span from left to right with the 5 ton load leading. Such a case represents, for instance, the passage of a 15-ton traction engine over a 50-foot span. Consider the loads as occupying a number of successive positions. In the figure, ten positions distant 5 feet apart have been taken.

The corresponding positions of the two loads for each 10 ft. of the span are indicated by circles of distinctive lining. When two concentrated loads roll over a span, the maximum B.M. is found to occur under the heavier load for certain positions of the loads, but under the lighter load for certain other positions. It is consequently necessary

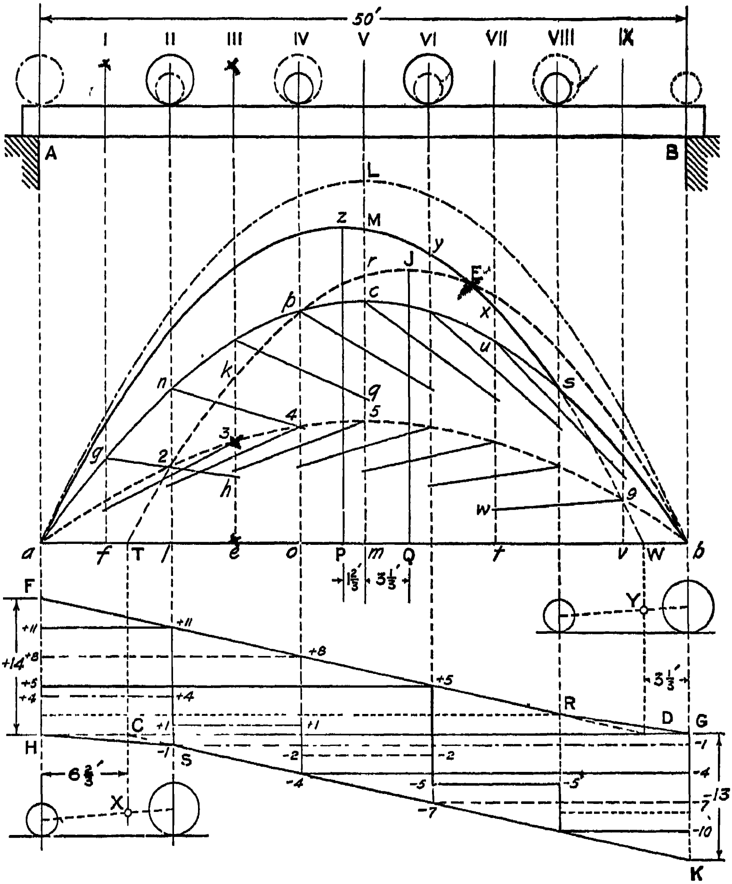


FIG. 31.

to trace out the bending moments which occur beneath each load, two distinct curves being obtained, an inspection of which at once indicates the positions of the loads which give rise to the maximum moments, as also the value of the moments. First draw the two parabolas *acb* and *a5b*, representing the moments due to each load rolling over the span separately. For the 10 tons load, $mc = \frac{10 \times 50}{4} = 125$ ft.-tons, and for the 5 tons load $m5 = \frac{5 \times 50}{4} = 62\frac{1}{2}$ ft.-tons. Next consider the

loads approaching the span from the left hand. The 5 tons load rolls over the first interval of 10 ft. before the 10 tons load reaches the span. The arc *a*2 therefore constitutes the curve of moments under the smaller axle for the interval A II. As the smaller load passes beyond section II the larger load also rolls on to the span and causes additional B.M. at the section occupied by the smaller load. When the smaller load is at section III, the B.M. at III due to its *own* weight = *e*3. The larger load is then at section I, 10 ft. behind. Here, it *alone* creates a moment = *fg*. Joining *g* to *b*, the height *eh* cut off on the vertical through III (the position of the leading load) gives the additional moment at section III due to the 10 tons load at section I. The total moment at section III due to *both* loads then = *e*3 + *eh*. Making 3*k* = *eh*, *ek* represents this total moment, and the curve is sketched from 2 to *k*. When the smaller load arrives at section IV, the moment due to its own weight alone = *o*4. The moment at section II, ten feet behind, due to the larger load = *ln*. Joining *n* to *b*, the height *o*4 cut off on the vertical through IV gives the additional moment at IV due to the 10 tons load at II. Making 4*p* = *o*4, *p* gives another point on the curve. Repeating the construction for the remaining sections, *m*5 = moment due to smaller load at V, and *mq* the additional moment at V due to larger load at III. Adding *m*5 and *mq*, point *r* is obtained, and so on. The curve *a*2*kprb* shown by the heavy dotted line indicates the bending moments which occur under the smaller load during the complete transit of both the loads across the span.

A similar construction is now made for ascertaining the moments which occur under the larger load. If the loads be assumed to run back over the span from right to left, the larger one leading, the construction is identical, but is worked in the reverse direction. Thus for the first 10 ft. from B to VIII, only the 10 tons load is on the span, and the curve from *b* to *s* gives its moments. When the larger load arrives at VII, its own moment = *tu* and the moment of the smaller load at IX = *v*9. Joining 9 to *a*, *tw* is the additional moment at VII due to the smaller load at IX, which being added to *tu* at *ux* gives another point along the curve from *s*. Repeating the construction, the curve *bsxyza*, shown by the heavy full line, indicates the bending moments under the heavier load during the transit of the two loads. The two curves intersect at E, and it is at once seen that the maximum moment occurs under the 10 tons load from *a* to E; but from E to *b*, the greater moments occur under the 5 tons load as shown by the dotted curve from E to *b* falling outside the full curve E*sb*. The complete curve of maximum moments is therefore *az*E*b*. If the loads are free to pass over the span in the reverse direction, that is from left to right with the *larger* load leading, the curve *az*E*b* would be reversed left for right, when the branch to the right of the centre line *m*M would be similar to the left-hand branch *az*M. This will generally be the case in practice.

If the two loads be added together and considered as a single concentrated load of 15 tons, the outermost curve *a*L*b* would represent the bending moments due to such a load rolling over the span. Its central height $m\text{L} = \frac{15 \times 50}{4} = 187\frac{1}{2}$ ft.-tons. The height of this

curve will not greatly exceed the height Pz of the actual curve of moments, provided the distance between the two loads is small compared with the span. In that case the curve aLb is sometimes substituted for the real curve of moments having two halves each similar to azM.

The S.F. diagrams are drawn in the lower figure corresponding to the positions II, IV, VI, VIII and B of the smaller load, the same type of line being employed as used for the circles denoting the positions of the loads in the upper figure. With the larger load at A and the smaller at II, the reaction and therefore the shear at A $= 10$ tons $+ \frac{40}{50}$ of 5 tons $= + 14$ tons. The S.F. diagram for this position is indicated by the chain-dotted lines. HF $= + 14$ tons, which immediately to the right of A is reduced by 10 tons, giving $+ 4$ tons, which remains constant between A and II. At II the shear is further reduced by 5 tons, giving $+ 4 - 5 = - 1$ ton, which remains constant from II to B. The other diagrams will be readily traced, remembering that the shear at A is, in every case, equal to the reaction at the support due to any position of the loads. Thus taking smaller load at VIII and larger at VI, indicated by the heavy line, the shear at A $= \frac{20}{50}$ of 10 tons $+ \frac{10}{50}$ of 5 tons $= + 5$ tons, which is constant from A to VI, then reduced by 10 tons, giving $- 5$ tons up to VIII, where it is again reduced by 5 tons, giving $- 10$ tons from VIII to B. The lines FRG and HSK enclosing all the possible shear diagrams complete the figure. It should be noticed that the boundary lines are broken at R and S distant 10 ft. or the load interval from the ends of the span. Also that if KS and FR be produced to meet the horizontal base line HG in points C and D respectively, the points C and D fall at distances of $6\frac{2}{3}$ and $3\frac{1}{3}$ ft. respectively from H and G. These distances are the segments into which the load interval of 10 ft. is divided by the centre of gravity of the two loads, and the loads are indicated by circles in the lower figure, in positions for projecting the points C and D on to the base line. It will be seen the loads are in the reverse position to that in which they were supposed to cross the span. The shear force diagram may be expeditiously drawn by setting off the reactions HF and GK, placing the loads in the positions indicated and projecting the centres of gravity X and Y to C and D respectively. By joining FD and RG, KC and SH, the complete diagram is obtained without drawing the individual shear diagrams. The end shears are unequal, being $+ 11$ and $- 13$ tons, since the loads are unequal. If, however, the loads may cross the span in the reverse order, the shear force diagram would be inverted. In such a case, which commonly occurs in practice, the larger portion of the shear diagram—here, the upper portion—would be employed on each side of the horizontal base line HG.

If X and Y be projected to T and W on the base line ab of the B.M. diagram, these points coincide with the ends of the parabolas aEW and TEb. The vertical centre lines of these parabolas, on which the maximum moments under each load are measured, are therefore displaced $1\frac{2}{3}$ ft. and $3\frac{1}{3}$ ft. respectively to left and right of the centre line mM of the span. The maximum moment under the 10 tons load $=$ Pz $= 163{\cdot}3$ ft.-tons and under the 5 tons load $=$ QJ $= 140{\cdot}8$ ft.-tons.

Beam supported at Both Ends and carrying Two Equal Concentrated Rolling Loads at a Constant Distance Apart. This is a special case of the last example. In Fig. 32 a span of 50 feet is taken with two equal loads of 10 tons each, separated by a distance of 20 feet. The construction is similar to that in Fig. 31. The parabola *acb* is first drawn having $mc = \frac{10 \times 50}{4} = 125$ ft.-tons. This serves for both

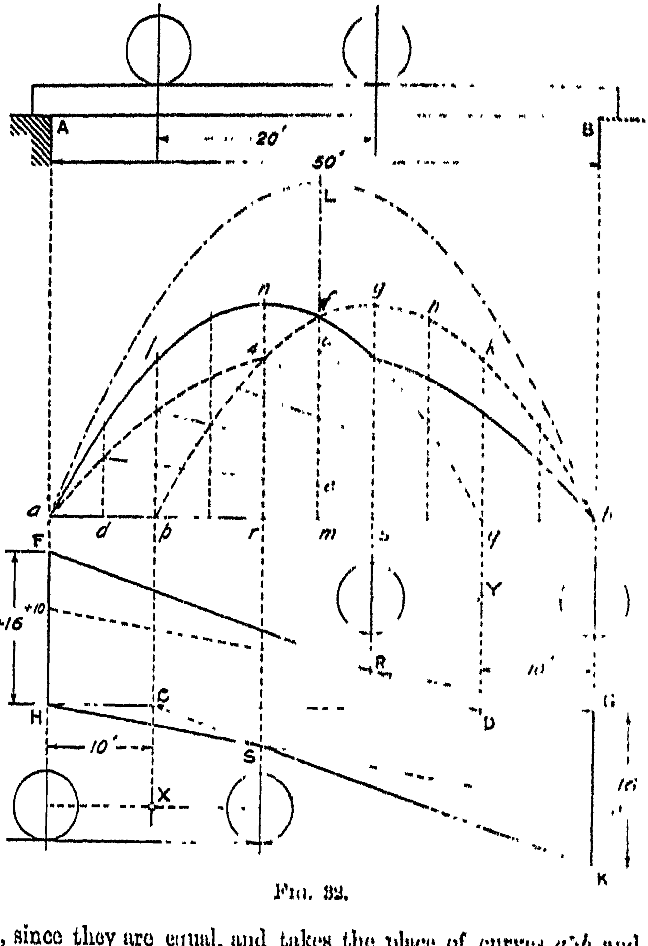


FIG. 32.

loads, since they are equal, and takes the place of curves *a5b* and *acb* in Fig. 31. Assuming the loads to roll over from left to right, the curve *a4* gives the moments under the leading load before the following load comes on to the span. With leading load at *m* and following load at *d*, the additional moment $= me = cf$, giving the total moment $= mf$. Other points *g*, *h*, and *k* are obtained similarly, and the curve *a4ghb* gives the maximum moments under the leading load. The curve *alng*, giving the moments under the following load, will be

similar and equal since the loads are equal. The total moments for either direction of transit are given by the curve *alnfgkb*. The outermost curve *a*L*b*, having $m\text{L} = \frac{20 \times 50}{4} = 250$ ft.-tons, indicates the moments which would result from adding the two loads, and treating them as a single concentrated load of 20 tons, which in this case would be inadmissible, since *m*L greatly exceeds the actual maximum bending moment, *rn* or $sg = 160$ ft.-tons. This is due to the load interval of 20 feet being so large relatively to the span. For the shearing force diagram, the maximum end shears will occur when one load is just over the abutment, and the other slightly more than 20 feet along the span, so that FH and GK each $= 10$ tons $+ \frac{30}{50}$ of 10 tons $= 16$ tons. Placing the loads in the extreme positions, with centres of gravity at X and Y, points C and D are obtained by projection and the boundaries FRG and HSK of the shear force diagram determined. Note that if GR and HS be produced to the opposite abutments, they cut off heights representing $+10$ and -10 tons respectively.

The preceding method becomes very tedious if the number of axle loads exceeds four or five, and the following construction is applicable to any number of concentrated loads, or to a system of concentrated and distributed loads combined. Fig. 33 shows the application of the method to the case of the Atlantic engine and tender, indicated by the axle loads in Fig. 33, rolling over a span of 60 feet. AB represents the span of 60 feet, and $w_1, w_2, w_3 \ldots w_8$ the positions of the axle loads when the engine is standing with the leading bogie axle over the right-hand abutment B. Multiplying each load by its distance from A, the following moments are obtained :—

$w_1 \times 60$ feet 0 inches	$= 8{\cdot}5 \times 60$	$= 510$	ft.-tons	$= \text{A}a$
$w_2 \times 53$,, 9 ,,	$= 8{\cdot}5 \times 53{\cdot}75$	$= 456{\cdot}8$	,,	$= ab$
$w_3 \times 48$,, 6 ,,	$= 18 \times 48{\cdot}5$	$= 873$	,,	$= bc$
$w_4 \times 41$,, 8 ,,	$= 18 \times 41{\cdot}66$	$= 750$	,,	$= cd$
$w_5 \times 33$,, 8 ,,	$= 13 \times 33{\cdot}66$	$= 437{\cdot}6$	,,	$= de$
$w_6 \times 21$,, 7 ,,	$= 11{\cdot}5 \times 24{\cdot}58$	$= 282{\cdot}7$	,,	$= ef$
$w_7 \times 18$,, 1 ,,	$= 11{\cdot}5 \times 18{\cdot}08$	$= 262{\cdot}2$	,,	$= fg$
$w_8 \times 11$,, 7 ,,	$= 15 \times 11{\cdot}58$	$= 173{\cdot}7$	,,	$= gh$

These values are set off to any convenient scale on the vertical line A*h* and *h*B joined. The total length A*h* represents the upward moment about A, of the reaction at B, due to all the loads in the position indicated. Consequently, any intermediate ordinate as *kl* represents the moment of the reaction at B about the point *k*. Next, join *a*B, project w_2 to *m* on *a*B and join *bm*. Project w_3 to *n* on *bm* and join *cn*, and w_4 to *o* on *cn* and join *do*. Repeating this construction until the last load w_8 has been utilized, the broken line B*mnopqrsh* is obtained, which is such that the *vertical* ordinates intercepted between it and the base line *h*B give the bending moments at any point of the span for the axle loads in the given position. This may be proved as follows :—

Ah = total reaction at B $\times$ AB, $\therefore xt = \text{R}_\text{B} \times \text{B}x$, since triangles BA*h* and B*xt* are similar.

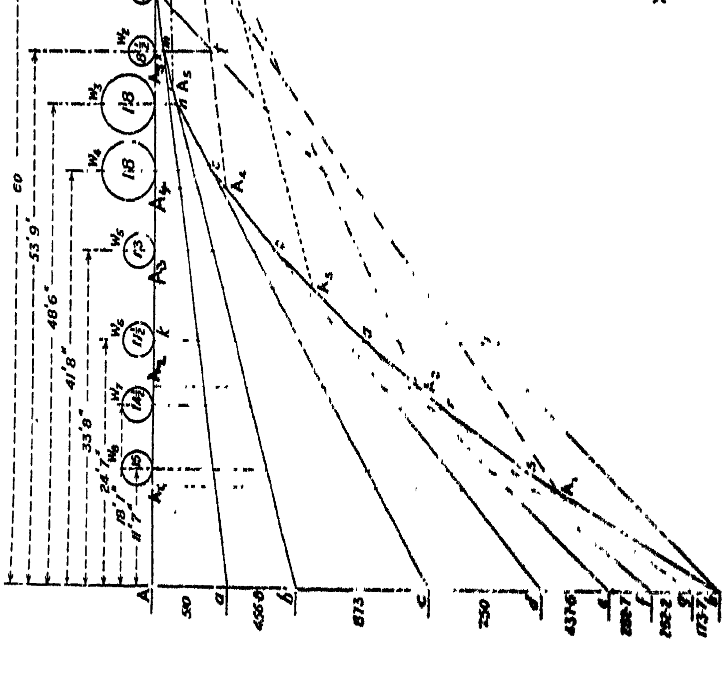


Fig. 33.

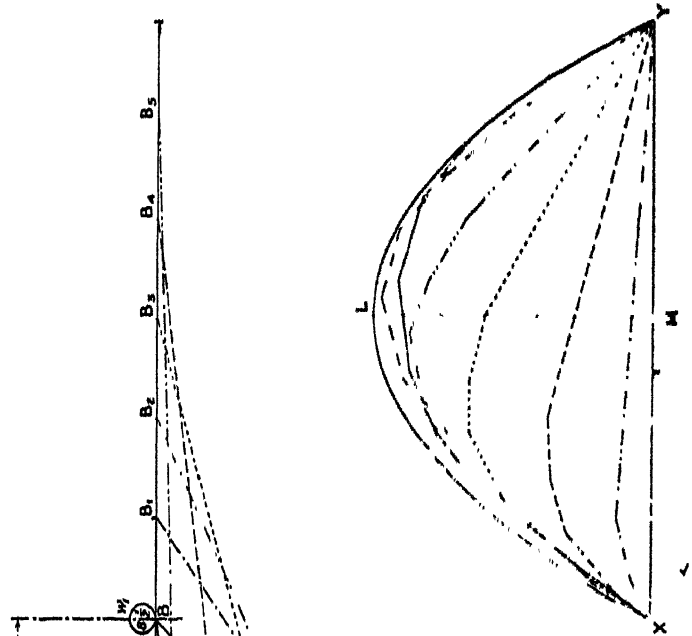


Fig. 34.

$Aa = w_1 \times AB$, $\therefore xm = w_1 \times Bx$, since triangles B$Aa$ and Bxm are similar.

But $mt = xt - xm = R_B \times Bx$ (upward moment) $- w_1 \times Bx$ (downward moment) = bending moment at section x.

Similarly at any section k, kl = upward moment $R_B \times Bk$, and kq = sum of downward moments of $w_1, w_2, w_3 \ldots w_5$ about k, whence $kl - kq = ql$ = bending moment at k.

The moments under each axle for this position of the loads are transferred to the horizontal base line XY in Fig. 34, being indicated by the full line. (The vertical scale of moments for Fig. 34 is twice that of Fig. 33.) If now the loads be supposed to roll backward from B towards A, or what amounts to the same thing, the span AB be supposed to move forward towards the right, the loads meantime remaining stationary, a new position A_1B_1 of the span may be assumed 10 feet (or other convenient distance) in advance of AB. The *vertical* ordinates between $A_1srqponm$BB_1 and the new base line A_1B_1 now give the bending moments under the various axles for this new position of the loads. These are also transferred to the common base line XY, being indicated by the single chain-dotted line corresponding with A_1B_1. Other positions of the span, A_2B_2, $A_3B_3 \ldots A_5B_5$, each 10 feet in advance of the preceding one, are taken, and the bending moments transferred to XY, Fig. 34, the resulting moment diagrams being indicated by similar lining to that adopted for the corresponding base lines in Fig. 33. By moving the span to the *left* of the loads it may readily be ascertained if any greater moments are created than those already plotted in Fig. 34. In this case no greater moments occur than those plotted, and the maximum moments are those indicated by the outer limits of the overlapping diagrams of Fig. 34. If an enveloping parabola XLY be drawn just including these diagrams, such a parabola will constitute the B.M. diagram due to a certain *distributed* load of w tons per foot run which may be substituted for the actual axle loads considered. The central height LM of this parabola scales 900 ft.-tons.

$$\text{Hence } \frac{wl^2}{8} = \frac{w \times 60 \times 60}{8} = 900, \therefore w = 2 \text{ tons.}$$

That is, the *equivalent distributed load* for this type of locomotive for a span of 60 feet is 2 tons per foot run.

The equivalent distributed load which will create at least the same *shearing force* as the concentrated axle loads when placed in any position on the span will be somewhat higher than that deduced from the bending moments Since, however, the shear force diagram for any position of the axles is simply constructed, the maximum shear at any point of the span is easily obtained after one or two trials.

When the load consists of a number of concentrated axle loads followed by a distributed load of given intensity, the following modification of the construction in Fig. 33 is to be observed.

In Fig. 35 four axle loads of 16 tons each are followed, after a 10 feet interval, by a distributed load of 1·6 tons per foot run, the span being 50 feet. Placing the leading axle over B, the preceding construction is repeated, but the distributed load is first treated as a load

of $25 \times 1.6 = 40$ tons, concentrated at its centre of gravity, 12·5 feet from A. The resulting B.M. diagram is then *abcd* . . . B. Project C to *c* on *bd*, and join *ac*. Since the load between A and C is actually distributed, bisect *bc* in *m*, and draw in a parabolic arc through *amc*. The required B.M. diagram is then *amcd* . . . B, which is utilized after the manner of Fig. 33, for determining the varying moments as the loads roll back from B towards A. The remainder of the construction being similar to that of the last example is here omitted.

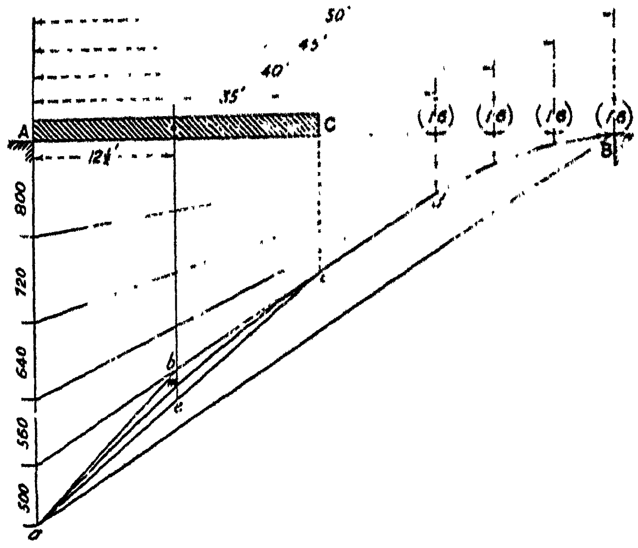


FIG. 35.

Beam supported at both Ends, and carrying a uniformly distributed Rolling Load.—This case is one which occurs very frequently in practice, corresponding closely with that of the passage of a train over a bridge span. A widely followed mode of procedure in such cases is to substitute for the actual loads on the different axles of the train, a uniform load per foot run sufficiently large to cause at least the same bending moments at every point of the span as would be caused by the concentrated axle loads. This materially reduces the labour involved in drawing out the curves of moments by the method just described, by substituting one parabola enveloping the several curves obtained by the former construction.

In Fig. 36, a span of 50 feet is taken with a load of one ton per foot run advancing over the span from left to right. When the load covers the length A–I, the B.M. diagram is *a*1*b*. As the load successively covers the lengths A–II, A–III, and A–IV, the corresponding moment diagrams are *a*2*b*, *a*3*b*, and *a*4*b*. These diagrams are obtained by the method shown in Fig. 24. When the load covers the whole span from A to B, the B.M. diagram becomes the parabola *acb*, having a central height $mc = \frac{1}{8} \times 1 \times 50 \times 50$

= 312·5 ft.-tons. The bending moment therefore gradually increases as the load advances and the maximum moments occur when the span is fully loaded, their value then being the same as for a stationary load of the same intensity covering the whole span. The B.M. remains constant so long as the moving load covers the whole span, but gradually diminishes as the tail end of the load rolls off towards B.

With AI only covered by the load (10 tons), the reactions at A

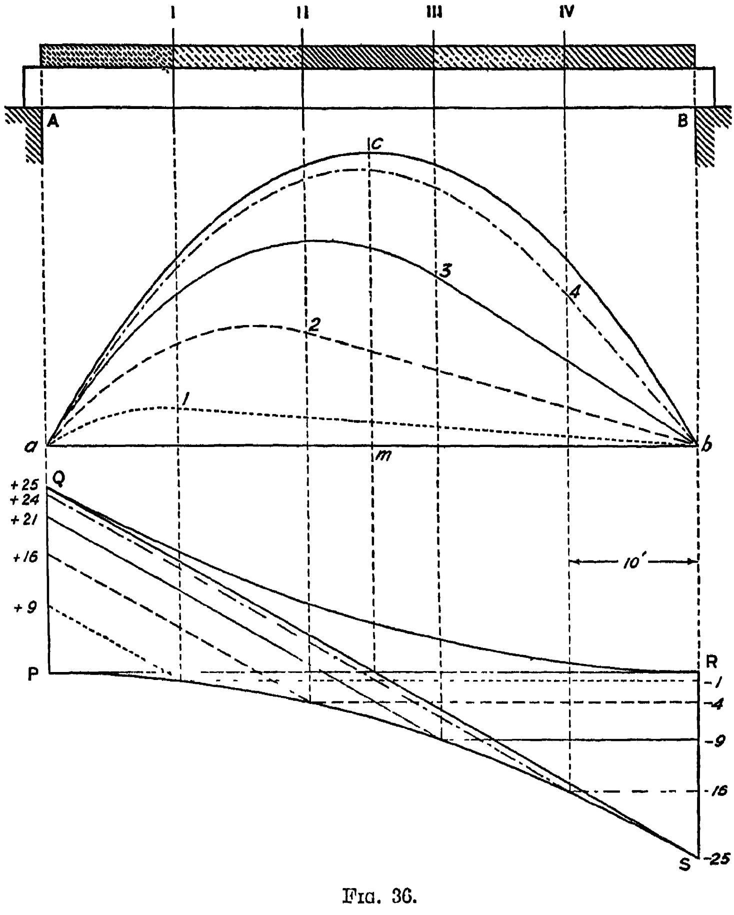


FIG. 36.

and B are respectively 9 tons and 1 ton, and the shear force diagram is obtained by drawing a horizontal through -1 as far as vertical section I, and then an inclined line upwards to $+9$. (See Fig. 24.) In the upper figure the sectional shading showing the positions to which the head of the load has advanced is drawn in distinctive lines corresponding with those representing the B.M. and S.F. for those positions. With load from A to II (20 tons), the reactions at A and B

are respectively 16 and 4 tons, and the corresponding S.F. diagram is shown by the horizontal through − 4, and inclined line to + 16. The three remaining diagrams from − 9 to + 21, − 16 to + 24, and − 25 to + 25 will readily be traced from the reactions in a similar manner. The curve PS enveloping these five diagrams will include all other possible shear diagrams for any intermediate positions of the load. Its outline is a semi-parabola touching the base line PR at P, since the depths 1, 4, 9, 16, and 25 at equal horizontal intervals equal the squares of the numbers 1, 2, 3, 4, 5. A similar curve, QR, indicates the shearing force as the tail end of the load rolls *off* the span from A to B, or as the load rolls *on* to the span from B towards A. The four intermediate diagrams in this case are omitted. It will be seen that the shearing force diagram may be rapidly drawn by setting off PQ and RS, each equal to half the total load required to cover the whole span, and inserting the semi-parabolas PS and QR by the geometrical method.

Continuous Beams.—A beam or girder is said to be continuous when it bridges over more than one span, and rests on one or more intermediate supports in addition to the two end supports. A continuous beam bends in the manner shown in Fig. 37. From A to a

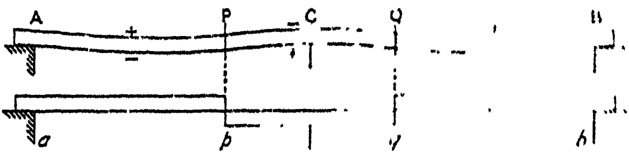


FIG. 37.

certain point P, the upper surface is concave and in compression whilst the lower surface is convex and in tension. From P to Q the curvature and bending are reversed, the upper portion being in tension and the lower in compression. At Q another reversal of bending takes place, with a corresponding change in the character of the stresses. The points P and Q are known as *points of contra-flexure*, and at these points the *bending moment is nothing*. The position of these points depends on the character of the loading and section of beam. The continuous beam ACB is equivalent to a system of two simple beams *ap* and *qb*, and a cantilever *pq*, as indicated in the lower figure, the points P and Q being projected to *p* and *q*, fixing the lengths of the equivalent beams and cantilever. The stresses in the lower combination of simple beams are then identical with those in the continuous girder alone. It is apparent from the lower figure that the pressures on the various supports differ considerably from those which would obtain if the two openings were spanned by separate beams with an interval at C. Thus, if the load be supposed uniformly distributed from A to B, the support at A will carry half the load on the length AP, and the support at B will carry half the load on the length QB. The central support will carry the whole of the load on PQ, together with the remaining halves of the loads on AP and QB. In the case of two *independent* spans AC and BC, the central support

would carry just half the total load from A to B, and each end support one quarter of the total load. The central support beneath the continuous beam thus carries a greater proportion of the total load than if supporting two simple beams only. When the positions of the points of contra-flexure are determined, the B.M. at any section of the beam, and the pressures on the supports, may readily be deduced by reference to the equivalent system of simple beams and cantilevers.

The following solutions, which illustrate typical cases, are based on the assumptions that the supports are all at the same level and the beam of uniform cross-section. They are therefore *strictly* applicable to rectangular timber beams, rolled sections used as beams, and plate girders of uniform section, whilst the results will be *approximately* correct for the generality of plate girders. The exact solution for beams of very variable section is considerably involved, and beyond the scope of this work. *It should be distinctly remembered that a slight subsidence in one or another of the supports of a continuous girder may considerably modify the bending moment, and consequently the stress, at any section.* Thus in Fig. 37, lowering of the central support C would throw more of the load on the end supports A and B and shorten the length of the convex portion PQ. The lengths *ap* and *bq* of the equivalent simple beams would be thereby increased with a corresponding increase in the bending moment upon them. Conversely, subsidence of A or B or both, would increase the pressure on C and cause the points P and Q to move outwards from the centre, with a corresponding increase in the bending moment at the centre of the cantilever *pq*. Variation in the loading further causes alteration in the positions of the points P and Q, and therefore in the bending moment also.

In Fig 38, if the span BC carry a much heavier load than the span AC, the effect is to cause relatively large deflection of BC and to spring

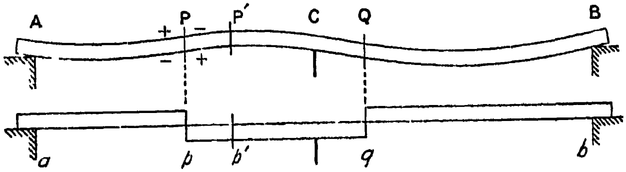


FIG 38.

up the length AC. The points of contraflexure P and Q would then move towards the left and the equivalent system of simple beams and cantilever be as shown at *apqb*, with correspondingly modified bending moments. Such action takes place in the case of a continuous girder bridge with the live or rolling load advancing from one end support. If P′ denote the previous position of P when the girder was symmetrically loaded, it will be seen that the upper and lower flanges of the girder between P and P′ undergo reversal of stress. In Fig. 37 the upper flange from A to P is in compression under symmetrical loading, whilst in Fig. 38, a portion of this flange, PP′, is now in tension under

the unsymmetrical load. The stress in the lower flange between P and P′ is also reversed from tension to compression.

For the above reasons, continuous girders are not very frequently adopted, especially for moving loads. The reversal of stress over a certain length of the girder may be suitably provided for, but a relatively slight alteration in the level of the supports, after erection, gives rise to unknown and possibly dangerous stresses. Several bridges, in fact, originally erected as continuous girders have, on account of unequal subsidence in the piers, been cut through in the neighbourhood of the points of contra-flexure and so converted into actual simple beams and cantilevers, the stresses in which are independent of slight differences in level of the supports.[1] As will be seen, however, a continuous girder is more economical of material than several independent spans together aggregating the same length, since the continuous girder under similar loading is subject to less bending moment and may therefore be designed of lighter section. It further possesses certain advantages relative to ease of erection in lofty situations, which, however, cannot be here considered in detail.

Characteristic Points of Bending Moment Diagrams. The bending moments on continuous beams of uniform section may be readily determined after finding what are called the *characteristic points* of the simple bending moment diagrams for each span considered independently. These characteristic points are obtained as follows. In Fig. 39 let AB represent the span, and the parabola ACB the B.M. diagram due to a uniformly distributed load. Divide the span AB into three equal parts in points E and F, and at these points erect perpendiculars EP and FQ. The height of points P and Q above the base line AB is fixed by the condition that the moment of the area ACB about one end of the span shall equal the moment of the rectangle AKHB about the same point, or area ACB $\times \frac{AB}{2}$ = area AKHB $\times \frac{AB}{2}$.

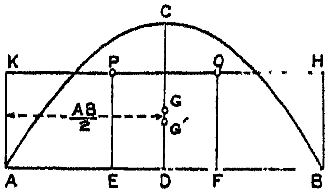


FIG. 39.

$\frac{AB}{2}$ being a common factor, may, in this case, be eliminated, leaving area ACB = area AKHB, and this condition is sufficient for fixing the heights of P and Q in all cases where the B.M. diagram is symmetrical about the vertical centre line, since G and G′, the centres of gravity of areas ACB and AKHB will both fall on CD, and the moment arm for each area will equal $\frac{AB}{2}$, or the half span. In the case of the parabola—

$$\text{Area } ACB = \tfrac{2}{3}\,CD \times AB, \text{ and area } AKHB = EP \times AB$$
$$\therefore EP \times AB = \tfrac{2}{3}\,CD \times AB, \text{ or } EP = FQ = \tfrac{2}{3}\,CD$$

The characteristic points for a parabolic diagram are therefore

[1] *Mins. Proceedings Inst. C. E.*, vol. clxii. p 245.

obtained by erecting perpendiculars at each $\frac{1}{3}$ of the span, and cutting off a height equal to $\frac{2}{3}$ the central height of the parabola.

In Fig. 40, ACB is the B.M. diagram for a concentrated load at the centre of span AB. The triangular area ACB, and the rectangle AKHB, will obviously be equalized by drawing KH at half the height CD. The characteristic points P and Q are therefore here situated at one-half the central height CD above the base. Fig. 41 illustrates another

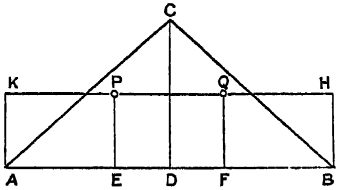


FIG 40

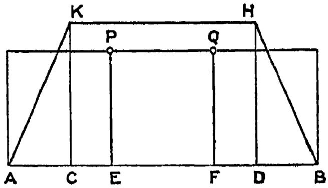


FIG. 41.

symmetrical case. The span AB is 20 feet, and equal loads of 6 tons are carried at 4-feet distances from each end. Reactions at A and B each equal 6 tons, and moments CK and DH = 6 × 4 = 24 ft.-tons, AKHB being the B.M. diagram. Its area, measured by the scales used for distance and bending moments, = AD × DH = 16 × 24 = 384. The height of the rectangle of equal area on base AB = $\frac{384}{20}$ = 19·2 units on the B.M. scale. Marking E and F at the one-third points of the span, the characteristic points are located at P and Q.

Characteristic Points for Unsymmetrical Bending Moment Diagrams.—In Fig. 42, AB = 30 feet, and a concentrated load of 15 tons is applied at D, distant 6 feet from B.

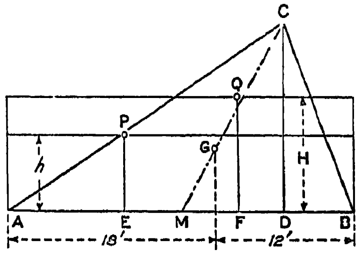


FIG. 42.

$R_A = \frac{6}{30}$ of 15 = 3 tons, the bending moment at D = 3 × 24 = 72 ft.-tons, and the B M. diagram is the triangle ACB, having its centre of gravity at G, such that MG = $\frac{1}{3}$ MC. The moment of area ABC will now be greater about the end A of the span than about B, and the characteristic points will no longer be at the same height above AB. Taking moments about A, and calling H the height of the rectangle, having the same moment about A as the triangle ACB—

$$\text{Area ABC} \times 18' = H \times 30' \times 15'$$

$$\text{or } \frac{30 \times 72}{2} \times 18 = H \times 450$$

$$\text{whence } H = 43{\cdot}2$$

which fixes the height of the characteristic point Q. Taking moments about B, and calling h the height of the rectangle of equal moment—

$$\frac{30 \times 72}{2} \times 12 = h \times 450$$

$$\text{whence } h = 28{\cdot}8$$

which fixes the height of the characteristic point P. Any unsymmetrical B.M. diagram may be treated in a similar manner.

Method of using the Characteristic Points for determining the Bending Moment on Continuous Girders.—*General Example.*—A girder is continuous over three spans of 30, 40, and 30 feet, and carries a load of 2 tons per foot run over the first span, and 1·5 tons per foot run over the second and third spans. To draw the B.M. and S.F. diagrams and determine the points of contra-flexure and pressures on the supports.

In Fig. 43 set out the spans AB, BC, and CD to scale, and upon them draw the B.M. diagrams for the stated loads, assuming the spans

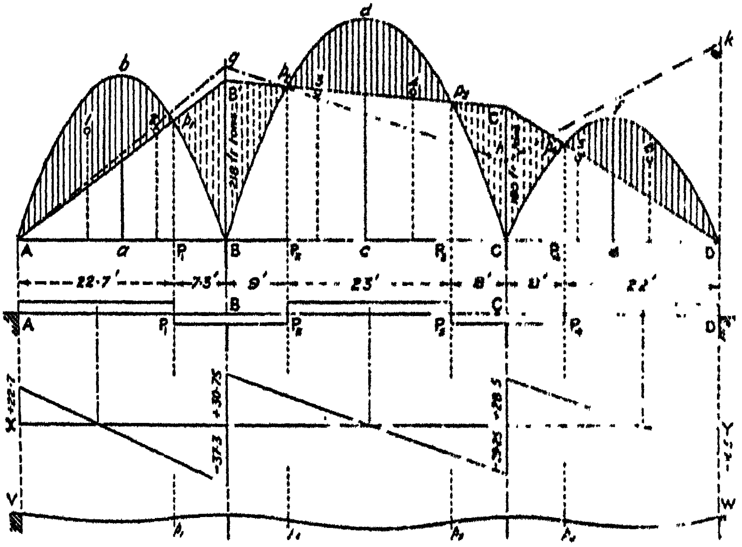


FIG. 43.

to be bridged by three independent girders instead of one continuous girder. The central bending moments for the three spans are respectively—

$$\frac{2 \times 30 \times 30}{8} = 225 \text{ ft.-tons}, \quad \frac{1·5 \times 40 \times 40}{8} = 300 \text{ ft.-tons}$$

and $\frac{1·5 \times 30 \times 30}{8} = 168·75$ ft.-tons. These values are set off to a convenient scale at *ab*, *cd*, and *ef*, and the parabolas A*b*B, B*d*C, C*f*D, drawn through them. Next mark the characteristic points of each parabola by dividing each span into three equal parts, and making the heights of 1 and 2 = $\frac{8}{9}$ *ab*, 3 and 4 = $\frac{8}{9}$ *cd*, and 5 and 6 = $\frac{8}{9}$ *ef*. If the *outer* ends of the girder rest freely on the supports at A and B, no further use is made of the extreme characteristic points 1 and 6, which may be disregarded. Commencing at A, a series of straight lines, AB′, B′C′, C′D, require to be drawn, such that any two lines as AB′, B′C′,

meeting over the intermediate support B, will pass at vertical distances *above or below* the characteristic point 2 and *below or above* the characteristic point 3, which are in inverse ratio to the adjacent spans AB and BC. Similarly the lines B'C' and C'D must pass at vertical distances above or below the characteristic point 4 and below or above the characteristic point 5, which are in inverse ratio to the adjacent spans BC and CD, and the closing line C'D must terminate at D. Obviously only one possible system of lines will fulfil these conditions, and their directions are easily located after one or two trials. A trial system of lines is indicated at A*ghk*, which does not fulfil the above conditions, since *hk* fails to close on D. The initial line A*g* evidently passes at too great a vertical distance above 2, so that by tentatively lowering A*g* the correct directions AB'C'D are ultimately obtained. If the spans adjacent to an intermediate support be equal, the vertical distances between the trial lines and the characteristic points on opposite sides of that support must also be equal. It should be noted that any two lines meeting over an intermediate support *must* pass on *opposite* sides of the two characteristic points adjacent to that support, but that it is immaterial whether they pass above *or* below either the right- or left-hand point. A line is occasionally found to pass *through* one of the characteristic points, in which case, the vertical interval being nothing, the succeeding line beyond the next support must pass *through* the corresponding adjacent characteristic point.

The broken line AB'C'D so found, constitutes a new base line from which to measure the bending moments which actually obtain for the *continuous* girder. The points p_1, p_2, p_3, p_4, where this new base line intersects the parabolic diagrams, determine the positions of the points of contra-flexure, and projecting them to P_1, P_2, P_3 and P_4 on AD, their horizontal distances apart may be scaled off. These are indicated in the lower figure, which also shows the manner in which the continuous girder may be divided into an equivalent system of simple beams and cantilevers. The vertically shaded portions of the upper figure indicate the bending moments on the continuous girder, the full lines denoting positive moments, and the dotted, negative moments. At the points of contra-flexure, the B.M. is of course zero, which necessitates these points being made the points of junction between the simple beams and cantilevers in the lower figure. The pressures on the supports A, B, C, and D are readily deduced from the lengths of the cantilever and simple girder spans. Thus—

Pressure on A $= \frac{1}{2}$ load on $AP_1 = \frac{1}{2} \times 22{\cdot}7 \times 2 = 22{\cdot}7$ tons.

,, B $= \frac{1}{2}$ load on AP_1 + load on $P_1P_2 + \frac{1}{2}$ load on P_2P_3
$= 22{\cdot}7 + (7{\cdot}3 \times 2) + (9 \times 1{\cdot}5) + (\frac{1}{2} \times 23 \times 1{\cdot}5)$
$= 68{\cdot}05$ tons.

,, C $= \frac{1}{2}$ load on P_2P_3 + load on $P_3P_4 + \frac{1}{2}$ load on P_4D
$= (\frac{1}{2} \times 23 \times 1{\cdot}5) + (16 \times 1{\cdot}5) + (\frac{1}{2} \times 22 \times 1{\cdot}5)$
$= 57{\cdot}75$ tons.

,, D $= \frac{1}{2}$ load on $P_4D = \frac{1}{2} \times 22 \times 1{\cdot}5 = 16{\cdot}5$ tons.

The sum of these pressures, 165 tons, of course equals the total load on the beam. The continuous girder deflects in the manner shown by

the curved line VW. The shearing force diagram readily follows from a consideration of the loads and pressures on supports. Note that the inclined lines indicating the shear force cut the base line XY beneath the central points of the equivalent girders AP_1, P_2P_4 and P_4D, where the shear is zero, and the positive B.M. a maximum.

A few special cases may be noticed. In Fig. 44 a beam is continuous over two equal spans, AB and BC, and carries a uniform load

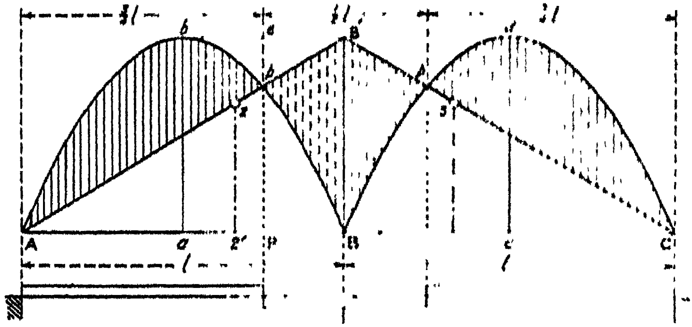


Fig. 44.

of w tons per foot run throughout. The parabolas AbB and BdC having $ab = cd = \frac{wl^2}{8}$, represent the moments for the spans considered independently. The characteristic points adjacent to the support B are 2 and 3. AB′ and B′C are the only possible lines fulfilling the conditions above mentioned, and they must obviously pass *through* the points 2 and 3. But since $22' = \frac{2}{3}\,ab$, and $A2' = \frac{2}{3}\,AB$, $22'$ also $= \frac{2}{3}\,BB'$, whence $BB' = ab$, or the negative B.M. over the pier B is equal in amount to the positive moment of $\frac{wl^2}{8}$ at the centre of either span considered independently. The points of contra-flexure, p, p, evidently occur at $\frac{3}{4}\,l$ from A and C, since pP will then $= \frac{3}{4}\,BB'$, and $ep = \frac{1}{4}\,BB'$, which

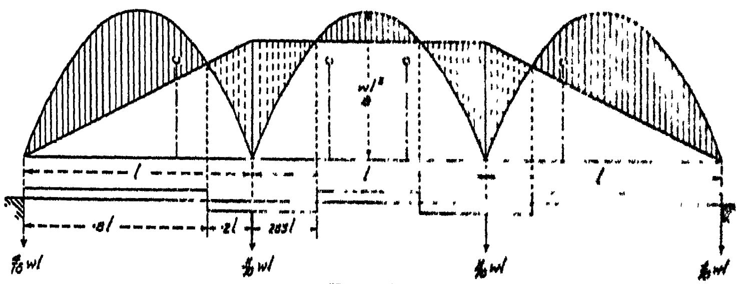


Fig. 45.

in a parabola is the condition for e to be situated halfway between b and B′. Hence the pressure on each end support A and C $= \frac{1}{2}$ of $\frac{3}{4}\,wl = \frac{3}{8}\,wl$, and on B $= \frac{3}{8}\,wl + \frac{1}{2}\,wl + \frac{3}{8}\,wl = 1\frac{1}{4}\,wl$, or the central

support carries $3\frac{1}{3}$ times the load on each end support. The equivalent system of two simple beams and a cantilever is shown below.

Fig. 45 shows the moment diagram for three equal spans of l feet, bridged by a continuous girder carrying a uniform load of w tons per foot, from which the indicated pressures on the supports may be deduced as shown.

A useful practical application occurs in the case of an open trough-shaped conduit for carrying a canal, or a continuous pipe line conveying water supply over several spans. Fig. 46 shows the character of the moment diagram where six equal spans are involved.

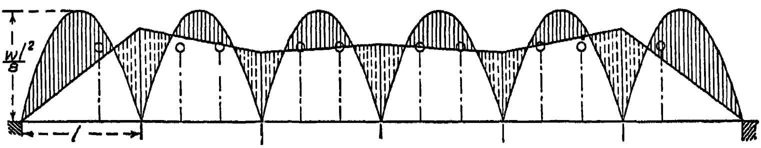


FIG. 46.

EXAMPLE 9.—*A girder is 42 feet long and is supported on walls at either end and by a column at the centre. At 6 ft. intervals it carries rolled joists, each of which imposes a floor load of 7 tons on the girder. Required the B.M. diagram for the girder and the pressures on the supports.* Fig. 47.

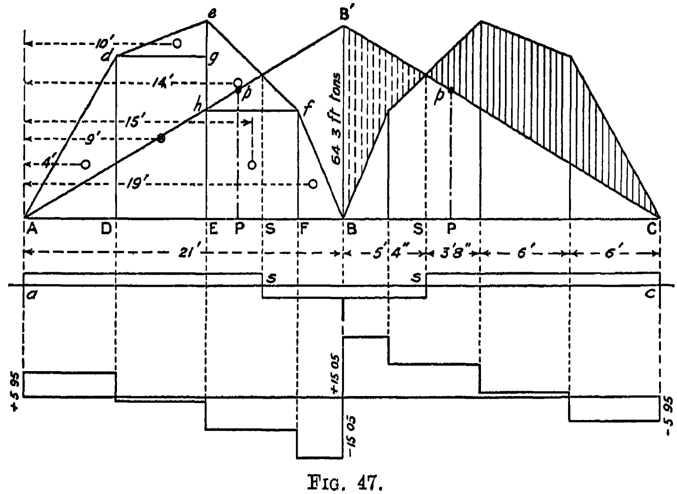


FIG. 47.

Regarding the span AB as independent,

$R_A = 7\left(\frac{15}{21} + \frac{9}{21} + \frac{3}{21}\right) = 9$ tons.

B M. at D $= 9 \times 6 = 54$ ft.-tons.

B.M. at E $= 9 \times 12 - 7 \times 6 = 66$ ft.-tons.

B.M. at F $= 9 \times 18 - 7 \times 12 - 7 \times 6 = 36$ ft -tons.

These are plotted at Dd, Ee, and Ff, giving AdefB as the moment diagram for an *independent* girder between A and B. Since the ends

A and C are free, only the characteristic points adjacent to B are required, and the loading being symmetrical, these will be situated at the same height above AC. Set off BP' = ½ AB. The diagram A*def*B is divided up into constituent rectangles and triangles by the full lines, and the sum of the moments of these areas about A will equal the moment of the whole diagram about A. The individual centres of gravity are located and marked by the small circles.

Moment of $\triangle$ AD*d*	about A	$\frac{6 \times 54}{2} \times 4$	=	648
„ $\triangle$ *deg*	„	$\frac{6 \times 12}{2} \times 10$	=	360
„ $\triangle$ *ehf*	„	$\frac{6 \times 30}{2} \times 14$	=	1260
„ $\triangle$ *f*FB	„	$\frac{3 \times 36}{2} \times 19$	=	1026
„ rect. DE*gd*	„	$6 \times 54 \times 9$	=	2916
„ „ EF*fh*	„	$6 \times 36 \times 15$	=	3240

$\therefore$ Mt. of A*def*B about A = 9450 units.

For the height H of the rectangle on AB having the same moment about A, $21 \times H \times \frac{21}{2} = 9450$, whence H = 42·85. Cut off P*p* = 42·85 units on the B.M. scale employed when *p* is the characteristic point required. Since the diagram is symmetrical, the new base line will be obtained by joining A to *p* and producing to B'. The maximum B.M. is obviously BB' = $1\frac{1}{2} \times$ P*p* = $1\frac{1}{2} \times 42{\cdot}85$ = 64·3 ft.-tons, and is negative, that is, the upper flange will be in tension and the lower in compression over the support B. The points of contra-flexure are at S, S, distant 15 ft. 8 in. from A or C. The lower figure shows the equivalent system of simple beams and cantilever, and from the positions of the two 7 ton loads which rest on *sc*, the reaction at C = $\frac{3' \, 8''}{15' \, 8''}$ of 7 + $\frac{9' \, 8''}{15' \, 8''}$ of 7 = 5·95 tons. A similar reaction exists at A, whence pressure on central column = total load − pressures on A and C = 42 − 11·9 = 30·1 tons.

The S.F. diagram readily follows from the pressures, each step scaling 7 tons.

Fixed Beams. A beam or girder is said to be *fixed* at the ends when it is so firmly built in or anchored down that a tangent, AB, Fig. 48, to the curve of the bent beam at A is horizontal. In order to realize this condition, it is evident there must be a sufficiently large downward pressure or pull P applied to the portion AC of the beam, as will create a reversed bending moment capable of balancing that caused at A by the loads on the beam. The holding down force P may be applied by the weight of masonry in the wall above AC, or by anchor rods taken down to a suitable depth. If, in the lower figure, P' be not sufficient to create the same amount of moment as would exist at A' if the beam were actually *fixed*, the end of the beam will tilt up to some extent and bend as shown at A'C', when the tangent A'B' to

the curve at A′ will no longer be horizontal, and the beam will not fulfil the condition of fixity of ends. In this case the point of contraflexure, which previously was located at p, will move to some point p' nearer to A′, and the B.M. at A′ will be reduced to that which P′ is capable of producing when acting at the leverage P′A′. This will be accompanied by a correspondingly increased B.M. at the centre of the beam. A little consideration will show the fallacy of assuming a beam to be *fixed* at the ends, simply because it is *apparently* firmly built into a wall at either end.

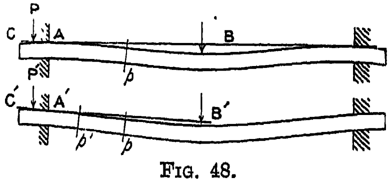


FIG. 48.

In Fig. 48 suppose the beam to carry a central load of 2 tons over a span of 20 feet. The B.M. at A, if the beam be actually *fixed*, will be $\frac{Wl}{8} = \frac{2 \times 20}{8} = 5$ ft.-tons or 60 inch-tons. If the beam project, say, 18 inches into the wall and be fixed by the weight of brickwork resting on AC, then $P \times \frac{18''}{2}$ must $= 60$ inch-tons, or $P = 6\frac{2}{3}$ tons. Assuming a breadth of flange of 12 inches, the bearing area from A to C $= 1\frac{1}{2}$ square feet, and the height h of the column of brickwork resting on this area and weighing $6\frac{2}{3}$ tons, will be given by $h \times 1{\cdot}5 \times \frac{112}{2240} = 6\frac{2}{3}$ tons, whence $h = 89$ feet. This is supposing the column of brickwork to actually rest on the end of the beam, whereas a portion of it would probably be supported by the bond in the wall. Assuming a reasonable height of wall above AC, say 30 feet, the beam would require to be firmly built in for a minimum distance of 2 feet 7 inches at each end, in order to realize fixed conditions, still supposing the weight of the 30 feet of brickwork to be wholly resting on AC. Probably the majority of *so-called fixed beams* fall far short of the required degree of fixation, with the result that if calculated as fixed beams they may be stressed to nearly double the intensity intended in their design. The moment of the holding-down force P about A, or $P \times \frac{1}{2}$ CA, in Fig 48, is called the **moment of fixation**, and the B.M. on the beam section at A cannot exceed this moment of fixation Consequently no beam or girder should be assumed as having fixed ends, unless the actual pressure upon the built-in or anchored-down ends is definitely known to be equal to that required to produce the necessary moment of fixation for balancing the B.M. due to the loading under consideration. Relatively few girders in practice are intentionally designed as fixed beams. Where it is necessary to *fix* the end of a girder, the necessary fixing moment is provided by *properly* loading the end of the girder with a definite balance weight, or by attaching to it anchor ties capable of exerting a predetermined downward pull.

The bending moments on fixed beams of uniform section are readily determined after locating the positions of the characteristic points of the B.M. diagram for the beam considered as simply supported. The straight line drawn *through* the two characteristic points constitutes the new base line above and below which to measure the positive and negative moments on the fixed beam.

Fig. 49 illustrates the case of a fixed beam of span l feet with a central concentrated load W. The triangle ACB of height $= \frac{Wl}{4}$ is the moment diagram for the simply supported beam. The height of the characteristic points $p, p, = \frac{1}{2}$ CD, and joining these by EF the moment at the centre is $\frac{1}{2}$ CD $= + \frac{Wl}{8}$, whilst EA $=$ FB $= - \frac{Wl}{8}$ is the moment at the fixed ends. The points of contra-flexure are S, S, distant $\frac{l}{4}$ from each end of the span. The fixed beam AB is equivalent to two fixed cantilevers *as* and *sb* with a simply supported span *ss* carried between them, as shown in the lower figure.

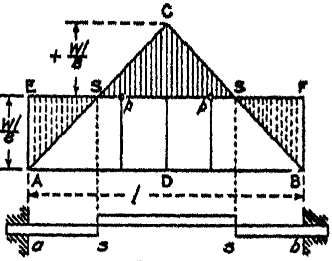


FIG. 49.

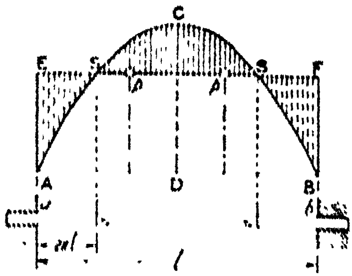


FIG. 50.

Fig. 50 gives the diagram of moments for a fixed beam of span l feet carrying a uniform load of w tons per foot run. The parabola ACB having CD $= \frac{wl^2}{8}$ ft.-tons, has the characteristic points p, p, at a height above AB $= \frac{2}{3}$ CD. The central B.M. on the fixed beam $= \frac{1}{3}$ CD $= \frac{1}{3} \times \frac{wl^2}{8} = + \frac{wl^2}{24}$, and the end moments AE and BF are each $= - \frac{2}{3} \times \frac{wl^2}{8} = - \frac{wl^2}{12}$ ft.-tons. The length *as* may be found as follows:

The bending moment at the centre of the independent span ss $= \frac{w \times ss^2}{8} = \frac{wl^2}{24}$, whence

$ss = \frac{l}{\sqrt{3}} = 0{\cdot}578l$. $\therefore$ *as* and *sb* together $= 0{\cdot}422l$ and *as* $=$ *sb* $= 0{\cdot}211l$.

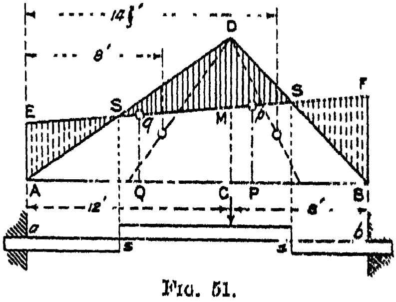


FIG. 51.

For any case of unsymmetrical loading the same construction holds. Thus in Fig. 51, the fixed beam AB of 20 feet span carries a concentrated load of 5 tons at C, distant 8 ft. from B. $R_A = \frac{8}{20}$ of $5 = 2$ tons, and the moment CD for the beam simply supported $= 2 \times 12 = 24$ ft.-tons. For the characteristic points—

$$\text{Moment of } \triangle \text{ ADC about A} = \frac{12 \times 24}{2} \times 8 = 1152$$
$$\text{,, } \triangle \text{ BDC ,, } = \frac{8 \times 24}{2} \times 14\tfrac{2}{3} = 1408$$
$$\therefore \text{ ,, } \triangle \text{ ADB ,, } = 2560$$

For the height H of rectangle on AB having the same moment about A, $H \times 20 \times \frac{20}{2} = 2560$, $\therefore H = 12{\cdot}8$. Mark P and Q at one-third the span and make $Pp = 12{\cdot}8$ units on the B.M. scale.

$$\text{Similarly, moment of } \triangle \text{ ADC about B} = \frac{12 \times 24}{2} \times 12 = 1728$$
$$\text{,, } \triangle \text{ BDC ,, } = \frac{8 \times 24}{2} \times 5\tfrac{1}{3} = 512$$
$$\therefore \text{ ,, } \triangle \text{ ADB ,, } = 2240$$

For the height h of rectangle on AB having the same moment about B, $h \times 20 \times \frac{20}{2} = 2240$. $\therefore h = 11{\cdot}2$ units. Make $Qq = 11{\cdot}2$ and join the characteristic points p and q. EqpF is the new base line for moments giving a maximum positive moment DM beneath the load and negative moments EA and FB at A and B respectively. The equivalent system of two cantilevers and a simple beam is shown at *assb*.

Beams fixed at One End and supported at the Other.—These are not of much practical interest. Fig. 52 shows the B.M. diagram for a beam fixed at A and supported at B carrying a distributed load of w tons per foot run, and Fig. 53, the same beam with a concentrated load

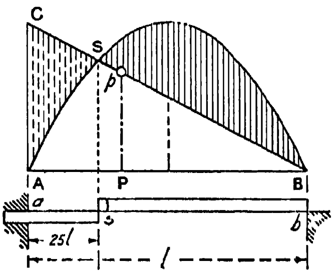


Fig 52.

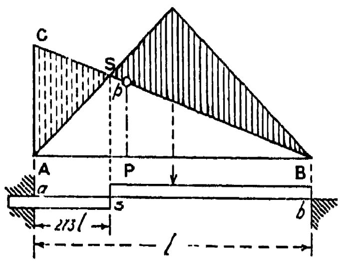


Fig 53.

of W tons at the centre. In these cases, the characteristic point adjacent to the freely supported end B is neglected, since the B.M. at this end is zero. The new base line BC is therefore drawn from B *through* the characteristic point p adjacent to the fixed end The beam AB is, in each case, equivalent to a cantilever *as* and supported beam *sb* with one point of contra-flexure at S.

Beam fixed at Both Ends and continuous over Intermediate Supports.—Fig. 54 represents a beam 50 ft. long fixed at each end and supported at 20 feet from one end, carrying a load of 2 tons per foot run.

The central heights of the parabolas ADB and BEC are given by $\frac{2 \times 30^2}{8} = 225$ ft.-tons and $\frac{2 \times 20^2}{8} = 100$ ft.-tons respectively.

The position of the new base line FGH is obtained by drawing two lines *through* the characteristic points p and q adjacent to the fixed ends

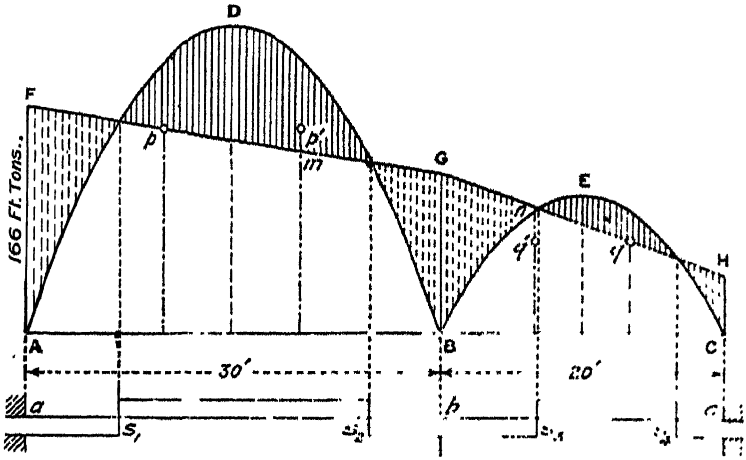


FIG. 54.

A and C, meeting at G vertically over B and passing below p' and above q', so that

$$\frac{p'm}{q'n} = \frac{20}{30}$$

Projecting down the four points of contra flexure, the equivalent system consists of three cantilevers as_1, s_2s_3 and s_4c and two supported beams s_1s_2 and s_3s_4. The negative moment AF is the maximum, scaling 166 ft.-tons.[1]

[1] For a demonstration of the proof of the method of characteristic points, see *A Practical Treatise on Bridge Construction*, by T. Claxton Fidler, M.Inst.C.E.

CHAPTER IV.

BEAMS.

Moment of Resistance.—Vertical forces acting on a horizontal beam produce a bending action in the beam. At any cross-section the bending action is proportional to the bending moment.

Suppose a loaded beam, Fig. 55, to be hinged at the centre. The bending action would tend to close the portion between P and C and

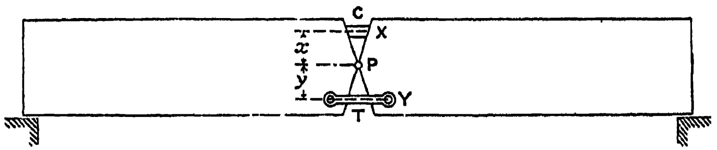


FIG. 55.

open the lower portion P to T. If a block of material be placed at X and a tie at Y to prevent movement about the hinge, it is evident that the block at X would be compressed and the tie at Y stretched. Equilibrium having been established, the bending moment at the section must be equal to the moment of the forces in the block and tie about the hinge at P.

Let C = compression in the block,
T = tension in the tie.

Then the *moment* of these forces about P

$$= T \times y + C \times x$$
$$= \text{the bending moment on the section.}$$

If the beam be made continuous, the material at the vertical section through P would be subject to stresses similar to those in the block and tie. All the material above some horizontal plane, such as that passing through P, would be in compression and all below that plane in tension. Let the small arrows in Fig. 56 represent the stresses in the material at the vertical section CPT, and R_C and R_T the resultants of the compressive and tensile stresses. Since all the forces causing bending must act normally to the horizontal plane through P, the only forces acting parallel to that plane are the forces R_C and R_T, which must therefore be equal

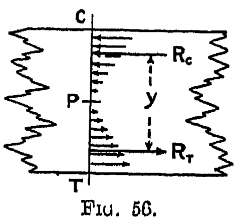


FIG. 56.

to produce equilibrium. The bending moment at the section CPT will therefore be equal to the moment of the couple $R_1 \times y$ or $R_T \times y$. The moment of this couple is a measure of the strength of the beam at the section and is known as the *moment of resistance*.

Suppose the block *abdc*, Fig. 57, be compressed to *efhg*. The upper edge *ef* is decreased in length to a greater extent than the lower edge *gh*, and as the stress must be proportional to the decrease of length, the stress at *ef* is greater than the stress at *gh*. The decrease in length is proportional to the distances of the edges *ab* and *cd* from the point P. Therefore the stresses must also be proportional to the distances from P. If the block had extended from P to C no alteration of length

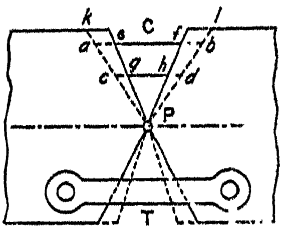


FIG. 57.

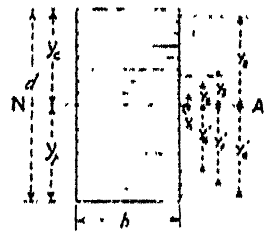


FIG. 58.

would have occurred at P, demonstrating that the material at P is not subject to any bending stress, whilst the maximum change in length and therefore the greatest stress occurs at C. In a similar manner it may be shown that the tensile stress below P varies from nothing at P to a maximum at T, and at any point in the section is proportional to the distance of that point from P.

At every vertical section of the beam there is some point P where there is no direct stress. The plane containing all such points is known as the *neutral plane*.

Position of the Neutral Axis. The intersection of the neutral plane with any cross-section of a beam is termed the neutral axis of the cross-section. Let Fig. 58 represent the cross-section of a rectangular beam divided into horizontal layers. If the intensity of compressive stress at the upper surface (usually called the skin stress) $= f_c$, and the intensity of tensile stress at the lower surface $= f_t$, then the average intensities of stress in the layers above the neutral axis NA will be

$$= f_c\frac{y_8}{y_c},\ f_c\frac{y_7}{y_c},\ \ldots\ f_c\frac{y_1}{y_c}$$

Let a = sectional area of each layer.

Then the total stresses in the separate layers *above* NA

$$= af_c\frac{y_8}{y_c},\ af_c\frac{y_7}{y_c},\ \ldots\ af_c\frac{y_1}{y_c}$$

and the total compression above the neutral axis

$$= \frac{f_c}{y_c}(ay_8 + ay_7 + \ldots + ay_1)$$

Similarly, the total tension *below* the neutral axis

$$= \frac{f_t}{y_t}(ay'_8 + ay'_7 + \ . \ . \ . \ + ay'_1)$$

Since total tension must equal total compression

$$\frac{f_c}{y_c}(ay_8 + ay_7 \ . \ . \ . \ + ay_1) = \frac{f_t}{y_t}(ay'_8 + ay'_7 \ \ . \ . + ay'_1)$$

But $$\frac{f_c}{y_c} = \frac{f_t}{y_t}$$

$$\therefore ay_8 + ay_7 \ . \ . \ . \ + ay_1 = ay'_8 + ay'_7 \ . \ . \ . \ + ay'_1$$

or the sum of the moments of the areas in tension is equal to the sum of the moments of the areas in compression. This is the condition for the neutral axis passing through the centre of gravity of the cross-section. So long as the moduli of elasticity of the material in tension and compression be the same, the neutral axis must always pass through the centre of gravity of the cross-section whatever be its shape.

Moment of Resistance.—The stress in the top layer, Fig. 58, was shown to be $= af_c\frac{y_8}{y_c}$.

Its moment about the neutral axis

$$= af_c\frac{y_8}{y_c} \times y_8$$
$$= \frac{f_c}{y_c}ay_8^2$$

The total moment of the stresses above the neutral axis will therefore

$$= \frac{f_c}{y_c}(ay_8^2 + ay_7^2 + \ \ . \ . + ay_1^2) \quad . \quad \quad . \ . \ . \ . \quad (1)$$

Moment of the stresses below the neutral axis

$$= \frac{f_t}{y_t}(ay'^2_8 + ay'^2_7 + \ . \ . \ . \ + ay'^2_1) \quad . \quad \quad . \ . \ . \quad (2)$$

The moment of resistance of the section is equal to the sum of the expressions (1) and (2) when the layers are taken infinitely thin. It will be seen that the portions of the expressions in the brackets are the sums of all the small areas multiplied by the square of their distances from the neutral axis. The moment of resistance may therefore be written—

$$\text{M.R.} = \left\{\begin{array}{c}\text{Sum of all the small}\\ \text{areas multiplied by}\\ \text{the square of their}\\ \text{distances from the}\\ \text{neutral axis}\end{array}\right\} \times \frac{\text{skin stress}}{\text{distance of skin from N.A.}} \quad (3)$$

In sections symmetrical about the neutral axis the skin stress in tension will be equal to the skin stress in compression, but for

unsymmetrical sections these stresses will not be equal. In unsymmetrical sections, the skin stress, in the above expression, is that of tension *or* compression, according as the denominator is the distance from the neutral axis, of the skin subject to the stress adopted.

Moment of Inertia.—For any section, the sum of all the small areas into which the section may be divided, multiplied by the square of their distances from the neutral axis, is termed the moment of inertia of the section, and is usually denoted by the letter I. The expression (3) may then be written

$$\text{M.R.} = \frac{\text{moment of inertia} \times \text{skin stress}}{\text{distance of skin from N.A.}}$$

$$= \text{I}\frac{f_c}{y_c} = \text{I}\frac{f_t}{y_t}$$

The bending moment being equal to the moment of resistance,

$$\text{B.M.} = \text{I}\frac{f_c}{y_c} = \text{I}\frac{f_t}{y_t}$$

The value of I is dependent on the distribution of the material about the axis considered. The calculation of the moment of inertia involves the use of the calculus, and it is not proposed to give here the mathematical proof. The formulæ for a number of simple cases will be found in Table 25, and from them I, for most ordinary sections, may be calculated.

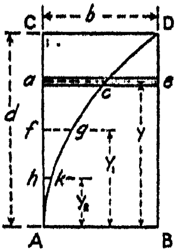


FIG. 59.

The following graphical method of obtaining I will prove the accuracy of the formulæ given for rectangles.

To find the moment of inertia of the rectangle ABCD, Fig. 59, about the side AB. Consider a very thin horizontal strip *ae* of the rectangle at a distance y from AB and of area l.

I of the strip $= ly^2$.

I for a similar strip at CD $= ld^2$.

If the area l be reduced to l', so that

$$l' = l\frac{y^2}{d^2}$$

then

$$l'd^2 = ly^2$$

If each horizontal strip of the rectangle be reduced in the same ratio, *i.e.* $\frac{\text{the square of its distance from AB}}{d^2}$, the sum of all such reduced areas multiplied by d^2 will be the moment of inertia about AB.

Since the strips are very thin the length may be taken to represent the area. The reduced length of *ae* will

$$= ae \times \frac{y^2}{d^2}$$

$$= b \times \frac{y^2}{d^2}$$

$$= ac$$

The reduced lengths

$$fg = b \times \frac{y_1^2}{d^2}$$
$$hk = b \times \frac{y_2^2}{d^2}$$

Taking a large number of strips and joining the extremities k, g, c, etc., the points k, g, c, will be found to lie on a parabola passing through A and D. The moment of inertia will then be equal to the area $\mathrm{CD}cgk\mathrm{A}$ multiplied by d^2. The area of the parabolic segment $\mathrm{D}g\mathrm{AB}$

$$= \tfrac{2}{3}bd$$

The "inertia area" $\mathrm{CD}cgk\mathrm{A}$ will therefore

$$= \tfrac{1}{3}bd$$
$$\text{and } \mathrm{I} = \tfrac{1}{3}bd \times d^2$$
$$= \tfrac{1}{3}bd^3$$

Let the dimensions of the rectangle be—

$$b = 6'', \ d = 12''$$

Then I about the side AB

$$= \frac{bd^3}{3}$$
$$= \frac{6 \times 12^3}{3} = 3456 \text{ in.}^4$$

To obtain the moment of inertia about the neutral axis N.A.

Treating each half of the rectangle by the above graphic method, two inertia areas, shown shaded, Fig. 60, are obtained, the area of each being $\tfrac{1}{3}b\frac{d}{2}$.

$$\text{I for each half} = \tfrac{1}{3}b\frac{d}{2} \times \left(\frac{d}{2}\right)^2$$
$$= \tfrac{1}{24}bd^3$$

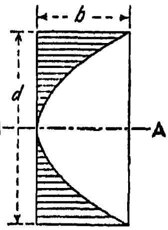


FIG 60.

For the whole rectangle

$$\mathrm{I} = 2 \times \frac{bd^3}{24}$$
$$= \frac{bd^3}{12}$$

Again, let $b = 6''$ and $d = 12''$.

Then the moment of inertia of the rectangle about N.A.

$$= \frac{bd^3}{12}$$
$$= \frac{6 \times 12^3}{12} = 864 \text{ in.}^4$$

EXAMPLE 10.—*To find the moment of inertia of a rolled beam section.*

Let the section be $12'' \times 6'' \times \tfrac{1}{2}''$ metal with parallel sides, Fig. 61.

Since the section is symmetrical the neutral axis will be situated 6 in. from the top and bottom.

The moment of inertia may be obtained by either of the following methods :—

(1) Calculate the moment of inertia for the rectangle 12″ × 6″ and subtract the moments of inertia of the two rectangles $b_1 \times d_1$.

(2) Calculate separately and add together the moments of inertia of the two flanges and the web.

By the first method

$$\begin{aligned}
\text{M.I.} &= \text{I of } 12'' \times 6'' \text{ rectangle} - 2\ (\text{I of } 11'' \times 2\tfrac{3}{4}'' \text{ rectangle}) \\
&= \frac{bd^3}{12} - 2\frac{b_1d_1^3}{12} \\
&= \frac{6 \times 12^3}{12} - \frac{2 \times 2\frac{3}{4} \times 11^3}{12} \\
&= 253{\cdot}96 \text{ in.}^4
\end{aligned}$$

To find the moment of inertia by the second method it will be necessary to consider the moment of inertia of a section about an axis other than that passing through its centre of gravity.

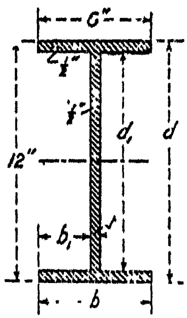


FIG. 61.

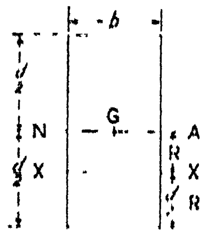


FIG. 62.

Moment of inertia of a section about any axis XX parallel to the axis through its centre of gravity G, Fig. 62.

It has already been proved that for a rectangle :

$$\text{I about axis through the centre of gravity} = \frac{bd^3}{12}.$$

$$\text{I about one side} = \frac{bd^3}{3}.$$

In Fig. 62 let the axis XX be parallel to and distant R from the neutral axis.

Treating the rectangle as composed of two rectangles, one above and one below the axis XX, the sum of the moments of inertia of such rectangles about XX will be the moment of inertia of the whole rectangle about XX.

For the rectangle above the axis XX—

$$I_{XX} = \frac{b\left(\frac{d}{2} + R\right)^3}{3}$$

For the lower rectangle

$$I_{XX} = \frac{b\left(\frac{d}{2} - R\right)^3}{3}$$

For the whole rectangle

$$I_{XX} = \frac{b\left(\frac{d}{2} + R\right)^3}{3} + \frac{b\left(\frac{d}{2} - R\right)^3}{3}$$

$$= \frac{b}{3}\left\{\left(\frac{d}{2} + R\right)^3 + \left(\frac{d}{2} - R\right)^3\right\}$$

$$= \frac{b}{3}\left(\frac{d^3}{4} + 3dR^2\right)$$

$$= \frac{bd^3}{12} + bdR^2$$

But $b \times d$ = area of whole rectangle

and $\frac{bd^3}{12}$ = I of rectangle about the axis through its centre of gravity.

Therefore the moment of inertia of the rectangle about the axis XX is equal to its moment of inertia about the axis through its centre of gravity plus the area of the rectangle multiplied by the distance between the axes squared—

$$I_{XX} = I_{CG} + AR^2$$

This is true for all sections whatever may be the shape

Returning to Example 10, second method. I of section = I of web + I of two flanges. As the neutral axis passes through the centre of gravity of the web, the moment of inertia of the web

$$= \frac{td_1^3}{12} = \frac{\frac{1}{2} \times 11^3}{12}$$

$$= 55 \cdot 16 \text{ in.}^4$$

From the above proof the moment of inertia of *each* flange

$$= \frac{6'' \times (\frac{1}{2})^3}{12} + 6'' \times \tfrac{1}{2}'' \times (5\tfrac{3}{4})^2$$

$$= 99 \cdot 25 \text{ in.}^4$$

The total moment of inertia for the section

$$= 55 \cdot 16 + (2 \times 99 \cdot 25)$$

$$= 253 \cdot 96 \text{ in.}^4$$

This result agrees with that of the preceding method.

The above method demonstrates the small resistance which the web offers to the bending action. It was shown on p. 92 that the moment of resistance was proportional to the moment of inertia, therefore the proportion of the resistance to bending exerted by the web will be $\frac{55\cdot46}{253\cdot96}$ or with an area $= \frac{5\cdot5}{6} = 0\cdot91$ of that of the flanges, its resistance as compared with that of the flanges is only $\frac{55\cdot46}{198\cdot5} = 0\cdot28$. Hence it is desirable that the material which has to resist the bending action be placed, within practical limits, as far from the neutral axis as possible.

Modulus of Section.—It has already been proved that

$$\text{M.R.} = \frac{fI}{y}$$

f, the skin stress, is dependent only on the material of which the beam is composed, but I and y are wholly dependent on the shape of the cross-section of the beam. The quantity $\frac{I}{y}$ is known as the *modulus* of the section, and is a relative measure of the strength of a section. The moment of resistance may then be written

$$\begin{aligned}\text{M.R.} &= \text{skin stress} \times \text{modulus of section} \\ &= f \times Z\end{aligned}$$

For sections symmetrical about the neutral axis the modulus of section is equal to the moment of inertia divided by half the depth of the section. Thus for a rectangular section

$$y = \frac{d}{2}$$

$$\therefore Z = \frac{I}{y} = \frac{\frac{bd^3}{12}}{\frac{d}{2}}$$

$$= \frac{bd^2}{6}$$

The modulus of section may be found graphically by the following method.

Graphical Method of obtaining the Modulus of Section. Consider a very thin layer, AB, in the flange of the beam section, Fig. 63, at a distance y_1 from the neutral axis. If the intensity of skin stress be equal to f, the intensity of stress on the layer AB $= f \times \frac{y_1}{y}$

$$\text{If the area of the strip} = l$$

$$\text{total stress on the layer} = lf\frac{y_1}{y}$$

If the area l be reduced in the ratio of $\frac{y_1}{y}$ to l', and the total stress on the layer be considered to be distributed over the area l', the intensity of stress on l' will

$$= f\frac{y_1}{y} \times \frac{l}{l'}$$

$$= f\frac{l\frac{y_1}{y}}{l\frac{y_1}{y}} = f$$

Reducing the area of all horizontal layers of the section in the ratio of their distances from the neutral axis divided by y, an area for the whole section will be obtained on which the intensity of stress is equal to f. The modulus of section will then be equal to the moment of that area about the neutral axis.

Draw a base line parallel to the neutral axis and at a distance y from it. Project the extremities of each layer on to the base line and join the points thus obtained to any point (say the centre of gravity of the section) on the neutral axis. Then the area of the layer between such lines will be the reduced area required. The projections of the ends of the layer AB on the base line are the points C and D. Join C and D to E. The area ab between the lines CE and DE is the area required.

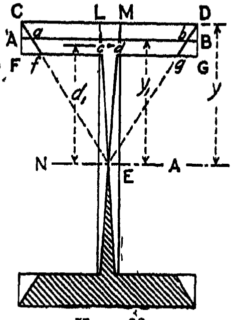


FIG. 63.

In the triangles CDE and abE

$$ab : CD :: y_1 : y$$

$$\therefore ab = CD \times \frac{y_1}{y}$$

But CD = AB

$$\therefore ab = AB \times \frac{y_1}{y}$$

The area of equal stress intensity, called the *modulus figure*, for the upper half will be $CfcEdgD$.

Let its area $= A_1$ and its centre of gravity be distant d_1 from the neutral axis. Then the total stress on the portion above the neutral axis—

$$= A_1 f$$

Moment about the neutral axis—

$$= d_1 A_1 f$$

Since the section is symmetrical the moment of the stress in the lower portion is equal to the moment of the stress in the upper portion.

Therefore the total moment of resistance of the whole section

$$= 2d_1 A_1 f$$

But
$$\text{M.R.} = fZ$$
$$\therefore Z = 2d_1 A_1$$

If D be the distance between the centres of gravity of the upper and lower modulus figures

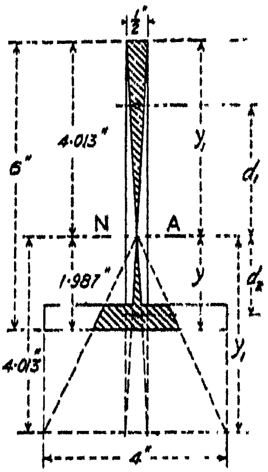


FIG. 61.

$$Z = DA_1$$

For sections symmetrical about the neutral axis

$$D = 2d_1$$

Since the total stresses above and below the neutral axis are equal, the area of the modulus figure below the neutral axis must be equal to the area of the modulus figure above the neutral axis.

Modulus Figure for Sections unsymmetrical about the Neutral Axis. In sections such as the tee, Fig. 61, the centre of gravity falls nearer to the lower surface than the upper, and consequently the intensity of skin stress at the lower surface will be less than at the upper.

Let f_l = intensity of skin stress at the lower surface.
y = distance of lower surface from the neutral axis.
f_o = intensity of skin stress at the upper surface.
y_1 = distance of upper surface from neutral axis.

$$\text{Then } f_l = f_o \frac{y}{y_1}$$

Two modulus figures may be drawn for the section, one having an intensity equal to f_o and the other an intensity equal to f_l.

The construction of the modulus figure for the upper skin stress, *i.e.* f_o, is shown in Fig. 61.

The base line for the upper portion is in the plane of the upper skin where the stress $= f_o$, but for the lower portion the base line being set out at a distance y_1 below the neutral axis (*i.e.* where the stress would equal f_o), falls below the section. All layers below the neutral axis must be projected on to the lower base line and joined to the point selected on the neutral axis. The shaded area is the modulus figure for the section.

Let the shaded area above the neutral axis = A.

$$\text{Then total stress above neutral axis} = f_o \text{A}$$

Let d_1 = distance of centre of gravity of upper shaded area from NA
d_2 = " " " lower " "
$D = d_1 + d_2$

Then the moment of resistance of the section

$$= f_c A(d_1 + d_2)$$
$$= f_c AD$$

Construction for Modulus Figure having an Intensity of Stress equal to f_t, Fig. 65.—The intensity of stress f_t occurs at a distance y below the neutral axis, therefore at a distance y above the neutral axis the intensity will also be f_t. The base line for the upper portion is therefore drawn through the plane ef. As the intensity of stress in the material above this base line is greater than f_t, the area $aefb$ must be increased. For any layer above the base line, say ab, project on to the base line in e and f; join e and f to a point on the neutral axis and produce these lines to cut the horizontal through ab in c and d. Then the length cd will be the increased length of ab required. For all layers between the base lines proceed as in the former construction.

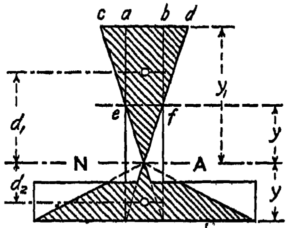


FIG. 65.

Let A_1 = area of each portion of the modulus figure

D_1 = distance between centres of gravity of shaded areas $= d_1 + d_2$

$$\text{Then M.R.} = A_1 D_1 f_t$$

Knowing the bending moment at a vertical section of a beam, the suitability of the cross-section for resisting it may be determined.

Bending moment = moment of resistance
$$= f_c AD$$
or $$= f_t A_1 D_1$$

If either quantity $f_c AD$ or $f_t A_1 D_1$ be less than the bending moment (after inserting a suitable value for f_t or f_c) the cross-section is not strong enough to safely support the load on the beam and must be increased. For beams composed of mild steel which has an equal strength in tension and compression, it is only necessary to construct the modulus figure for the larger intensity f_c, as failure must occur where the material is the more highly stressed. For cast iron and other materials where the strength in tension does not equal the strength in compression, both the maximum intensities f_c and f_t produced by the bending moment, must be calculated and compared with the allowable safe intensities for the material employed.

EXAMPLE 11.—*To find the moment of resistance of a* $4'' \times 6'' \times \frac{1}{2}''$ **T** *with parallel sides*, Fig. 64.

The distance of the centre of gravity of the section from the lower edge

$$= \frac{\text{moment of all layers about lower edge}}{\text{total area of section}}$$
$$= \frac{4'' \times \frac{1}{2}'' \times \frac{1}{4}'' + 5\frac{1}{2}'' \times \frac{1}{2}'' \times 3\frac{1}{4}''}{4'' \times \frac{1}{2}'' + 5\frac{1}{2}'' \times \frac{1}{2}''}$$
$$= 1{\cdot}987''$$

Therefore y (Fig. 64) $= 1{\cdot}987''$

$y_1 \quad = 6 - 1{\cdot}987 = 4{\cdot}013''$

Construct the modulus figure as in Fig. 64. Then the area of the figure above the neutral axis

$$= \text{area of shaded triangle.}$$
$$= \tfrac{1}{2}'' \times \frac{4\cdot013}{2}$$
$$= 1\cdot003 \text{ sq. in.}$$

Distance d_1 of centre of gravity above the neutral axis NA

$$= \tfrac{2}{3} \times 4\cdot013$$
$$= 2\cdot675''$$

Moment of triangle about neutral axis

$$= 1\cdot003 \times 2\cdot675$$
$$= 2\cdot68 \text{ in.}^3$$

The modulus area below the neutral axis must equal the area above = 1·003 sq. in.

The centre of gravity of the lower area may be found by calculation or by cutting out the figure in cardboard and suspending from two points.

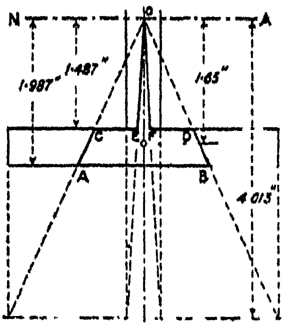


Fig. 66.

In the enlarged Fig. 66,

$$AB = 4 \times \frac{1\cdot987}{4\cdot013} = 1\cdot98 \text{ in.}$$

$$\text{Length } CD = 4 \times \frac{1\cdot487}{4\cdot013} = 1\cdot48 \text{ in.}$$

$$\text{Area } ABDC = \frac{1\cdot98 + 1\cdot48}{2} \times \tfrac{1}{2}$$
$$= 0\cdot865 \text{ sq. in.}$$

$$\text{Length } EF = \tfrac{1}{2}'' \times \frac{1\cdot487}{4\cdot013} = 0\cdot185 \text{ in.}$$

Area of triangle

$$OEF = 0\cdot185 \times 1\cdot487 \times \tfrac{1}{2}$$
$$= 0\cdot138 \text{ sq. in.}$$

Total area below the neutral axis

$$= 0\cdot865 + 0\cdot138 = 1\cdot003 \text{ sq. in.}$$

which corresponds with the area obtained for the upper portion.

Distance of centre of gravity of triangle OEF below neutral axis

$$= \tfrac{2}{3} \times 1\cdot487 = 0\cdot991 \text{ in.}$$

Moment of triangle OEF about neutral axis

$$= 0\cdot138 \times 0\cdot991 = 0\cdot137 \text{ in.}^3$$

Distance of centre of gravity of ABDC below CD

$$= \frac{\tfrac{1}{2}(1\cdot48 + 2 \times 1\cdot98)}{3(1\cdot48 + 1\cdot98)} = 0\cdot27 \text{ in.}$$

Distance of centre of gravity of ABDC from neutral axis

$$= 0\cdot27 + 1\cdot487 = 1\cdot757 \text{ in.}$$

Moment of ABDC about neutral axis

$$= 1{\cdot}757 \times 0{\cdot}865 = 1{\cdot}52 \text{ in.}^3$$

and distance d_2 of centre of gravity of the lower modulus figure from the neutral axis

$$= \frac{1{\cdot}52 + 0{\cdot}137}{1{\cdot}003} = 1{\cdot}65 \text{ in.}$$

Moment of lower modulus figure about neutral axis

$$= 1{\cdot}52 + 0{\cdot}137 = 1{\cdot}657 \text{ in.}^3$$

Then moment of resistance of the section

$$\begin{aligned} &= f_c(1{\cdot}657 + 2{\cdot}68) \\ &= 4{\cdot}337\, f_c \\ \text{or}\quad &= f_c\{1{\cdot}003 \times (1{\cdot}65 + 2{\cdot}675)\} \\ &= 4{\cdot}337\, f_c \end{aligned}$$

EXAMPLE 12.—*What distributed load will a* **T** *4″ × 6″ × ½″ support over a span of 6 feet, the working stress (maximum skin stress) not to exceed 7 tons per square inch?*

(1) When the 4 in. leg is horizontal.

From the previous example

$$\begin{aligned} \text{M.R.} &= 4{\cdot}337\, f_c \\ &= 4{\cdot}337 \times 7 \\ &= 30{\cdot}359 \text{ inch-tons.} \end{aligned}$$

Let w = tons per foot run supported by the beam.

Maximum bending moment

$$\begin{aligned} &= \frac{wl^2}{8} \\ &= \frac{w \times 6^2}{8} \text{ ft.-tons} \\ &= \frac{w \times 36 \times 12}{8} = 54w \text{ inch-tons.} \end{aligned}$$

Note.—The moment of resistance being expressed in inches and tons, the bending moment must also be expressed in those terms.

Then

$$\begin{aligned} \text{B.M.} &= \text{M.R} \\ 54w &= 30{\cdot}359 \\ w &= 0{\cdot}551 \text{ ton per foot run.} \end{aligned}$$

(2) When the 4 in. leg is vertical

The modulus of section may be readily calculated since the neutral axis passes through the centres of gravity of both rectangles forming the **T**.

$$\begin{aligned} \text{I} &= \frac{5\frac{1}{2} \times (\frac{1}{2})^3}{12} + \frac{\frac{1}{2} \times 4^3}{12} \\ &= 2{\cdot}72 \end{aligned}$$

$$\text{Modulus of section} = Z = \frac{2{\cdot}72}{2''} = 1{\cdot}36$$

$$\text{M.R.} = 1{\cdot}36 \times 7 = 9{\cdot}52 \text{ inch-tons.}$$

The bending moment will be the same as above

$$\therefore \quad B.M. = M.R.$$
$$54w = 9\cdot52$$
$$w = 0\cdot176 \text{ ton per foot run.}$$

Massing up of Sections.—The resistance to bending of a section depends only on the disposition of the material normally to the neutral axis and not to its relative position along the axis. Sections of inconvenient shape, such as the channel of Fig. 67, are massed together along the neutral axis before constructing the modulus figure.

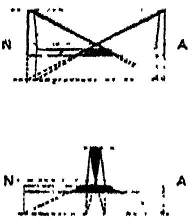
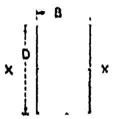
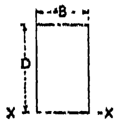


FIG. 67.

Construction of Modulus Figure for a Rail Section. The centre of gravity of the section, Fig. 68, is most readily found by cutting out the section in good quality cardboard, suspending it from two points and finding where the verticals through those points intersect. The centres of gravity for the modulus figures may also be found in this way.

For any horizontal layer, say 4_1 4_1, project the extremities on to the base line in IV, IV. Join IV, IV to a point in the neutral axis, such lines cutting the layer 4_1 4_1 in 4, 4. Then the points 4, 4 will be on the boundary of the modulus figure. The areas of the modulus figures are obtained by the aid of a planimeter or calculated by the aid of squared tracing paper. The rail section in the figure is a 100-lbs. rail drawn full size, and the modulus works out 11·46 inch units.

TABLE 25. MOMENTS OF INERTIA AND MODULI OF SECTIONS.

Section.	Moment of inertia about axis XX	Modulus of section about XX
	$\frac{BD^3}{12}$	$\frac{BD^2}{6}$
	$\frac{BD^3}{3}$	

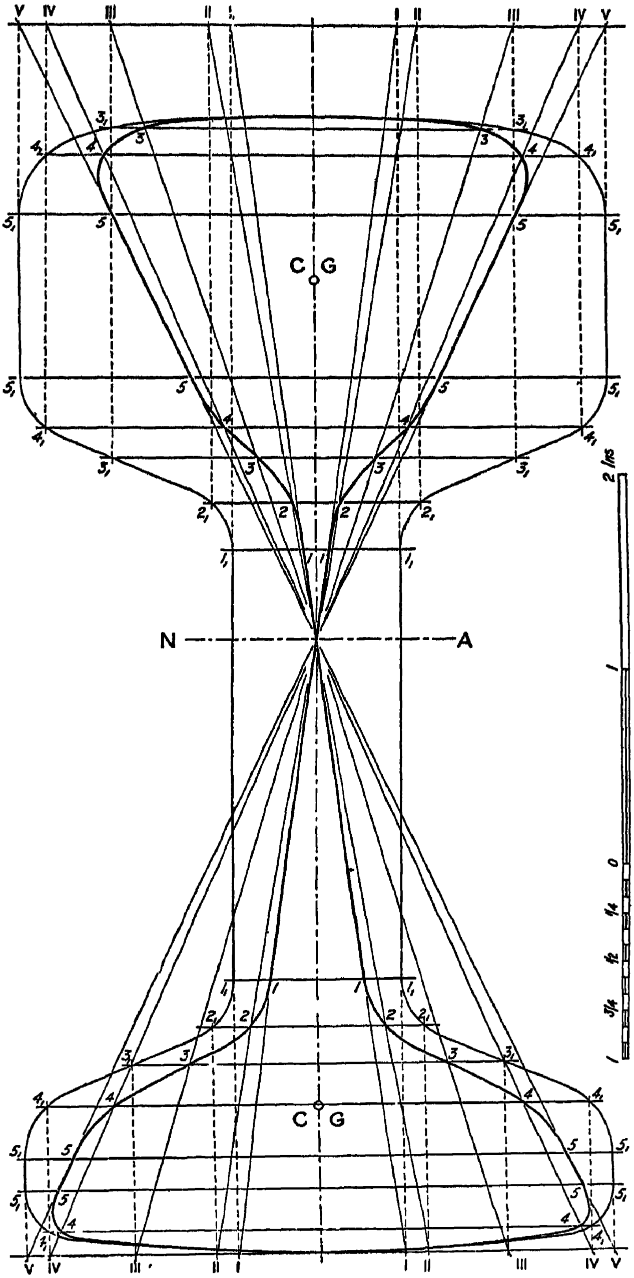
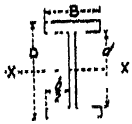
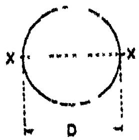
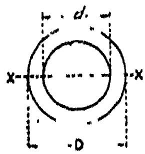
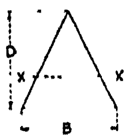
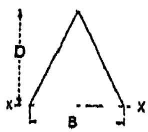
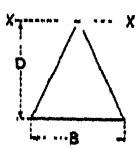


FIG. 68.

Section.	Moment of inertia about axis XX.	Modulus of section about XX.
	$\frac{BD^3 - bd^3}{12}$	$\frac{BD^3 - bd^3}{6D}$
	$\frac{\pi D^4}{64}$ 0·0491D^4	$\frac{\pi D^3}{32}$ 0·0982D^3
	$\frac{\pi(D^4 - d^4)}{64}$	$\frac{\pi(D^4 - d^4)}{32D}$
	$\frac{BD^3}{36}$	$\frac{BD^2}{24}$
	$\frac{BD^3}{12}$	
	$\frac{BD^3}{4}$	...

Shear.—When any system of forces acts on a beam it produces a vertical shearing action which tends to shear the beam in vertical planes, as in Fig. 69, A. The bending action creates differences of stress in the horizontal layers of the beam and thereby produces a horizontal shearing action between the layers. If the beam were composed of a number of separate plates, they would slide upon each other as in Fig. 69, B. In solid beams the tendency for the layers to slide upon each other is resisted by the shear stress in the material.

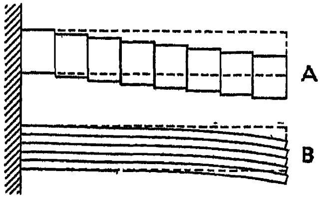


FIG. 69.

The method of calculating the vertical shearing force at any vertical section of a beam has been explained in Chapter III.

In Fig. 70 consider the equilibrium of a portion of a beam *acdb* lying between two vertical sections very close together. The horizontal forces acting on it are, the horizontal stress on *ac* caused by the bending, the horizontal stress on *bd* acting in the opposite direction, and the

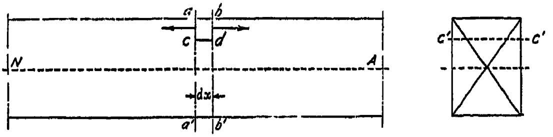


FIG. 70.

shear stress on *cd*, which is equal to the difference of the horizontal stresses on *ac* and *bd*. The horizontal stress above $c'c'$ at the section aa' is equal to the area of the modulus figure above $c'c'$ multiplied by the skin stress f at that section.

But
$$\text{B.M.}_a = \frac{\text{I}}{y} f$$
$$\therefore f = \text{B.M.}_a \times \frac{y}{\text{I}}$$

Similarly at the section bb' the skin stress f_1 will be

$$f_1 = \text{B.M}_b \times \frac{y}{\text{I}}$$

Let A_1 be the area of the modulus figure above $c'c'$. Then the difference of stress at the sections aa' and bb'

$$= \text{A}_1(f - f_1)$$
$$= \text{A}_1\frac{y}{\text{I}}(\text{B.M.}_a - \text{B.M.}_b) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let a_1 = area of a thin horizontal strip distant y_2 from the neutral axis.

The area of the modulus figure for this strip

$$= a_1\frac{y_2}{y}$$

Let A_1 be the sum of all such areas between a and c;

then $$A_1 = \Sigma a_1 {}^{y_2}_{y}$$

The moment of the area a_1 about the neutral axis $= a_1 y_2$

The total area of the section between a and $c = A = \Sigma a_1$

and its moment about the neutral axis $= \Sigma a_1 y_2$

Let Y be the distance of the centre of gravity of this area from the neutral axis.

Then $$A \times Y = \Sigma a_1 y_2$$

Also $$\frac{A_1}{A \times Y} = \frac{\Sigma a_1 {}^{y_1}_{y}}{\Sigma a_1 y_2}$$

from which $$A_1 = \frac{A \times Y}{y}$$

The total shear along cd will therefore

$$= \frac{A \times Y}{I}(\text{B.M.}_a - \text{B.M.}_b) \text{ from (1)}$$

Let the width of section $cc' = w$;

Then the *intensity* of shear on the plane cd

$$= f_s = \frac{A \times Y}{Iw} \cdot \frac{\text{B.M.}_a - \text{B.M.}_b}{dx}$$

In the limit $\text{B.M.}_a - \text{B.M.}_b = d\,(\text{B.M.})$.

But the total vertical shear S on any section

$$= \frac{d(\text{B.M.})}{dx}$$

Therefore $$f_s = \frac{AYS}{wI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Consider a small rectangular prism of material in a loaded beam (Fig. 71). The load and reaction produce shear stresses, acting in opposite directions on two vertical sides of the prism.

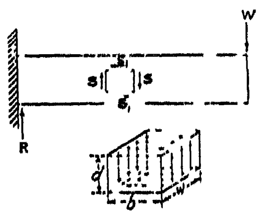


FIG. 71.

To establish equilibrium there must be another couple acting on the horizontal faces.

Let the total vertical stress $= S$

" " horizontal " $= S_1$

Let the intensity of vertical stress $= s$

" " horizontal " $= s_1$

Then $$Sb = S_1 d$$
$$swdb = s_1 wbd$$
$$\therefore s = s_1$$

That is, the intensity of horizontal shear on the material must be equal to the intensity of vertical shear.

Therefore the intensity of vertical shear at any point in the section of a beam must

$$= f_s = \frac{AYS}{wI}$$

Intensity of Vertical Shear Stress on a Rectangular Beam Section.—To find the intensity at the neutral axis.

A = area of section above NA $= \frac{bd}{2}$

Y = distance of centre of gravity of area above NA from the NA $= \frac{d}{4}$

$$w = b$$

$$I = \frac{bd^3}{12}$$

$$\therefore f_s = \frac{bd}{2} \times \frac{d}{4} \times S \times \frac{1}{b} \times \frac{12}{bd^3}$$

$$= \frac{3}{2} \cdot \frac{S}{bd}$$

But $\frac{S}{bd}$ is the *mean* intensity of shear on the section; therefore the intensity at the neutral axis is one and a half times the mean intensity. Let the section of the beam be 10″ × 6″ and the vertical shear be 10 tons.

Then the mean shear intensity

$$= \frac{10}{10 \times 6} = 0{\cdot}166 \text{ ton per sq. in.}$$

Intensity at the neutral axis

$= \frac{3}{2} \times 0{\cdot}166$ ton per sq. in.
$= 0{\cdot}25$ ton per sq in.

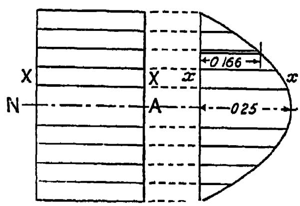


FIG 72

By calculating the values of f_s for a number of other planes and plotting them to a vertical line as in Fig. 72, a diagram of shear intensity for the section is obtained. The bounding curve will be a parabola.

Distribution of Shear Stress in a Beam Section (Fig. 73).—The stresses at a number of horizontal planes may be calculated by the above formula and the values plotted to a vertical line, or the values may be found from the modulus of section.

It has already been proved that

$$A_1 = \frac{AY}{y}, \text{ whence } A = A_1\frac{y}{Y}$$

Substituting this in the expression

$$f_s = \frac{AYS}{wI}$$

$$f_s = \frac{A_1}{w} \cdot \frac{yS}{I}$$

The quantity $\frac{vS}{I}$ is constant for any particular vertical section. Therefore the intensity of shear stress on any horizontal plane is proportional to the area A_1 of the portion of the modulus figure above that plane divided by the width of the section at the plane. At any plane *cd*, Fig. 73, the intensity will be equal to the shaded modulus area divided by the width *cd* and multiplied by the constant $\frac{vS}{I}$. The

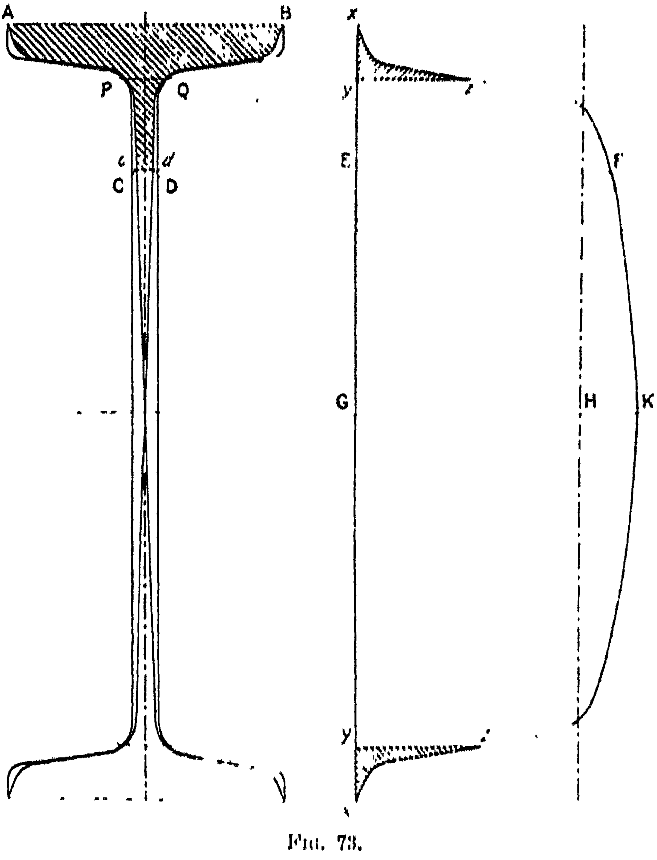


Fig. 73.

intensity diagram may be constructed by calculating a series of values of *f*, by the above method and plotting them to the line *xx*.

The diagram demonstrates (1) the small intensity of shear stress in the flanges (section lined on the diagram), and (2) the almost even distribution of stress over the web area. The maximum intensity = GK; the mean intensity over the whole section = GH. When designing beams with deep webs, the resistance to shear offered by the flanges is usually neglected, and the web designed to resist the whole shearing action.

EXAMPLE 13.—*To find the pitch of rivets in the flanges of a plated girder* (Fig. 74).

Let the section of the girder be, one 18″ × 7″ rolled beam with one 12″ × $\frac{5}{8}$″ plate riveted to each flange. Span of girder = 24 feet. Load = 48 tons distributed.

The maximum vertical shear will occur at the supports, and be equal to 24 tons.

Shear intensity at the horizontal plane between the plates and rolled beam

$$= f_s = \frac{\mathrm{AYS}}{w\mathrm{I}}$$

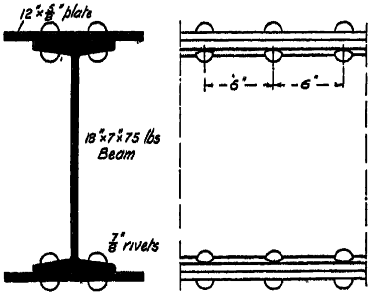


FIG. 74.

where A = sectional area of plate.
Y = distance of centre of gravity of plate from N.A.
S = total vertical shear on section.
I = moment of inertia of section.

w has two values

= width of plate when calculating the intensity in the plate.
= „ flange of beam „ „ „ joist.

Suppose the shear intensity along the plane under consideration to remain constant for a horizontal length of 12 inches. The total shear for this length would then be equal to the shear intensity multiplied by the area of the plane.

Area for 12 inches length = $w \times 12$.

Total shear stress for 12 inches length

$$= f_s \times 12w = \frac{\mathrm{AYS}}{w\mathrm{I}} \times 12w$$

$$= \frac{12\mathrm{AYS}}{\mathrm{I}}$$

$$= \frac{12 \times 12 \times \frac{5}{8} \times 9\frac{5}{16} \times 24}{2225}$$

$$= 9{\cdot}04 \text{ tons.}$$

Let the rivets be $\frac{7}{8}$ inch diameter.

Resistance to single shear of one rivet = 3 tons. (@ 5 Tons □″)

Number of rivets required per foot length

$$= \frac{9{\cdot}04}{3} = 3{\cdot}01$$

Four rivets would therefore be used, and being in pairs the pitch would be 6 inches.

The shear decreases to nothing at the centre of span, and therefore the pitch required would increase to a maximum at the centre of the span. It is not advisable, however, to make the pitch of rivets in

such girders more than 6 inches, to avoid local buckling of plate, so a uniform pitch would be kept throughout the full length of the girder.

NOTE.—If the bearing resistance of the rivets be less than the shearing resistance, the bearing resistance must be used in the above calculation in place of the shearing resistance.

Types of Beams.—Where long spans have to be bridged beams of correspondingly deep sections are necessary. Such beams will take the form of lattice or plate girders and the methods of design of these will be treated in subsequent chapters. The present chapter will be restricted to comparatively short span beams of practically constant section throughout their length.

Timber Beams are necessarily restricted to rectangular cross sections, as the material does not lend itself to the shaping of other forms as does steel or cast iron. Knowing the bending moments and shearing forces to be resisted a suitable rectangular section can be directly determined by equating the moment of resistance to the bending moment.

$$\begin{aligned} \text{Bending moment} &= \text{moment of resistance} \\ &= \text{modulus of section} \times \text{skin stress} \\ &= \frac{\text{breadth} \times (\text{depth})^2}{6} \times \text{skin stress} \end{aligned}$$

The depth will usually be made some suitable fraction of the span (see subsequent discussion in this chapter on ratio of depth to span), the skin stress will be a known quantity depending on the kind of timber used, leaving the breadth the only unknown factor in the foregoing equation. In the following table the *ultimate* skin stresses for the more commonly used timbers are given. A factor of safety suitable to the conditions under which the beam is employed must be used in conjunction with these values to determine the safe skin stress for insertion in the above equation.

Timber.	Ultimate skin stress	Timber.	Ultimate skin stress
	cwts. per sq. in.		cwts. per sq. in.
Ash, English .	114	Spruce . . .	78
Ash, American .	96	Oak, English.	108
Birch . . .	102	Pine, yellow .	70
Beech . . .	90	" red . .	72
Deal . . .	84	" Memel	72
Elm	60	" pitch . .	96
Greenheart . .	174	Teak	132

Steel Beams, with their advantages of additional strength and equal resistance in both tension and compression, are now adopted throughout structural work in preference to the cast and wrought iron beams formerly in use. The great variety of forms into which steel is rolled and the ease of riveting together combinations of these forms, presents the designer with a wide range from which to select the composition of his beam. Single joists or channels, where suitable to the loading conditions, make economical beams and lend themselves to simple and efficient connections. In cases where a wide bearing is required to support the loading, as in the case of lintels carrying thick walls over door or window openings, two or more joists can be

placed side by side as at (*a*), Fig. 74A, and connected together at intervals by distance pieces or separators.

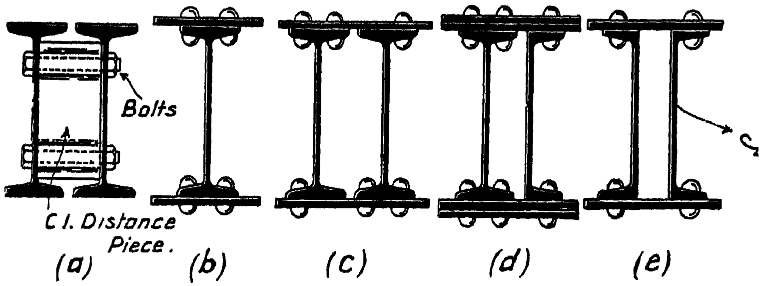


FIG 74A.

The strength of such a beam is equal to the strength of one joist multiplied by the number of joists in the section. The cast-iron distance pieces in no way affect the strength and are provided only to prevent relative motion of the joists.

Where a single section is inadequate to resist the bending moment a combination of sections, examples of which are shown in Fig. 74A, (*b*) to (*e*), can often be advantageously employed. (*b*) is formed by riveting together a joist and two or more plates. Care must be taken when designing such sections to provide a suitable depth for the span of the beam and also adequate shearing resistance in the web. (*c*) having a double web is suitable for comparatively short spans with heavy loading where a high shearing resistance is required. It has the disadvantage that the surfaces between the webs are inaccessible for painting after erection. In beams of this type only three rows of rivets can be driven in each flange, leaving an undesirable width between plate and joist loose. This objection is overcome in types (*d*) and (*e*), where one or two channels are substituted for one or both joists.

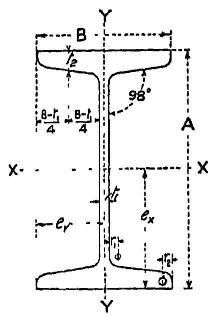


FIG 75.

Variations of strength can be obtained by adding to the number of plates, joists or channels. The flange plates need only extend along part of the length of the beam as required by the varying bending moment The method of determining the required lengths of flange plates is explained in Chapter VI. when considering the similar case of flange plates for the plate girders. The riveting of these compound girders must conform to the shear requirements as already explained, and to prevent buckling of the plates and liability of corrosion between adjacent surfaces the pitch of the rivets should in no case exceed six inches.

In calculating the moment of resistance of compound beam sections labour is saved by using the tables of properties of the standard rolled sections issued by the Engineering Standards Committee and generally to be found in the handbooks published by the leading structural contractors. The following table together with Fig. 75 shows the usual method of tabulation.

1	2	3	4	5	6	7	8	9	10
Reference No. and code word.	Size.	Standard thickness.		Radii.		Weight per foot, w	Sectional area, a.	Centre of gravity.	
	A × B	t_1	t_2	r_1	r_2			C_x	C_y
N.B.S.B. 1. Abscission	In. 3 × 1½	In. 0·160	In. 0·248	In. 0·260	In. 0·130	lbs. 4·00	In.² 1·176	In 0	In 0

11	12	13	14	15	16	17
Moments of inertia.		Radii of gyration.		Moments of resistance.		B.S.B. No.
I_x	I_y	r_x	r_y	R_x	R_y	
In.⁴ 1·657	In.⁴ 0·124	In 1·187	In 0·325	In 1·105	In 0·165	1

In column 1 the code word and reference number for use when ordering the section are given. Columns 2 to 10 contain the physical properties of the section. In columns 11 and 12 are given the moments of inertia about the axes X-X and Y-Y. In ascertaining the strength of a beam section to resist certain forces, the moment of inertia used will be that about the axis normal to the forces. The least moment of inertia of a section is also required in column calculations. Columns 15 and 16 are here called moments of resistance. This term must not be confused with the moment of resistance defined in this chapter as being the modulus of section multiplied by the skin stress. The tabular value headed Moment of Resistance is actually the modulus of the section, or $\frac{I}{y}$. The values are again given about both axes.

To use the above table to find what uniformly distributed load the $3'' \times 1\frac{1}{2}''$ beam would support over a span of L_f feet, the web of the section to be vertical.

$$\text{B.M.} = \text{M.R.} \times f$$

Let $f = 7$ tons per square inch,

Then
$$\frac{wL_f^2 \times 12}{8} = 1·105 \times 7$$

$$\therefore w = \frac{5·156}{L_f^2} \text{ tons per foot run.}$$

Note.—When using the formula $\frac{wL^2}{8}$ care must be taken to express L in the correct units. If w be given, as is usual, in tons per foot run, the total load on the beam will be equal to wL_f tons where L_f is in feet. The bending moment in inch-tons will be $\frac{wL_f \times L_i}{8}$ where L_i

is the span in inches. Or taking L throughout in feet the formula must be written $\frac{wL^2 \times 12}{8}$.

Columns 13 and 14 contain the radii of gyration. This property and its use will be explained in Chapter V.

EXAMPLE 14.—*To find the modulus of section for a compound beam.*

Let the beam be composed of one $20'' \times 7\frac{1}{2}''$ rolled beam and two $12'' \times \frac{5}{8}''$ plates riveted to the rolled beam by $\frac{7}{8}$ in. diameter rivets (Fig. 76).

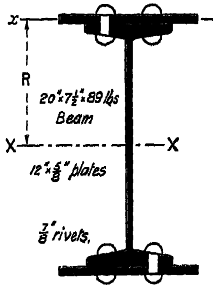


FIG. 76.

Neglecting for the present the effect of the rivets.

The moment of inertia of the rolled beam about the horizontal axis, from the tables

$$I_X = 1671{\cdot}291$$

Moment of inertia for the plates—

$$\begin{aligned} I_X &= 2(I_x + AR^2) \\ &= 2\left(\frac{12 \times (\frac{5}{8})^3}{12} + 12'' \times \frac{5}{8} \times (10\tfrac{5}{16})^2\right) \\ &= 1595{\cdot}7 \text{ in.}^4 \end{aligned}$$

Total I_X for the section

$$\begin{aligned} &= 1671{\cdot}291 + 1595{\cdot}7 \\ &= 3266{\cdot}991 \text{ in.}^4 \end{aligned}$$

The rivets in the flanges are staggered, so that not more than two rivets appear at any cross-section. If the rivet in the compression flange completely fills up the hole, the total area of the compression flange is not affected, but the liability of having rivets imperfectly fitted makes it advisable, to ensure safety, to deduct the area of the holes from the flange area when calculating the strength of the section. It is apparent that the holes through the tension flange *will* reduce the strength of that flange and *must* be taken into consideration. The moment of inertia for the section will therefore be $3266{\cdot}991 - I_x$ of two rivet holes

For $\frac{7}{8}$ in. rivets the holes are drilled $\frac{15}{16}$ in. diameter.

The mean thickness of the flanges, from the tables, $= 1{\cdot}01$ in., and may be taken to represent the mean length of the rivet in the beam. The total length of the rivet will be $1{\cdot}01 + 0{\cdot}625 = 1{\cdot}635$ in.

Area of cross-section of hole $= 1{\cdot}635 \times \frac{15}{16} = 1{\cdot}53$ sq. in.

$$\begin{aligned} I_X \text{ of 2 rivet holes} &= 2\left\{\frac{0{\cdot}94 \times (1{\cdot}635)^3}{12} + 1{\cdot}53 \times (9{\cdot}8)^2\right\} \\ &= 294{\cdot}567 \text{ in.}^4 \\ I_X \text{ of section} &= 3266{\cdot}991 - 294{\cdot}567 \\ &= 2972{\cdot}424 \\ \text{Modulus of section} &= \frac{I}{y} = \frac{2972{\cdot}424}{10\frac{5}{8}} \\ &= 279{\cdot}75 \text{ in.} \end{aligned}$$

Note.—Rivet holes, if not symmetrically placed, will change the position of the centre of gravity of the section, and therefore the position of the neutral axis. In such cases the tables are of little benefit.

Ratio of Depth to Span of Beams.—When selecting the form of a beam section due consideration must be paid to the depth, as it is upon this property of the section that the deflection of the beam depends. The allowable deflection varies with the class of work and is measured in terms of the span. The ratio of deflection to span for first-class bridge work is as low as 1 to 2,000, whilst for small girders and rolled steel joists in ordinary buildings the ratio may be as high as 1 in 400.

From Table 27, page 303, it will be seen that the maximum deflection of a beam of constant cross section

$$= c\frac{WL^3}{EI} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where c is a factor varying with the methods of loading and end conditions of the beam. Since the bending moment = moment of resistance

$$\frac{Wl}{c'} = \frac{fI}{y}$$

or

$$Wl = \frac{c'fI}{y}$$

Substituting for Wl in equation (1):

$$\text{maximum deflection} = cc'\frac{L^2 f}{Ey}$$

$$= \text{say } a\frac{L^2 f}{Ey} \quad . \quad . \quad . \quad (2)$$

For beams symmetrical about the neutral axis $y = \frac{D}{2}$, where D is the depth of the section.

For such beams the maximum deflection

$$\Delta = a\frac{2L^2 f}{ED} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The values of c are given in Table 27 and those of c' in Chapter III. As an example take the case of a cantilever with a concentrated load at its outward end.

Then $\qquad c = \frac{1}{3}$ and $c' = 1$

therefore $\qquad a = c \times c' = \frac{1}{3} \times 1 = \frac{1}{3}$

Other values of a:

cantilever with uniformly distributed load . . . . . . $a = \frac{1}{4}$

beam simply supported at ends, and central load . . . $a = \frac{1}{12}$

" " " " distributed load . . $a = \frac{5}{48}$

To find the depth of a mild steel beam centrally loaded, if the ratio of deflection to span must not exceed 1 to 1000, the working stress to be 7 tons per square inch, and E to be 13,000 tons.

$$\Delta = a \cdot \frac{2fL^2}{ED}$$

$$\frac{\Delta}{L} = a \cdot \frac{2fL}{ED}$$

$$\frac{1}{1000} = \frac{1}{12} \cdot \frac{2 \times 7}{13500} \cdot \frac{L}{D}$$

$$\therefore \frac{D}{L} = \frac{1}{11 \cdot 57}$$

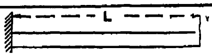
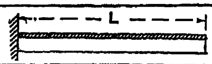
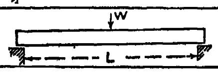
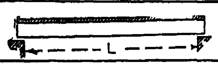
i.e. the depth must be equal to $\frac{1}{11 \cdot 57}$ of the span, or if the span be 100 feet the depth must be—

$$= \frac{100}{11 \cdot 57} = 8 \cdot 65 \text{ feet.}$$

The following table gives the ratios of D to L for mild steel beams for various ratios of deflection to span—

f has been assumed = 7 tons per square inch
E „ = 13,500 tons.

TABLE 26.—RATIOS OF DEPTH TO SPAN OF MILD STEEL BEAMS.

	Deflection formula.	Ratios of deflection to span.			
		1 to 400	1 to 600	1 to 1000	1 to 1500
		Ratios of $\frac{D}{L}$			
	$\Delta = \frac{1}{3} \cdot \frac{fL^2}{Ey}$	$\frac{1}{7}$	$\frac{1}{4 \cdot 8}$	$\frac{1}{2 \cdot 9}$	$\frac{1}{1 \cdot 2}$
	$\Delta = \frac{1}{4} \frac{fL^2}{Ey}$	$\frac{1}{9 \cdot 6}$	$\frac{1}{6 \cdot 4}$	$\frac{1}{3 \cdot 9}$	$\frac{1}{2 \cdot 6}$
	$\Delta = \frac{1}{12} \cdot \frac{fL^2}{Ey}$	$\frac{1}{29}$	$\frac{1}{19}$	$\frac{1}{11 \cdot 6}$	$\frac{1}{7 \cdot 7}$
	$\Delta = \frac{5}{48} \frac{fL^2}{Ey}$	$\frac{1}{23}$	$\frac{1}{15 \cdot 4}$	$\frac{1}{9 \cdot 2}$	$\frac{1}{6 \cdot 2}$

The values of $\frac{D}{L}$ for beams irregularly loaded will lie between those given for a single concentrated load at the centre, and a uniformly distributed load.

The values of $\frac{D}{L}$ for other values of f and E, say f_1 and E_1, may be obtained by multiplying the ratios by $\frac{f_1 E}{f E_1}$.

Flange Width.—Long beams unsupported laterally may deflect horizontally an undesirable amount due to the action of lateral wind or centrifugal loading if sufficient lateral stiffness is not provided. Such stiffness depends chiefly on the flange width, and since, in the majority of cases, the necessary stiffness cannot be theoretically determined, a

generally adopted practical rule is to make the flange width at least $\frac{1}{40}$ of the spacing between lateral supports. Usually secondary members, flooring joists, etc., will act as lateral supports at short intervals along the beam, in which case the necessary flange width to resist the primary bending moment will be in excess of the above practical requirements.

Connections.—In all framed structures the different members are fastened together by means of angles, plates, etc., and connecting rivets or bolts. The available methods of connection will in some cases determine the best cross-section of members to be employed. The duty of a connection is to transmit forces from one member of a structure to another, and all connections must receive the same care in design as the members themselves.

The simplest method of connecting beams is to allow one beam to rest on the flange of another and bolt them securely through the flanges. Such a connection is shown in Fig. 77, A. If there are two beams in line resting on the main girder, it is usual to fasten them together by means of fish plates in the webs. This increases the lateral stiffness of the beams and reduces the tendency to twist. Tapered washers should be placed under the heads and nuts of all bolts having a bearing

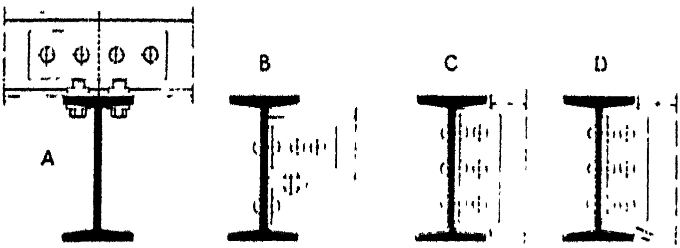


Fig. 77.

on the inside faces of beam flanges, otherwise only a very small part of the head or nut will be bearing on the flange. It is not always convenient to allow the secondary beams to rest on the top flange of the main beam, and in such cases web connections have to be employed. Fig 77, B, shows the connection of a comparatively small beam to a larger beam. An angle or tee riveted to the web of the main beam forms a bracket on which the small beam rests, and to increase the stiffness of the connection cleats are bolted to the webs. Figs. 77, C, D, show other connections where the beams are equal or nearly so in depth. Although unusual, the lower flange of a beam is sometimes joggled, so as to rest upon the tapered portion of the main beam flange (Fig. 77, D). This method is costly, and does not greatly increase the efficiency of the connection. The angle connections between the webs of rolled beams, and the number and spacing of rivets in them have been standardized, and may be found in any maker's section book. When using such standard connections, the strength of the rivets or bolts should be checked to ensure that the strength is at least equal to the shearing force at the connection. Bolts or rivets in such connections may fail either by shearing or crushing. The shearing strength of a rivet is equal to the area of its cross-section multiplied

by the shearing strength of the material. For rivet steel the shearing strength is 21 to 22 tons per square inch, and factors of safety of from 4 to 6 are usually adopted, giving a working stress of 4 to 5 tons per square inch. A rivet is said to be in single shear when for failure of the connection to occur, it is only necessary to shear the rivet at one section, as at *a* (Fig. 78). For the joint to fail in Fig. 78, *b*, the rivet must be sheared along two planes, or is said to be in double shear. Although the area of material sheared at *b* is twice that sheared at *a*, the strength of a rivet in double shear is found, in practice, to be less than twice the strength of a rivet in single shear. The strength of a rivet in double shear is from $1\frac{1}{2}$ to $1\frac{3}{4}$ times the strength of the rivet in single shear, and in the following calculations will be assumed as $1\frac{1}{2}$ times the strength in single shear.

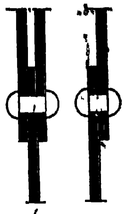


FIG. 78.

Let d = diameter of rivet in inches.
f_s = safe shearing stress of material in tons per square inch.
S = vertical shear at connection in tons.

Then the strength of a rivet in single shear $= \frac{\pi}{4} d^2 f_s$

" " " double " $= \frac{3}{2} \cdot \frac{\pi}{4} d^2 f_s$

Let n = number of rivets in single shear required to transmit the shearing force S.

Then $$S = n\left(\frac{\pi}{4} d^2 f_s\right)$$

or $$n = \frac{4S}{\pi d^2 f_s}$$

If n' = number of rivets required in double shear

$$n' = \tfrac{2}{3}\left(\frac{4S}{\pi d^2 f_s}\right) = \tfrac{8}{3} \cdot \frac{S}{\pi d^2 f_s}$$

The resistance to crushing offered by a rivet is equal to the crushing (or bearing) resistance of the material multiplied by the area of the rivet normal to the force. The safe bearing resistance of rivet steel is from 7 to 10 tons, say 8 tons per square inch.

Let t = thickness of plate bearing on rivet,
f_b = bearing stress of material.
Then the bearing resistance of one rivet $= dtf_b$
The number of rivets required to transmit the shearing force S

$$= n'' = \frac{S}{dtf_b}$$

The number of rivets required at a connection will be the larger value of n in the expressions—

$$n = \frac{4S}{\pi d^2 f_s} \text{ if the rivets are in single shear}$$

or $$n' = \frac{8}{3} \cdot \frac{S}{\pi d^2 f_s} \quad ,, \quad ,, \quad \text{double} \quad ,,$$

and $$n'' = \frac{S}{d t f_b}$$

In practice rivet holes are punched or drilled $\frac{1}{16}$ in. larger in diameter than the rivets, and on closing the rivets should fill up the holes, thus increasing the sectional area of the rivets, but calculations for shear should be based on the original diameter of the rivets. Bolt holes are drilled similarly, unless specified to be a driving fit, and there is always a little uncertainty as to the number really in action. For this reason more than are theoretically necessary are usually employed at connections.

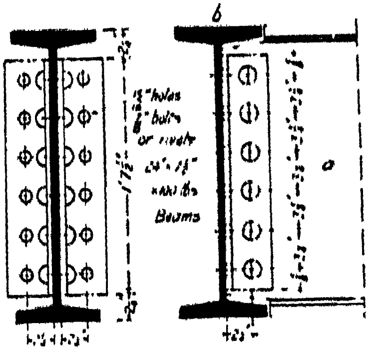


FIG. 79.

EXAMPLE 15. *To find the number of $\frac{7}{8}$ in. bolts and rivets required at the connection of a 24" × 7½" rolled steel beam supporting a uniformly distributed load of 50 tons, to another beam of similar dimensions.*

The connecting angles to be riveted to the beam a, Fig. 79, and bolted to the beam b.

The vertical shear at the connection is equal to the reaction, *i.e.* 25 tons.

The rivets being in double shear, the shearing resistance of one rivet

$$= \tfrac{3}{8}\pi d^2 f_s$$

Let $f_s = 5$ tons per square inch,

Then $$n = \frac{25}{\frac{3}{8} \times \pi \times (\frac{7}{8})^2 \times 5} = \text{(say) } 6$$

The bearing resistance of one rivet $= d t f_b$

where t = thickness of web = 0·6 inch.

Let $f_b = 8$ tons per square inch,

Then $$n = \frac{25}{\frac{7}{8} \times 0{\cdot}6 \times 8} = \text{(say) } 6$$

The bolts through the web of beam b being in single shear, the shearing resistance of one bolt

$$= \frac{\pi}{4} d^2 f_s$$

Therefore $$n = \frac{25}{\frac{\pi}{4} \times (\frac{7}{8})^2 \times 5} = \text{(say) } 9$$

The bearing resistance of one bolt $= dtf_b$

where t = thickness of angle $= \frac{1}{2}$ inch,

Therefore $$n = \frac{25}{\frac{7}{8} \times \frac{1}{2} \times 8} = \text{(say) } 8$$

The theoretical number of bolts required is therefore 9, but as 12 may be conveniently employed this number will be adopted to allow for a number being out of action.

The shearing resistance of the angles may be taken to be equal to the minimum area, *i.e.* along a vertical section through the rivets or bolts, multiplied by the resistance to shear of the material, although this value will be somewhat small on account of the resistance offered by the rivets or bolts to fracture at such a section.

In the above example the sectional area of the angles along the vertical section through the rivets or bolts

$$= 2\left(19\tfrac{1}{2} - 6 \times \tfrac{15}{16}\right) \times \tfrac{1}{2} = 13{\cdot}875 \text{ sq. in.}$$

The shearing resistance will therefore $= 13{\cdot}875 \times 5 = 69{\cdot}375$ tons.

This is greatly in excess of the vertical shear, but the thickness of angle cannot be much reduced, as such reduction would mean an increased number of rivets required in bearing.

Joints in Tension Members.—Structural members subject to a purely tensile stress are usually butt-jointed, with single or double covers at the joints, as B and C, Fig. 80. When only one cover is employed, there is a tendency for the member to bend near the joint, as at E. The better construction is to employ two covers at all such joints.

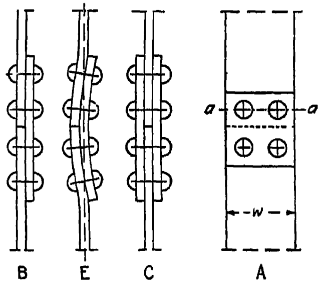


FIG 80.

Failure of the joint may occur—

(1) By shearing all the rivets to either side of the joint.

(2) By crushing the rivets.

(3) By pulling apart the cover plate or plates along the weakest section.

(4) By tearing of the main plate.

Consider the joint Fig. 80, A.

Let t = thickness of member.

t_1 = " cover plate or plates.

d = diameter of rivets.

d_1 = " " holes.

f_s = safe shearing intensity on rivet.

f_b = " bearing " "

w = width of member.

f_t = safe tensile intensity on plates.

T = tension in member.

Then the shearing resistance of the rivets to either side of the joint

$$= 2(d^2\frac{\pi}{4}f_s) \text{ for single cover}$$
$$= 2(\tfrac{5}{12}d^2\pi f_s) \text{ for double covers.}$$

The bearing resistance of the rivets to either side of the joint

$$= 2(dtf_b) \text{ in member.}$$
$$= 2(dt_1f_b) \text{ in single cover.}$$
$$= 2(2dt_1f_b) \text{ in double covers.}$$

The resistance of the cover plate or plates to tension along the section a–a

$$= (w - 2d_1)t_1f_t \text{ for single cover.}$$
$$= 2(w - 2d_1)t_1f_t \text{ for double covers.}$$

The resistance of the member to tension along the section a–a

$$(w - 2d_1)tf_t$$

The resistance in each of the above cases must be at least equal to the tension in the member.

EXAMPLE 16.—*A mild steel tension member is subject to a pull of 70 tons. Design a suitable section for the member and also a butt joint with double cover plates.*

The intensity of tensile stress not to exceed 7 tons per sq. in.
" " bearing " " 8 " "
" " shear " " 5 " "
Double shear to be taken equal to $1\frac{2}{3}$ times single shear.
Adopting a rivet diameter of $\frac{7}{8}$ inch.
The shearing resistance of one rivet = 5 tons.
The number of rivets required to either side of the joint

$$= \frac{70}{5} = 14$$

Let t = thickness of member.

Then the bearing resistance of one rivet $= \frac{7}{8} \times t \times 8$

and for the bearing resistance of the rivets to be equal to the pull in the member

$$(\tfrac{7}{8} \times t \times 8)14 = 70 \text{ tons}$$

from which $t = \frac{5}{7}$ inch, say $\frac{3}{4}$ inch.

Arranging the rivets as in Fig. 81, the weakest section at which the member might fail would occur at either a–a or b–b. If the section at a–a be assumed for the present as the weakest, let w = width of the member,

Then $(w - \frac{15}{16})\frac{3}{4} \times 7 = 70$ tons,
from which $w = 14{\cdot}27$ inches, say $14\frac{1}{2}$ inches.

For failure to occur along the section b–b, the plate must be torn along that section and the leading rivet sheared. The strength along b–b will therefore

$$= (14{\cdot}5 - 2 \times \tfrac{15}{16})\tfrac{3}{4} \times 7 + 5$$
$$= 71{\cdot}3 \text{ tons.}$$

The member is therefore strong enough along this section to resist the pull.

The strength along the section c–c is greater than along the section b–b, since the reduction of plate area for the extra rivet in the section

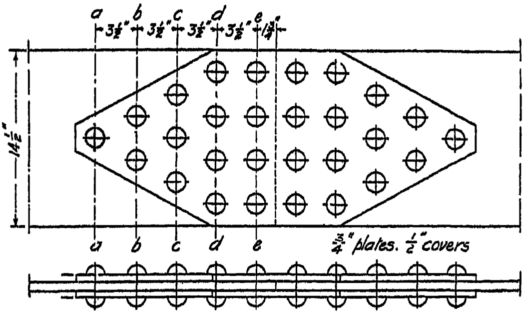


FIG. 81.

c–c does not weaken that section to the same extent as the shearing of the two extra rivets along b–b increases the resistance to failure along c–c. For the same reasons failure would not take place by tearing of the member along the sections d–d or e–e.

The joint may fail by tearing the cover plates along e–e, or by the crushing of the rivets in the covers.

Let t_1 = thickness of each cover.

The resistance to tearing of the covers along e–e must be equal to the pull on the member, or

$$(14\tfrac{1}{2} - 4 \times \tfrac{15}{16})2t_1 \times 7 = 70 \text{ tons}$$
$$\text{from which } t_1 = 0{\cdot}46, \text{ say } 0{\cdot}5 \text{ inch.}$$

Since the combined thickness of the covers is greater than the thickness of the member, the bearing resistance of the rivets in the covers will be greater than the bearing resistance in the member, and failure would not occur by crushing of the rivets in the covers.

Examples of such joints applied to the ties of lattice girders will be found in Fig. 210.

Riveted Connections subject to Bending Stresses.—Suppose, in Fig. 82,

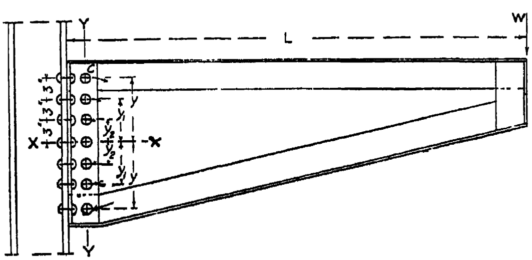


FIG. 82.

a cantilever L inches long to be loaded at its outward end with W tons.

Then the bending moment at the connection of the cantilever with its support = WL inch-tons.

This bending moment tends to produce a rotation about a horizontal axis perpendicular to X-X, and subjects the rivets along the section Y-Y to a horizontal shearing stress, in addition to the vertical stress due to the vertical shear at the section. The moment of the horizontal shearing stresses in the rivets along Y-Y about the horizontal axis through X-X, must equal the bending moment at the section. The moment of inertia of rivet c about the axis through X–X = $I_0 + ay^2$,

where I_0 = moment of inertia of the rivet section about its longitudinal axis.
a = area of cross-section of rivet in shear.
y = distance of centre of cross-section of rivet from the axis.

Since I_0 is very small compared with ay^2, the moment of inertia may be considered equal to ay^2.

The sum of the moments of inertia of the system of rivets will then—

$$= \Sigma ay^2 = a\,(y^2 + y_1^2 + y_2^2, \text{ etc.})$$
$$= I$$

Let f_s = maximum horizontal shear stress in outermost rivet.
y = distance of centre of that rivet from the axis.

Then the moment of resistance of the system of rivets

$$= \frac{f_s I}{y} = \frac{f_s}{y} a(y^2 + y_1^2 + y_2^2, \text{ etc.}).$$

Therefore the bending moment at the section must

$$= \text{B.M.} = \frac{f_s a}{y}(y^2 + y_1^2 + y_2^2, \text{ etc.}),$$

from which the maximum horizontal shearing stress in the rivets is obtainable. The vertical shear will be equally distributed amongst the rivets.

Let f_h = the maximum intensity of horizontal stress.
f_v = the intensity of vertical stress.

Then the maximum stress on the rivets will be the resultant of f_h, and $f_v = \sqrt{f_h^2 + f_v^2}$.

The bending moment also produces tension in the rivets above X–X, in the other plane of the connection.

Then $$\text{B.M.} = \text{M.R.} = \frac{f_t a}{y}(y^2 + y_1^2 + y_2^2, \text{ etc.}).$$

EXAMPLE 17.—*Let the span of the cantilever be 5 feet, and the load 2 tons.*

The bending moment at the connection

$$= 5 \times 12 \times 2 = 120 \text{ inch-tons.}$$

Assume a depth of 21 inches for the beam, and let it be connected

by two angle irons to the support. The moment of resistance of the seven rivets of $\frac{7}{8}$ in. diameter along the section X–X, Fig. 83.

$$= f_s \times a \times 2\left(\frac{3^2+6^2+9^2}{9}\right)$$

Since the rivets are in double shear, the area a will be equal to twice the area of the cross-section of a $\frac{7}{8}$-in. rivet $= 2 \times 0{\cdot}6 = 1{\cdot}2$ sq. in.

Then B.M. = M.R. to shearing.

$$120 = f_s \times 1{\cdot}2 \times 2\left(\frac{3^2+6^2+9^2}{9}\right)$$

$$\therefore f_s = 3{\cdot}57 \text{ tons per sq. in.}$$

Let f_b = maximum intensity of bearing stress on the rivets.
a_1 = the bearing area of one rivet.
t = thickness of web plate.

Then $a_1 = t \times \frac{7}{8}$ in.

The moment of resistance to bearing of the line of rivets will

$$= f_b \times a_1 \times 2\left(\frac{3^2+6^2+9^2}{9}\right)$$

Let the thickness of the web plate $= \frac{3}{8}$ in.

Then $a_1 = \frac{3}{8} \times \frac{7}{8} = 0{\cdot}33$ sq. in.

Again B.M. = M.R. to bearing

$$120 = f_b \times 0{\cdot}33 \times 2\left(\frac{3^2+6^2+9^2}{9}\right)$$

$$\therefore f_b = 12{\cdot}9 \text{ tons per sq. in.}$$

This is in excess of the safe bearing stress, and either more rivets must be used or the web plate thickened to give a greater bearing area. Suppose a second line of rivets be used, Fig. 83,

then $$\text{M.R.} = f_b \times 0{\cdot}33 \times 2\left(\frac{3^2+6^2+9^2+(\frac{3}{2})^2+(\frac{9}{2})^2+(\frac{14}{2})^2}{9}\right)$$

$$\therefore 120 = f_b \times 14{\cdot}48$$

and $$f_b = \frac{120}{14{\cdot}48} = 8{\cdot}28 \text{ tons per sq. in}$$

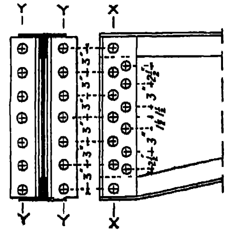


FIG. 83.

This stress would only occur on the two extreme rivets, and may be safely adopted.

The addition of the second row of rivets will decrease the horizontal shear stress in the outermost rivets to 2·36 tons per square inch. The total maximum shear in the rivets

$$= \sqrt{f_h^2 + f_v^2} = \sqrt{(2{\cdot}36)^2 + \left(\frac{2}{13 \times 1{\cdot}2}\right)^2}$$

$$= 2{\cdot}362 \text{ tons per sq. in.}$$

The moment of resistance of the rivets along the lines Y–Y

$$= f_t \times a \times 4\left(\frac{3^2 + 6^2 + 9^2}{9}\right)$$

$$\text{B.M.} = \text{M.R.}$$

$$120 = f_t \times 0{\cdot}6 \times 4\left(\frac{3^2 + 6^2 + 9^2}{9}\right)$$

$$\therefore f_t = 3{\cdot}57 \text{ tons per sq. in.}$$

i.e. the maximum tension on the rivets is much below the safe working stress, and a reduced number might be safely adopted. Only the rivets in the upper half of the connection are stressed under the bending action, the bearing of the back of the bracket against the column relieving the rivets in the lower half of the longitudinal stress. All the fourteen rivets are subject to a slight vertical shear stress of mean intensity $= \frac{2}{14 \times 0{\cdot}6} = 0{\cdot}24$ ton per square inch, so that the actual maximum intensity of stress will be slightly in excess of 3·57 tons per square inch.

EXAMPLE 18.—**Design of Floor for Warehouse.** Suppose the outline in Fig. 84 be the plan of a floor of a warehouse for which a design is required.

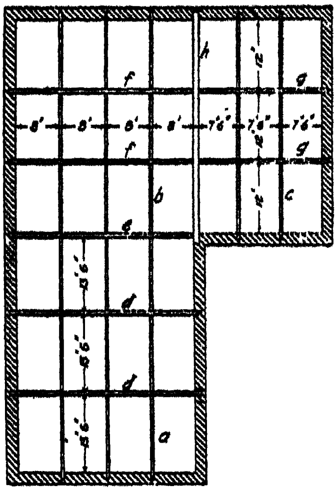


FIG. 84.

The first consideration is the kind of floor to be adopted; whether fireproof or not. It will be supposed that a fireproof floor is not required, and an ordinary timber floor supported on steel beams is decided upon.

Let the estimated live load on the floor be equivalent to a dead load of $1\frac{1}{2}$ cwts. per square foot of floor area. The dead load may be determined as the design proceeds.

A suitable covering for the floor would be $1\frac{1}{2}$ in. flooring secured to 11″ × 3″ timber joists spaced at 1′ 6″ centres. The span of the floor joists will be determined by their depth. For stiffness the span should not exceed 10 times the depth; therefore the maximum span will be 10 × 11″ = 9′ 2″. The arrangement shown in the figure makes the maximum span 8′ 0″, or 8·6 times the depth of the joists.

Stress in Timber Floor Joists.—Each floor joist supports an area of floor = $1\frac{1}{2}$′ × 8′ = 12 sq. feet. Load on each joist

$$\text{Weight of flooring} = 12 \times \frac{1{\cdot}5}{12} \times 35 \qquad = 52{\cdot}5 \text{ lbs.}$$

$$\text{,,} \quad \text{one joist} = \frac{11 \times 3}{12 \times 12} \times 8 \times 35 = 62{\cdot}5 \text{ ,,}$$

$$\text{Live load} = 12 \times 1\tfrac{1}{2} \times 112 \qquad = 2016 \text{ ,,}$$

$$\text{Total distributed load} = 2131$$

$$\text{say} = 19 \text{ cwts.}$$

Weight of timber has been taken as 35 lbs. per cubic foot.

$$\text{Max. B.M.} = \frac{wl}{8}$$

$$= \frac{19 \times 112 \times 8 \times 12}{8}$$

$$= 25{,}536 \text{ in.-lbs.}$$

$$\text{Modulus of section of joist} = \frac{3 \times 11 \times 11}{6}$$

$$= 60{\cdot}5$$

$$\text{Maximum stress in the timber} = \frac{25{,}536}{60{\cdot}5} = 422 \text{ lbs. per sq. in.,}$$

which gives a factor of safety of about 10. The timber joists in the offset bay having less span would be stressed to a less extent, but for uniformity 11″ × 3″ joists would be adopted throughout.

Let the working stresses for the steel beams be—

f_t (tension or compression) = 7 tons per sq. in.
f_s (shear intensity) not to exceed 3 tons per sq. in.

Primary Beams, *a*, 13′ 6″ *long*.—The maximum loading for these beams would be as shown in Fig. 85.

Reaction from each timber joist = 9·5 cwts.
Load at each bearing = 19 „
Reactions = 85·5 „

Maximum B.M. (at centre)

$$= 85{\cdot}5 \times 6{\cdot}75 - 19(1{\cdot}5 + 3 + 4{\cdot}5 + 6)$$
$$= 292{\cdot}125 \text{ ft.-cwts.}$$
$$= 175{\cdot}275 \text{ in.-tons}$$

$$\text{Modulus of section required} = \frac{175{\cdot}275}{7}$$
$$= 25{\cdot}04$$

The depth of the section is controlled by the allowable deflection, which may be taken in this case as $\frac{1}{400}$ of the span. The depth from Table 26 would require to be about $\frac{1}{23}$ of the span, since the loading approximates very nearly to a distributed load.

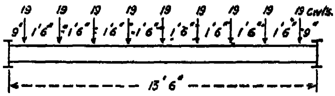


FIG 85.

$$\therefore \text{Min. depth} = \frac{13{\cdot}5 \times 12}{23} = 7 \text{ in.}$$

Referring to a list of properties of beam sections, it is found that an 8″ × 6″ × 35 lbs. section has a modulus of section 27·649, also a 10″ × 5″ × 30 lbs. section has a modulus of 29·137. The 10″ × 5″ beam would be stiffer than the 8″ × 6″ beam and there would be a saving in weight by adopting that section.

The dead load of the beam = 30 × 13·5 = 405 lbs.

Total reactions would then = 85·5 + 1·7 = 87·2 cwts.

Allowing for this extra dead load the modulus of section required would be increased to 25·51, but still remain below that of the beam.

$$\text{Area of the web of beam} = 0{\cdot}36 \times 8{\cdot}8$$
$$= 3{\cdot}168 \text{ sq. in.}$$
$$\text{Average shear on web} = \frac{87{\cdot}2}{3{\cdot}168 \times 20}$$
$$= 1{\cdot}37 \text{ tons per sq. in.}$$

The web is therefore strong enough to resist the shear.

Primary Beams, b, 12′ 0″ long (Fig. 86).—The load at each bearing of joists will be as in the previous case = 19 cwts.

Reactions = 76 cwts.

$$\text{Max. B.M. (at centre)} = 76 \times 6 - 19\,(0{\cdot}75 + 2{\cdot}25 + 3{\cdot}75 + 5{\cdot}25)$$
$$= 328 \text{ ft.-cwts.}$$
$$= 136{\cdot}8 \text{ in.-tons.}$$
$$\text{Modulus of section required} = \frac{136{\cdot}8}{7}$$
$$= 19{\cdot}54$$

A beam 8″ × 5″ × 28·02 lbs. section having a modulus equal to 22·339 would be strong enough, but the difference in weight of an

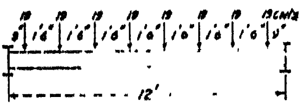


FIG. 86.

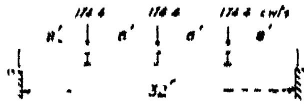


FIG. 87.

8″ × 5″ and a 10″ × 5″ section is only 1·97 lbs. per foot, and so for uniformity the 10″ × 5″ × 29·99 lbs. section would be adopted.

Weight of beam = 3·2 cwts.

Total reactions = 77·6 cwts.

Shear on web = 1·2 tons per sq. in.

Beams, d (Fig. 87). Reaction from each primary beam a = 87·2 cwts.

Load at each bearing = 174·4 cwts.

Reactions = 261·6 cwts.

$$\text{Max. B.M. (at centre)} = 261{\cdot}6 \times 16 - 174{\cdot}4 \times 8$$
$$= 2790{\cdot}4 \text{ ft.-cwts.}$$
$$= 1674{\cdot}24 \text{ in.-tons.}$$
$$\text{Modulus of section required} = \frac{1674{\cdot}24}{7} = 239{\cdot}18$$
$$\text{Min. depth} = \frac{32 \times 12}{23} = 16{\cdot}7 \text{ in., say 17 in.}$$

No standard beam section has the required modulus. A broad flanged beam 20″ × 12″ (nominal size) × 138 lbs. having a modulus of section of 272 might be adopted.

Weight of beam = 1·97 tons.

Bending moment at centre due to weight of beam

$$= \frac{1{\cdot}97 \times 32 \times 12}{8} = 94{\cdot}56 \text{ in.-tons.}$$

$$\text{Total B.M. at centre} = 1674{\cdot}24 + 94{\cdot}56 = 1768{\cdot}8 \text{ in.-tons.}$$

$$\text{Modulus of section required} = \frac{1768{\cdot}8}{7} = 252{\cdot}7$$

The 20″ × 12″ beam is therefore strong enough to resist the bending.
Area of web = 16·5 × 0·76 = 12·54 sq. ins.
Total shear = 14·065 tons.
Average shear on web = 1·1 tons per sq. in.
Beams, f (Fig. 88).—Reaction from each primary beam = 77·6 cwts.
Load at each connection = 155·2 cwts.

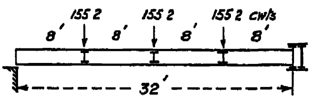


Fig. 88.

Reaction = 232·8 cwts.
Max. B.M. (at centre)
= 232·8 × 16 − 155·2 × 8
= 2483·2 ft.-cwts.
= 1489·92 in.-tons.

$$\text{Modulus of section required} = \frac{1489{\cdot}92}{7} = 212{\cdot}86$$

Min. depth of beam = 17 in.

A 24″ × 7½″ × 100 lbs. beam has a modulus = 221·231
Weight of beam = 1·43 tons.

$$\text{Max B.M. due to weight of beam} = \frac{1{\cdot}43 \times 32 \times 12}{8} = 68{\cdot}64 \text{ in.-tons.}$$

$$\text{Total max. B M.} = 1489{\cdot}92 + 68{\cdot}64 = 1558{\cdot}56 \text{ in.-tons.}$$

$$\text{Modulus of section required} = \frac{1558{\cdot}56}{7} = 222{\cdot}65$$

This is slightly in excess of the modulus of a 24″ × 7½″ beam, but as the maximum stress would only be $\frac{1558{\cdot}56}{221{\cdot}231} = 7{\cdot}04$ tons per square inch, this section may be adopted.

Total reactions = 12·355 tons.
Area of web = 9·6 sq. ins.
Average shear on web = 1·28 tons per sq. in.

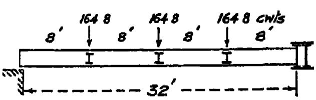


Fig. 89.

Beam, e (Fig. 89) —
Reaction from each primary beam b = 77·6 cwts.
„ „ „ a = 87·2 „

Load at each connection = 164·8 „

Reactions = 247·2 cwts.
Max. B.M. (at centre) = 247·2 × 16 − 164·8 × 8
= 2686·8 ft.-cwts.
= 1582·08 in.-tons.

$$\text{Modulus of section required} = \frac{1582 \cdot 08}{7} = 226 \cdot 01$$

Min. depth = 17 in.

A broad flanged beam will again be suitable; 19″ × 12″ × 128 lbs has a modulus of 241.

Weight of beam = 1·83 tons.

$$\text{Max. B.M. due to weight of beam} = \frac{1 \cdot 83 \times 32 \times 12}{8}$$
= 87·81 in.-tons.
Total B.M. = 1582·08 + 87·84
= 1669·92 in.-tons.

$$\text{Modulus of section required} = \frac{1669 \cdot 92}{7} = 238 \cdot 56$$

which is less than the modulus of the beam.
Total reactions = 13·275 tons.
Area of web = 10·35 sq.-in.
Average shear on web = 1·3 tons.

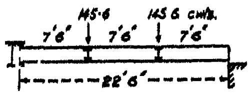


FIG. 90.

Beams, g (Fig. 90).

$$\text{Load on floor joists in offset} = \frac{19 \times 7 \cdot 5}{8} = 17 \cdot 8 \text{ cwts.}$$

Total load on each primary beam = 17·8 × 8 = 142·4 cwts.

$$\text{Weight of each primary beam} = \frac{12 \times 30}{112} = 3 \cdot 2 \text{ cwts.}$$

$$\text{Reaction from each primary beam} = \frac{142 \cdot 4 + 3 \cdot 2}{2} = 72 \cdot 8 \text{ cwts.}$$

Load at each connection = 145·6 cwts.
Reactions of beam *g* = 145·6 cwts.
Max. B.M. due to loads (between loads) = 145·6 × 7·5
= 1091·6 ft.-cwts.
= 654·96 in.-tons.

$$\text{Modulus of section required} = \frac{654 \cdot 96}{7} = 93 \cdot 56$$

Say 18″ × 7″ × 75 lbs. with modulus of 127·7.
Weight of beam = 0·75 ton.

$$\text{B.M. due to weight of beam} = \frac{0 \cdot 75 \times 22 \cdot 5 \times 12}{8}$$
= 25·31 in.-tons.
Total B.M. = 25·31 + 654·96
= 680·27 in.-tons.

$$\text{Modulus of section required} = \frac{680 \cdot 27}{7} = 97 \cdot 18$$

Total reactions = 7·655 tons.
Area of web = 8·8 sq. in.
Average shear on web = 0·87 ton per sq. in.

Beam h (Fig. 91).—

Reaction from beam f = 12·355 tons.
Reaction from beam g = 7·655 „

Total load at connection = 20·01 „

Reactions of beam h = 20·01 „

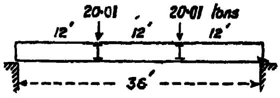


FIG. 91.

Max. B.M. due to loads (between loads) = 20·01 × 12
= 240·12 ft.-tons
= 2881·44 in.-tons.

$$\text{Modulus of section required} = \frac{2881{\cdot}44}{7} = 411{\cdot}64$$

$$\text{Minimum depth} = \frac{36 \times 12}{23} = \text{say } 19 \text{ in.}$$

A compound girder, composed of two 20″ × 7½″ rolled joists, with one 16″ × ⅝″ plate riveted on each flange, has a modulus of 460.

Weight per foot length of girder = 252 lbs.

Total weight of beam = 4 05 tons.

$$\text{B.M. due to weight of beam} = \frac{4{\cdot}05 \times 36 \times 12}{8}$$
$$= 218{\cdot}7 \text{ in.-tons.}$$
$$\text{Total max. B.M} = 2881{\cdot}44 + 218{\cdot}7$$
$$= 3100{\cdot}14 \text{ in.-tons.}$$
$$\text{Modulus of section required} = \frac{3100{\cdot}14}{7} = 442{\cdot}9$$
$$\text{Total reactions} = 20{\cdot}01 + \frac{4{\cdot}05}{2} = 22{\cdot}035 \text{ tons.}$$

Area of webs = 21·6 sq. in.
Average shear on webs = 1·02 tons per sq. in.

Pitch of rivets in flanges.

Number of rivets required per foot (see Example 13)

$$= \frac{12\text{AYS}}{\text{I}} \div \text{R} \ (\text{R} = 3 \text{ tons for } \tfrac{7}{8}'' \text{ rivets})$$
$$= \frac{12 \times (16 \times \frac{5}{8}) \times 10\frac{5}{16}'' \times 22{\cdot}035}{4888 \times 3}$$
$$= 1{\cdot}9 \text{ rivets } \tfrac{7}{8}'' \text{ diar.}$$

As the pitch should preferably not exceed 6 inches, a 6-inch pitch will be adopted.

Connections.—To avoid having too great a depth of floor the beams must be fastened together by web connections. A suitable arrangement is shown in Figs. 92 and 93.

The theoretical requirements for resisting the shear only have been exceeded to add lateral stability to the connections. For example, at the connections of the beams f to the girder h, the theoretical number of ¾ in. rivets in double shear required through the web of beams f

$$= \frac{\text{vertical shear}}{\text{resistance of one rivet}}$$

$$= \frac{12 \cdot 355}{3 \cdot 3} = \text{(say) 4 for shear.}$$

$$\text{For bearing} = \frac{12 \cdot 355}{8 \times 0 \cdot 6 \times 0 \cdot 75} = \text{(say) 4}$$

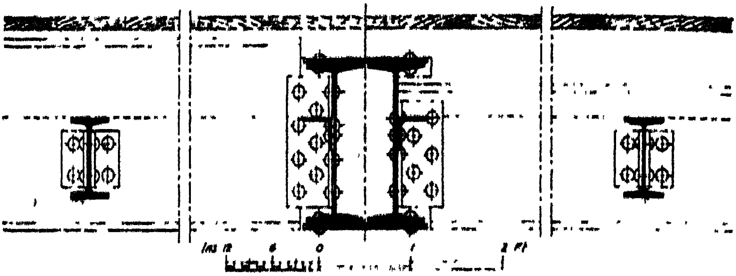


FIG. 92.

Number of rivets required through the web of girder h in single shear

$$= \text{for shear } \frac{12 \cdot 355}{2 \cdot 2} = \text{(say) 6}$$

$$= \text{for bearing } \frac{12 \cdot 355}{8 \times 0 \cdot 5 \times 0 \cdot 75} = \text{(say) 5}$$

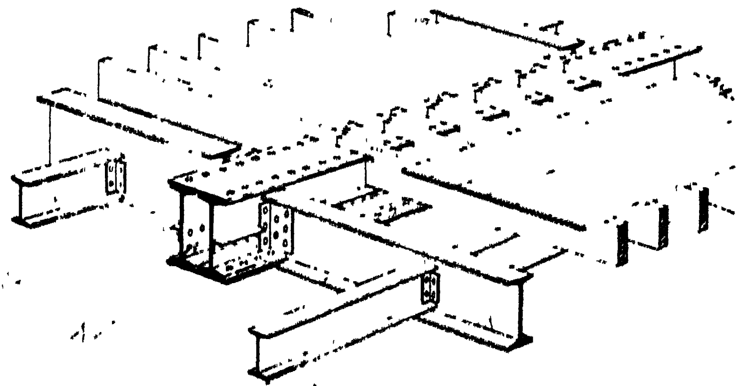


FIG. 93.

Bearing on Walls.—The length of bearing on walls should not be less than the depth of the beam, and a minimum length of 8 inches should be allowed for beams of less than 8 inches in depth. Suppose stone templates be used under the ends of all beams resting on walls, and the safe-bearing pressure on such templates be 15 tons per square foot. Dividing the reactions of the beams by 15 will give the bearing

area required, and again dividing by the flange width the required length of bearing is obtained. In each case of the present example, the length of bearing required by such calculation will be less than the depth of the beam, and therefore the bearing lengths will be made equal to the depths of the beams.

Angle runners are riveted to the girder *h* for supporting the ends of the timber joists.

Beams under the Action of Non-parallel Forces.—In the foregoing discussion on the resistance of beams all the loading forces have been acting parallel to one axis of the beam section, but cases frequently arise in which the forces are not all parallel, and the distribution of the stress on the section is then materially altered from that in the previously considered beams.

Suppose a beam, the section of which is shown in Fig. 93A, be simply supported at the ends and be acted upon by two forces F and F_1 acting in the directions indicated. The force F deflects the beam in a downward direction and produces compressive stress in the material along the top side AB and tensile stress along the lower side CD. Likewise the force F_1 deflects the beam sideways, causing compressive stress along the side AC and tensile stress along the side BD. At the corner A both forces create compressive stress, whilst at the corner B the forces induce stresses of opposite sign. The maximum compressive stress will evidently occur at A and be the sum of the stresses induced by the two forces. The stress at B will be the algebraic sum of the compressive and tensile stresses caused by the different forces and may be either compressive or tensile, depending on the comparative magnitudes of F and F_1 and the resistances of the section about its vertical and horizontal axes. Similarly the maximum tensile stress will occur at the corner D, and the stress at C may be found by summing the compressive and tensile stresses as at B.

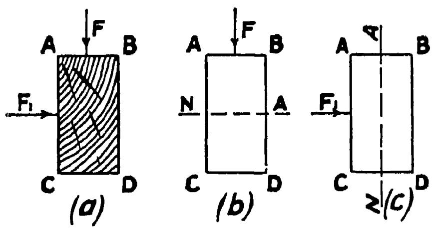


Fig. 93A.

Example.—*A greenheart beam 14″ deep, 6″ wide and 10 feet span is simply supported at the ends and loaded with a vertical load of 10 cwts. per foot run and also a horizontal load of 3 cwts. per foot run. Find the factor of safety if the ultimate skin stress of greenheart be assumed equal to 174 cwts. per square inch.*

Referring to Fig. 93A, the vertical load corresponds to the force F and the horizontal load to the force F_1.

The maximum bending moment produced by F

$$= \frac{10 \text{ cwts.} \times 10 \times 120 \text{ in.}}{8}$$

The modulus of section resisting this bending $= \dfrac{6 \times 14^2}{6}$

Therefore the maximum skin stress produced by F

$$= \frac{10 \times 10 \times 120}{8} \times \frac{6}{6 \times 14^2} = \pm 7{\cdot}6 \text{ cwts. per sq. in.}$$

By the same method of calculation the maximum skin stress produced by $F_1 = \frac{3 \times 10 \times 120}{8} \times \frac{6}{14 \times 6^2} = \pm 5{\cdot}3$ cwts. per sq. in.

The maximum stresses produced by the combined forces are :—

compressive at A = $+ 7{\cdot}6 + 5{\cdot}3 = + 12{\cdot}9$ cwts. per sq. in.
tensile at D = $- 7{\cdot}6 - 5{\cdot}3 = - 12{\cdot}9$ " "

Factor of safety = $\frac{174}{12{\cdot}9} = 13{\cdot}5$

The stress at B = $+ 7{\cdot}6$ cwts. per sq. in. induced by F
$- 5{\cdot}3$ " " " F_1
$+ 2{\cdot}3$ " " " F and F_1

The stress at C = $- 2{\cdot}3$ cwts. per sq. in.

The maximum shearing stresses at the ends of the beam are the resultants of the stresses produced by the loads separately. The average shearing stress at the ends in the foregoing example

$$= \frac{\sqrt{10^2 + 3^2}}{2} \times 10 \times \frac{1}{14'' \times 6''}$$

$$= 0{\cdot}62 \text{ cwt. per sq. in.}$$

The reactions at the ends of the beam have horizontal components $= \frac{3 \text{ cwts.} \times 10}{2} = 15$ cwts., and provision must be made for these reactions in securing the ends against lateral displacement.

Where considerable horizontal loading of a beam occurs, as in the above example, the necessary width of the beam should be determined in a similar manner as for the depth, as explained on page 116.

The loading forces on beams may be inclined to the principal axes of the section, as in the case of the dead load on the roof purlins in Example 35; but such forces can be resolved into components normal to the principal axes of the section and the components treated as separate loads corresponding to the loads in the foregoing example.

Combined Bending and Direct Stresses. The weight of individual members of framed structures, or loads imposed on such members between the panel points, may produce considerable bending stresses in the material in addition to the stresses resulting from the panel point loading. With perfectly designed connections the direct stresses arising from the frame loading are distributed equally over the cross section of the member, but any bending stresses there may be will vary in intensity and sign about the neutral axis of the section. Combining the direct and bending stresses will result in producing a maximum intensity of stress greater than the direct stress alone. The distribution of stress on any cross section can be obtained by calculating separately the bending and direct stresses and summing them algebraically.

The effect of secondary bending in compression members will be

more fully treated in a subsequent chapter. In tension members secondary bending stresses often restrict the length of such members, and in some cases make it advisable to introduce redundant members to reduce the effect of the bending action. (See example of roof principal, page 335, in which such a member is employed from the ridge to the middle tie.)

Improperly designed end connections are often a source of bending in members that otherwise would be free from bending. For the stress to be equally distributed over the cross section of a member of a frame, the rivets in the end connections must be distributed symmetrically about the line through the centre of gravity of the section.

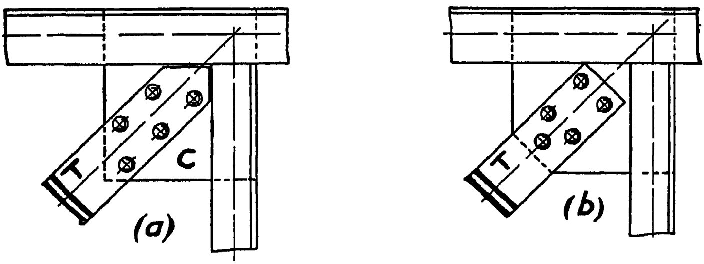


FIG. 93B.

In Fig. 93B (*a*) the tension member T is connected to other members of the frame by means of rivets and a junction plate C. The calculated number of rivets required is, say, five. If disposed as in (*a*) their combined resultant line of action falls to the right of the centre line of the member and will produce a downward bending action and resulting bending stresses in the member. The bending moment produced is equal to the total force on the member as found from the frame calculations multiplied by the eccentricity of the line of action of the rivets from the centre line of the member. The bending stresses can then be obtained as in previous beam examples, and the maximum stress on the member derived by summing the direct and bending stresses. If the rivets be redistributed as in (*b*) there will be no bending on the member, which will then have only the direct frame force to resist.

Machine frames, crane hooks, etc., in which the member itself is not straight throughout its length, are other examples in which secondary bending stresses play an important part, and must be taken into consideration when designing such structures.

CHAPTER V.

COLUMNS AND STRUTS.

STRUCTURAL members which are exposed to compressive stress in the direction of their length are classed generally as columns or struts, the term column, pillar, or stanchion being more usually applied to the main uprights of framed buildings, whilst compression members of girders and trusses are referred to as struts. The practical design of compression members, especially those in which the length is great compared with the cross-sectional dimensions, is relatively a more difficult problem than is the case with the majority of structural members, since theory does not furnish so reliable a guide and considerable judgment and experience are very essential.

The mathematical theory regarding the strength of columns has been ably and thoroughly developed by numerous investigators. It is based, however, on various assumptions which are never realized in practice, and the absence of one or more of these assumptions materially affects the capability of resistance of the column. The assumptions made in regard to the ideal or theoretical column are as follow:—

1. Perfect straightness of the physical axis.

2. The load is considered as a purely compressive stress acting along the axis of the column, or, in other words, centrally applied.

3. Uniformity of cross-section.

4. Uniform modulus of elasticity of the material of which the column is constructed, throughout the whole length of the column and over every part of any cross-section.

It is sufficiently difficult sensibly to realize these conditions in a carefully prepared and mounted test column, whilst it is certain that all practical columns and struts fail to comply with at least one and usually more than one of these conditions. Considerable discrepancy, therefore, exists between the theoretical strength of a column as deduced from mathematical considerations and the practical strength obtained by methods of testing, or from the observation of columns which have failed *in situ*. It is important, however, to bear in mind the above conditions, since the degree in which they are realized in any particular column furnishes a valuable aid to judgment in deciding to what extent the mathematical theory may be relied on.

The material employed and method of manufacture largely influence the degree in which a practical column will tend to realize the above conditions. No material, however carefully manufactured, is perfectly

uniform in structure and elasticity, although a very high degree of uniformity is realized by modern methods of manufacture in the case of mild steel, and too much emphasis has frequently been laid upon the variable nature of the material in accounting for the discrepancies between theory and practice. In the case of wrought iron and mild steel the effect of cold straightening is to locally strain the material beyond the limit of elasticity, with the result that the stiffness of the fibres overstrained in tension is considerably lowered as regards resistance to compressive stress, and the fibres overstrained in compression are affected similarly as regards their resistance to tensile stress. Permanent internal tensile and compressive stresses are thus set up in the material, the effect of which is by no means insignificant. Conclusions regarding the resistance of columns, deduced from experimental tests, may further be materially affected by the previous history of the material. The following instances of the influence of history of material have been given by the late Sir B. Baker in the case of experiments carried out on solid mild steel columns, thirty diameters in length, showing that the resistance varied according to previous treatment, as follows:—

	Tons per sq. in.
Annealed	14·5
Previously stretched 10 per cent.	12·6
„ compressed 8 „	22·1
„ „ 9 „	28·9
Straightened cold	11·8

The general adoption of machine riveting in the case of built-up members is another frequent cause of local initial stress, as well as of initial curvature. The effect of riveting up an assemblage of plates and bar sections is to cause the various bars to stretch and creep past each other in different degrees. This is most marked where light and heavy sections are adjacent to each other, the lighter sections being more severely stretched during riveting than the heavy. In symmetrical sections, the camber caused by riveting down one side of a long member will be sensibly neutralized by riveting along the opposite side, so that the finished member may be apparently straight, but the ultimate effect will be the creation of initial local stress in the material. In the case of unsymmetrical sections, permanent curvature or waviness in the direction of length with unavoidable occasional twisting results from the process of riveting, and these effects are difficult to minimize, however carefully the work may be executed. Columns of cast iron or cast steel are not subject to defects caused by riveting, but are influenced by the usual hidden defects inherent to all castings, as well as by initial stresses set up by unequal contraction. Hollow cast columns are especially liable to irregularity of cross-section due to the core getting slightly out of centre during casting, which defect will be more marked if the column be cast in a horizontal or inclined position.

In Fig. 94, let AB represent a hollow circular column having irregular horizontal cross-sections as indicated. The *geometrical* axis is the *straight* line AB, which, in the absence of defects of material and irregularity of cross-section, would be assumed to be the true or

physical axis of the column. In such a case the line of application AB of the load would coincide everywhere with the physical axis of the column, and the resulting stress on every cross-section would be purely compressive, with no tendency to bend the column. Further, if the physical axis of a long column were a perfectly straight line, it would be possible to apply a steadily increasing central load until the column failed by direct crushing of the material. In Fig. 94 the centres of gravity of the irregular cross-sections occur at the points a, b, c, etc., and these being transferred to their corresponding positions a', b', c', on the elevation, the physical axis of the column becomes the curved line A a' b' c' . . . B. This alteration in outline of the physical axis is due only to the considered defects of cross-section, but it will be borne in mind that non-uniformity of elasticity of the material and initial camber may still further modify the positions of points a', b', c', etc. The above defects are, in any practical column, quite unknown quantities, so that it is impossible to state with accuracy to what extent the physical axis does or does not coincide with the geometrical axis. The resulting effect of this deviation of the physical axis from the geometrical axis is that at cross-section No. 1 there is acting a direct compression = P, and also a bending moment = $P \times a'1$. At cross-section No. 2, a direct compression = P and a B.M. = $P \times b'2$, and so on. It is the presence of this bending moment which determines the tendency of the practical column to yield towards one side or the other, depending on the outline of the physical axis. The points a, b, c, etc., may be termed the centres of resistance of the various sections, being understood to represent the points through which the resultant compression *should* act in order to create uniform intensity of compression over the whole cross-section, after making allowance for defects of form of cross-section and variable elasticity of material. It follows that simple compression on every cross-section of a column might only be ensured if the line of action of the load coincided with the physical axis. Since, however, the line of action of the load is a straight line and the physical axis a curved or wavy line, it is *practically* impossible for any column not to be subject to more or less bending moment at several sections throughout its length.

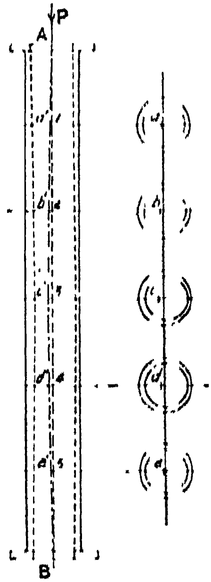


FIG. 94.

Method of Application of Load.—In practice, relatively few columns have the load centrally applied. In Fig. 95, A, a girder carrying similar loads over two adjacent equal spans will impose a resultant *central* load on the column. At B, two unequally loaded girders connected to opposite sides of the same column will impose a resultant load on the column, the line of action of which may be considerably out of coincidence with the axis of the column. At C, the load on a girder attached to one face of the column will impose a still

more eccentric load. This may be modified, as at D, by employing two girders, G, G, instead of one, and carrying them on brackets placed centrally on opposite faces of the column. This arrangement, although more satisfactory theoretically, is often inconvenient in

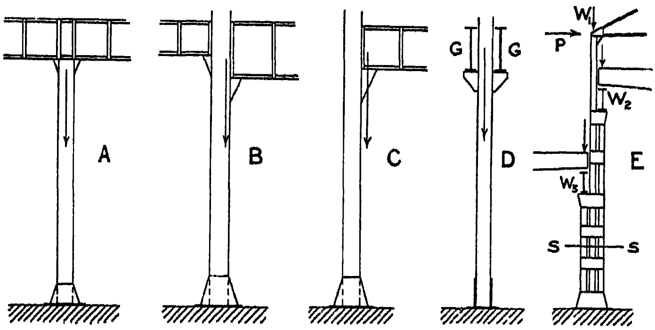


FIG. 95.

practice, since it multiplies and complicates the connections, and interferes with the arrangement of other members meeting on the same column. In the case of compound columns built up of two or three girder sections with tie-plates or lattice bracing as at E, the line of application of the load becomes more difficult to define. Such columns generally carry vertical loads W_1, W_2, and W_3, due respectively to roof weight and the loads handled by travelling cranes, whilst they are further subject to bending moment caused by the horizontal wind pressure P acting on the roof slope. The manner in which the resultant load is shared by the three columns at any horizontal section *ss*, will depend largely on the strength and rigidity of the bracing.

A further small amount of additional bending moment is caused by the deflection of the column itself. In Fig. 96, the straight line AB represents the original axis of the column before the imposition of the load. If a load P be applied at a small eccentricity e, the resulting B.M. will be $P \times e$, which will cause a small deflection of the column indicated by d. The ultimate B.M. at the central section of the column, after it has reached a state of equilibrium, will then be $P \times (e + d)$. In most practical cases, the deflection being very small, the additional moment $P \times d$, due to that deflection, may be neglected. The jibs of cranes and long horizontal or inclined struts in large bridge girders are subject to additional deflection caused by their own weight, which still further increases the B.M. upon them. Thus in Fig. 97 the straight line AB represents the axis of a crane jib when in an unstrained condition. An appreciable amount of sag will be caused by the dead

FIG. 96.

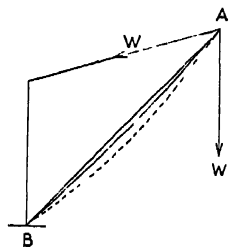


FIG. 97.

weight of the jib, due to its inclined position, which will cause it to assume some curved outline, as shown by the full line curve. When lifting a weight W, the pressure on the jib due to the tensions W, W, in the chain will augment the deflection, so that the axis of the jib takes up some new position indicated by the dotted curve. A similar action takes place in all horizontal and inclined structural members subject to end thrust.

From the above remarks it will be apparent that the manner in which the compressive load affects a column or strut is more complex than is usually the case with tension members, and cannot be dismissed by the assumption of a simple compressive stress acting at every cross-section.

Methods of supporting or fixing the Ends of Columns.—Whilst the above remarks apply to any column irrespective of the way in which its ends are supported, the manner of support or attachment of the ends of a column to adjacent members of a structure, greatly influences the load that the column will safely carry. Four well-defined methods of end support are easily recognized. These are indicated diagrammatically in Fig. 98.

The column at A is said to be hinged or pin-ended, and under the load deflects in a single curve. B illustrates a fixed-ended column which under the load is constrained to deflect in a treble curve with points of contra-flexure at P and Q. In column C the lower end is fixed and the upper end hinged or rounded, the deflection causing a double curvature with one point of contra-flexure at P. In cases A, B, and C the upper end of the column is supposed to be situated vertically over the lower end, and to be incapable of lateral movement, so that on deflection the upper end becomes slightly depressed along the vertical line. Fig. 98, D, represents a column fixed at the lower end, but free to move laterally at the upper end when deflection under the load takes place. Such a column will obviously bend in a single curve, and its behaviour will be sensibly similar to one-half of the round-ended column at A, as may be indicated by drawing in a similarly deflected lower half shown by the dotted line.

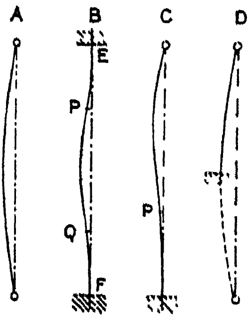


Fig. 98.

In practice, so-called round-ended columns are constructed by forming the ends to fit on round bearing pins or in hollow curved sockets. Examples of these occur in crane jibs and in the struts of pin-connected girders—a type very frequently adopted in American practice. With regard to "pin-ended" columns, Mr. J. M. Moncrieff, M. Inst. C.E., remarks, "It is quite useless to theorise with the view of showing their superiority to round or pivot ends, owing to the fact that their behaviour under load, even in a testing machine, depends very largely on the closeness of the fit between pin and hole, upon the smoothness or otherwise of the bearing surfaces, upon the diameter of the pin in relation to the radius of gyration (of the column section),

and upon the presence, either accidental or premeditated, of a lubricating medium."[1]

In actual practice, a truly fixed-ended column seldom, if ever, exists. Fixity of ends implies that the ends are so firmly held that, on bending, the portions EP and FQ of the column in Fig. 98, B, remain strictly tangent to the straight line EF. This is only possible where the head and foot of the column are so rigidly held, or attached to adjoining portions of a structure, as to be absolutely immovable laterally —a condition difficult to realize in experiments with a testing machine, and still more difficult of realization in practical structures. The nearest approach to a fixed-ended column in practice is probably exemplified in the case of the lowermost portion of a heavy column in a framed building. The foot is secured to a heavy foundation block of concrete sunk a considerable distance into the earth, and the head secured to relatively heavy and rigid girders supporting the first floor of the building. Even this, however, is not a truly fixed-ended column, since the girders, no matter how rigid they may be, must deflect to some extent under their load, and so permit of slight movement of the head of the column, whilst the whole building is subject to lateral movement due to wind pressure. The appearance of a column, either on a working drawing or *in situ* frequently gives a very false impression of its fixity. It is a commonly claimed advantage for riveted connections in structural work that the compression members are constrained to act as fixed-ended columns. This is in many cases a quite erroneous assumption, since the degree of approximation to fixity of ends depends entirely upon the relative stiffness of the column and the other members of the structure attached to it, and the estimation of this degree of fixity demands very careful consideration on the part of the designer. The following two cases cited by Mr. Moncrieff are instructive and suggestive. In Fig. 99, assume a series of stiff gantry

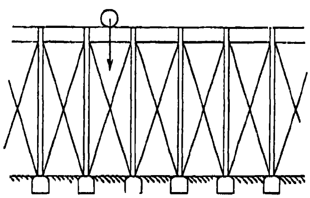


FIG. 99.

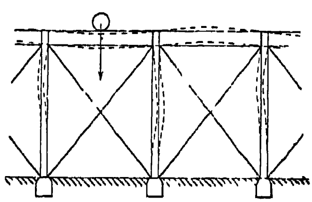


FIG. 100.

girders 2 ft. deep by 10 ft. span, riveted securely to the heads of columns 30 ft. high, firmly braced together to preserve their verticality. Assume also that the foundation blocks on which the columns rest are very rigid, that the columns have large well-bolted bases, and that the ratio of length to radius of gyration of these columns is very large, and the columns, therefore, slender in proportion. Then the imposition of load on any span will cause deflection in the girder, and the ends of the girder will deviate from the vertical to a slight degree, but the relative stiffness of the girders themselves, as compared with the

[1] *Transactions Am. Soc. C.E.*, vol. xlv. p. 358.

column, being high, the approximation to ideal fixity of ends would, practically speaking, be of a high degree.

In Fig. 100, let the columns be spaced at 30 ft. centres, retaining the same depth of girder, 2 ft., and merely increasing the girder sections to obtain the same value of working unit stress, while increasing the radius of gyration of the columns to provide much greater stiffness of column. Under these conditions the deflection of the girder under load, and consequently the slope of the ends of the girders where they are securely riveted to the column heads, would be increased largely, and the columns would be subjected to heavy bending stresses in addition to their direct load. These columns would be much less heavily stressed if they had pin-joint connections to the girders, and the apparent fixity of end given by a secure riveted connection would actually be accompanied by severely prejudicial secondary stresses.

A distinction requires to be drawn between "fixed-ended" and "flat-ended" columns. The assumption has generally been made that these two types of columns act in an identical manner, and formulæ giving the permissible loads for both in one expression are frequently quoted. This is quite erroneous, both theoretically and from the evidence of practical tests. In the case of a column with flat ends, that is, in which the ends are merely kept in contact with their bearing surfaces by pressure, no tensile stress can be developed at the ends, whilst with fixed ends a considerable amount of tension may be safely resisted. The two cases are illustrated in Fig. 101.

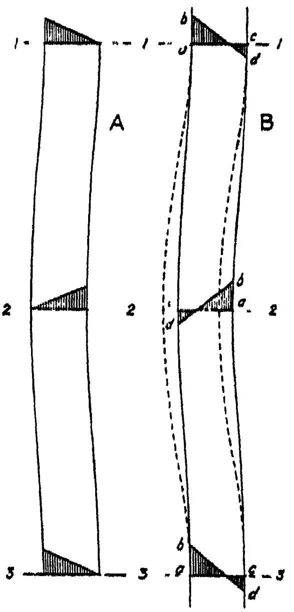


FIG. 101.

Fig. 101, A, represents a flat-ended column in a deflected condition, such that compressive stresses are set up at the three sections, 1-1, 2-2, 3-3, of intensities shown diagrammatically by the shaded areas. So long as the stress on the two end sections is entirely compressive, the ends of the column will remain in close contact with the bearing surfaces applying the load, and the column will behave in exactly the same manner as a fixed-ended column of similar dimensions. In such a case any bolts or other fastenings employed with a view to fixing the ends of the column will be quite inoperative. Fig. 101, B, represents a fixed-ended column such that compressive stresses of intensity *ab* are set up on the *left-hand* side of sections 1-1 and 3-3, and the *right-hand* side of section 2-2, and tensile stresses of intensity *cd* on the *right-hand* side of sections 1-1 and 3-3, and on the *left-hand* side of section 2-2. In this case the tension *cd* at section 2-2 will be resisted by the material of the column, whilst the tensile stresses *cd* at sections 1-1

and 3–3 will have to be resisted by the bolts or rivets which connect the column with neighbouring parts of the structure. If the connecting bolts on the right-hand side of sections 1–1 and 3–3 of column B were cut through so as to transform the column into a flat-ended column whilst under load, the deflection would immediately increase and the column alter its curvature, as indicated by the dotted lines. Any increase of the load in column A would *attempt* to set up a state of stress similar to that in column B, but column A being flat-ended, and therefore incapable of resisting tension at its ends, would deflect or spring further towards the left hand in a similar manner to the supposed case of column B with its end connections severed. A flat-ended column, although *apparently* as strong as a fixed-ended column, may actually be on the verge of failure by excessive deflection, if the load be of such a nature as to set up incipient tension along one edge of the bearing surfaces. This point is clearly evidenced by the results of tests of flat-ended columns made by Mr. Christie.

In actual structures it is not customary to employ purely flat-ended columns, bolts or rivets being invariably inserted to make connections with the foundation and upper members of the structure. These, however, are often employed more with a view to convenience in erection and to prevent lateral movement of the column, than to specifically resist tensile stresses which, under certain conditions of loading, may become very severe. It is important, therefore, in designing columns on the assumption of fixed ends, to ensure the bolted or riveted connections being sufficiently strong or numerous to resist the abovementioned tensile stress. Fig. 98, D, represents a type of column which most commonly occurs in connection with roof designs.

Fig. 102, A, illustrates the case of a detached roof carried by columns fixed to a substantial foundation at F, F. The columns carry a vertical load, W, due to the weight of the roof, whilst their upper ends are subject to appreciable horizontal movement due to the wind pressure P. Fig. 102, B, is another example of this type of column, supporting an "umbrella" or island platform roof. In both cases the columns deflect in a single curve.

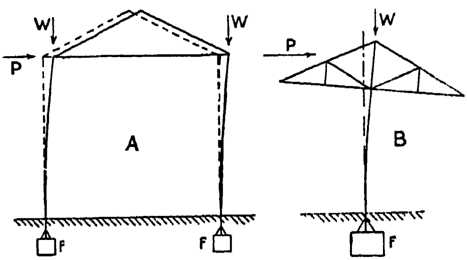


FIG. 102.

From the preceding remarks it will be apparent that no column in actual practice is ever subject to a uniform compressive stress per square inch, since the bending action in combination with the direct loading results in increasing the intensity of compression on the concave or hollow side of the column and in decreasing the intensity of compression on the convex side. Further, in cases where the bending moment is large compared with the direct compression, the stress on the convex side may be tensile instead of compressive. The maximum

stress per square inch on the section of a column under given conditions of loading may only be arrived at by a calculation of the bending moment as well as the direct compression per square inch. Most of the column formulæ in general use aim at giving the safe *uniform* compression per square inch for columns of given dimensions and material. Whilst this is the most convenient form in which to use such formulæ, it should be remembered that the *maximum stress* on the material of the column is usually considerably in excess of the *uniform* or *average* stress exhibited by the formulæ. Consideration will now be given to some of the many formulæ in common use.

Radius of Gyration.—Before proceeding to these, it is necessary to comprehend what is implied by the term Radius of Gyration of a column section. If, for any column section, the moment of inertia about *any* axis be divided by the sectional area, the square root of the resulting quotient gives the radius of gyration about that axis. Or, if

I = moment of inertia $\quad A$ = sectional area

and $\quad r$ = radius of gyration $\quad r = \sqrt{\frac{I}{A}}$.

As an example consider the two sections shown in Fig. 103, A and B. A is a girder section of 30 sq. in. area. Section B is a box-section having the same over-all dimensions and also the same sectional area.

For section A, moment of inertia about axis $Y-Y = 167 \cdot 5$ in. units.

$$\text{Sectional area} = 30 \text{ sq. in.} \quad \therefore r = \sqrt{\frac{167 \cdot 5}{30}} = 2 \cdot 36 \text{ in}$$

For section B, moment of inertia about axis $Y-Y = 272 \cdot 5$ in. units.

$$\text{Sectional area} = 30 \text{ sq. in.} \quad \therefore r = \sqrt{\frac{272 \cdot 5}{30}} = 3 \cdot 01 \text{ in.}$$

Whilst possessing the same sectional area, section B has a decidedly larger radius of gyration than section A. The increase in the radius of gyration of section B is evidently caused by the altered distribution of the web section, since the flange section is the same for both A and B. The radius of gyration is thus seen to be dependent on the *shape* of section or distribution of material about the axis in question. It will be obvious merely from an inspection of the two sections that B would form a stiffer column than A, provided equal lengths were taken, and it may be stated generally that the radius of gyration of a section affords a relative measure of its stiffness to resist a bending or "buckling" action such as occurs in columns. In order to form an estimate of the *actual* stiffness of a column, the length as well as the radius of gyration of the cross-

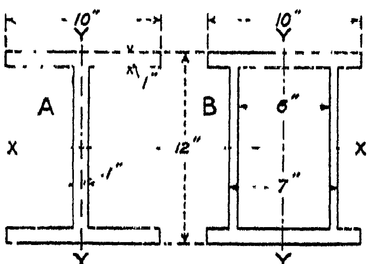


FIG. 103.

section must be taken into account. Thus, if a column of section B be twice the length of a column of section A, the former will be less capable of supporting a given load than the latter, notwithstanding its greater radius of gyration. In other words, the greater length of column B rendering it more slender than column A, will more than neutralize the advantage it possesses by reason of its greater radius of gyration. If the length of a column be divided by the radius of gyration of its cross-section, the resulting ratio $\frac{l}{r}$ may be regarded as a measure of its *slenderness*. Thus, if a column of section A be 15 ft. long, and one of section B be 30 ft. long, then

for column A, $\qquad \frac{l}{r} = \frac{15 \times 12''}{2 \cdot 36} = 76$

and for column B, $\qquad \frac{l}{r} = \frac{30 \times 12''}{3 \cdot 01} = 119,$

column B is considerably more slender than A, and consequently less capable of carrying so great a load. It will be seen presently that the ratio $\frac{l}{r}$ constitutes an important term in all formulæ which aim at giving the safe or breaking loads for columns.

In the above example the radii of gyration were calculated about the axis Y–Y. They may also be calculated about the axis X–X, or, if desired, about any other axis passing through the section. Considering the axis X–X of section A, the distribution of material is plainly different from that with regard to Y–Y. The moments of inertia about X–X and Y–Y will therefore have different values, and since the sectional area remains the same whatever axis be considered, the radius of gyration will necessarily vary with the moment of inertia. Thus

Moment of inertia of section A about X – X = 690 in. units.

Sectional area = 30 sq. in. $\therefore$ r about X – X = $\sqrt{\frac{690}{30}} = 4 \cdot 8$ in.

The previously calculated radius of gyration about Y–Y was 2·36″, and the significance of the two figures is that a column of section A is a little more than four times as stiff to resist bending about the axis X–X (or in the plane Y–Y), than about the axis Y–Y (or in the plane X–X). If a column is equally free to spring or buckle towards one side or another, it will naturally yield in that direction in which it is least stiff, or, in other words, it will bend in the plane at right angles to the axis about which its radius of gyration is least. Thus, columns A and B would both more readily bend *in the plane* XX than in the plane YY, since both have a smaller radius of gyration *about the axis* Y–Y than about X–X. The least value of the radius of gyration for any given section is commonly referred to as the **Least Radius** simply. In many column sections the axis about which the radius of gyration is least is easily recognized at sight. Such sections as a circle, hollow circle, square, etc., have the same radius of gyration about *any* axis passing through the centre. In some built-up sections, notably box sections,

as in Types 8 and 9, the radius of gyration may differ appreciably about the axes X–X and Y–Y, and it is advisable to calculate both rather than hastily assume the axis of least radius from inspection only. It will be obvious that the most economical forms of column sections, so far as load-bearing capacity is concerned, will be those having equal, or practically equal, radii of gyration about both principal axes. A built-up section may generally be arranged to give this result, although practical considerations, with regard to convenience of connections, sometimes preclude the employment of such sections. The radii of gyration of the elementary rolled sections will be found given in the list of Properties of British Standard Sections, as well as in most firms' section books. It should be noticed that equal angle sections have the *least* radius about an axis X–X (Fig. 104) passing through the centre of gravity of the cross-section, and that they will consequently bend most readily cornerwise, or in the plane Y–Y.

The radius of gyration of compound or built-up sections is readily calculated from the given properties of the elementary sections of which they are composed.

EXAMPLE 19.—*To calculate the radii of gyration of the section in Fig. 105, about the axes X–X and Y–Y.*

1. About X–X. Obtain from the section book the following

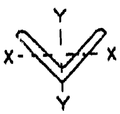


FIG. 104.

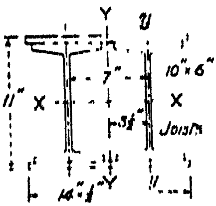


FIG. 105.

properties for a 10″ × 6″ × 42 lbs. joist. Sectional area = 12·4 sq. in. Moment of inertia about X–X = 212. Then

Mt. of inertia of two joists about X–X = 212 × 2 = 424
" " two plates " " = $\frac{1}{12}$ × 11(11³ − 10³) = 386

Total I of section about X–X = 810

Total sectional area = 2 × 11 × ½ + 2 × 12·4 = 38·8 sq.-in.

$$\text{and } r \text{ about X} - \text{X} = \sqrt{\frac{810}{38{\cdot}8}} = 4{\cdot}57 \text{ in.}$$

2. About Y–Y. From section book,

Mt. of inertia of one joist about y–y = 22·9
∴ " " one " " Y–Y = 22·9 + 12·4 × (3½)² = 174·8
" " two joists " " = 174·8 × 2 = 349·6
" " two plates " " = $\frac{1}{12}$ × 1 × 11³ = 228·6

Total I about Y–Y = 578·2

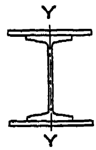
$$\text{and } r \text{ about Y–Y} = \sqrt{\frac{578{\cdot}2}{38{\cdot}8}} = 3{\cdot}86 \text{ in.}$$

The *least radius* is therefore about Y–Y and = 3·86 in. If desired the 7 in. spacing between the joists might be increased in order to make the radius of gyration about Y–Y equal to that about X–X. The necessary spacing to effect this will be readily found after one or two trials.

The following tables give the sectional area and least radius of gyration for the most useful types of built-up columns, the figures applying in every case to new British Standard Sections.

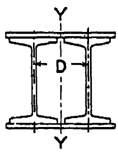
Type 1.—One joist with two or four plates.

Size of joist.	Size of plates.	Two plates		Four plates	
		Sectional area.	r, YY	Sectional area.	r, YY.
in.	in				
9 × 7 × 50 lbs.	10 × ½	24·71	2·24	34·71	2·44
10 × 8 × 55 „	12 × ½	28·18	2·66	40·18	2·92
12 × 8 × 65 „	12 × ½	31·12	2·59	43·12	2·86
14 × 8 × 70 „	14 × ⅝	38 09	3·04	55·59	3·39
16 × 6 × 50 „	12 × ½	26·71	2·50	38 71	2·83
18 × 8 × 80 „	14 × ⅝	41 03	2·94	58·53	3·31



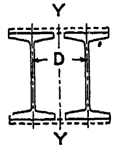
Type 2.—Two joists with two or four plates.

Joists	D	Plates	Two plates		Four plates	
			Area	r, YY.	Area	r, YY.
in	in	in				
10 × 6 × 40 lbs	7	14 × ½	37 54	3 86	51·54	3·91
12 × 5 × 30 „	6	12 × ½	29·65	3·29	41·65	3·34
14 × 5½ × 40 „	7	14 × ½	37 53	3·82	51·53	3 88
16 × 6 × 50 „	8½	16 × ½	45·41	4·49	61·41	4 53
18 × 8 × 80 „	10½	20 × ½	67·05	5 60	87·05	5·64
20 × 6½ × 65 „	8½	16 × ½	54·24	4·50	70·24	4·53

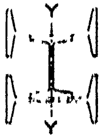


Type 3.—Two joists with tie-plates or lattice bracing.

Joists.	D	Area	r, YY
in.	in		
10 × 6 × 40 lbs.	7	27·54	3·75
12 × 5 × 30 „	6	17·65	3·16
14 × 5½ × 40 „	7	23·53	3 68
16 × 6 × 50 „	8½	29·41	4 43
18 × 8 × 80 „	10½	47·05	5·52
20 × 6½ × 65 „	8½	38·24	4·45

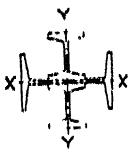


Type 4.—Three joists.



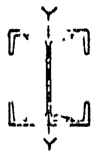
Two Joists.	One Joist.	Area.	r, YY.
in.	in.		
15 × 6 × 45 lbs.	15 × 6 × 45 lbs.	39·71	5·08
16 × 6 × 50 „	16 × 6 × 50 „	44·12	5·84
18 × 6 × 55 „	16 × 8 × 75 „	51·43	5·07
18 × 8 × 80 „	16 × 8 × 75 „	69·12	6·19
22 × 7 × 75 „	18 × 8 × 80 „	67·65	7·11
24 × 7½ × 90 „	22 × 7 × 75 „	74·99	8·11

Type 5.—Three joists.



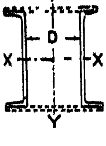
Two Joists.	One Joist.	Area	r, YY.	r, XX.
in.	in.			
6 × 5 × 25 lbs.	13 × 5 × 35 lbs.	25·00		3·15
8 × 6 × 35 „	15 × 6 × 45 „	33·84	3·96	–
9 × 7 × 50 „	16 × 6 × 50 „	44·13	3·98	
9 × 7 × 50 „	18 × 8 × 80 „	52·95		4·66
10 × 6 × 40 „	20 × 6½ × 65 „	42·66		5·04
12 × 8 × 65 „	24 × 7½ × 90 „	64·71		6·25

Type 6.—One joist with two channels.



Channels.	Joist	Area.	r, YY.
in	in		
12 × 3½ × 25·25 lbs.	15 × 6 × 45 lbs.	28·09	3·44
15 × 4 × 36·37 „	16 × 6 × 50 „	36·10	4·47
17 × 4 × 41·34 „	16 × 8 × 75 „	48·15	4·80
17 × 4 × 44·34 „	18 × 6 × 55 „	42·26	5·02
15 × 4 × 36·37 „	18 × 8 × 80 „	44·92	4·13
15 × 4 × 36·37 „	20 × 6½ × 65 „	40·51	4·25

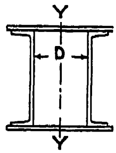
Type 7.—Two channels with lattice bracing.



Channels.	D.	Area	r, YY.
in.	in.		
7 × 3 × 14·22 lbs.	3½	8·36	2·77
8 × 3 × 15·96 „	3½	9·39	2·78
10 × 3½ × 24·46 „	4½	14·39	3·37
12 × 3½ × 29·23 „	6½	17·19	4·27
15 × 4 × 36·37 „	7½	21·39	4·85
17 × 4 × 44·34 „	9½	26·08	5·77

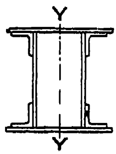
Type 8.—Two channels with two or four plates. Channel box-section.

Channels.	Plates.	D.	Two plates.		Four plates.	
			Area.	r, YY.	Area	r, YY.
in.	in	in.				
7×3	10×½	3½	18·36	2·88	28·36	2·85
8×3	10×½	3½	19 39	2 81	29·39	2·84
9×3½	12×½	4½	25·10	3·44	37·10	3·45
10×3½	12×½	4½	26 39	3·41	38·39	3·43
12×3½L	14×½	6½	28·85	4·13	42 85	4·10
15×4	16×½	7½	37·39	4·75	53·39	4·71
17×4	18×½	9½	44·08	5·54	62 08	5·45



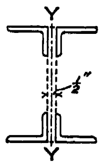
Type 9.—Built-up box section.

Angles	Web plates.	D.	Flange plates.	Area.	r, YY.
in.	in.	in	in		
3½×3½×½	20×½	5½	14×½	47 00	3 75
3½×3½×½	20×½	7½	16×½	49 00	4·60
4 ×4 ×½	22×½	8	18×½	55·00	5·02
4 ×4 ×½	22×⅝	8	18×⅝	65·00	5·03
5 ×5 ×⅝	24×⅝	9	21×⅝	79·69	5·85
5 ×5 ×⅝	24×⅝	12	24×⅝	83·44	7·11



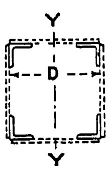
Type 10.—Four angles back to back with lacing.

Angles.	Area	r, YY
in		
3½×3 ×½	12·00	1·68
4 ×3 ×½	13 00	2·00
5 ×3 ×½	15 00	2·53
6 ×4 ×½	19·00	2 92
7 ×3½×⅝	24·68	3·63
8 ×4 ×⅝	28 43	4 09

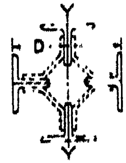


Type 11.—Lattice-box section. Four angles with lacing on each side.

Angles	D	Area	r, YY.
in	in		
2½×2½×⅜	8	6 93	3·32
3 ×3 ×⅜	10	8·44	4·21
3½×3½×½	12	13·00	5·05
4 ×4 ×½	14	15·00	5·95
5 ×5 ×½	18	19 00	7·72
6 ×6 ×⅝	20	28·43	8 49

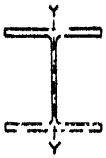


Type 12.—"Gray" column. Eight angles with tie-plates.

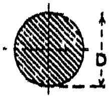


Angles.	D	Area.	r, YY.
in	in		
3 × 2½ × [illegible]	12	20·00	3·66
3½ × 2½ × [illegible]	14	22·00	4·28
4 × 3 × [illegible]	15	26·00	4·54
4 × 4 × [illegible]	16	30·00	5·04
5 × 5 × [illegible]	20	38·00	6·38
6 × 6 × [illegible]	20	56·87	6·25

Type 13.—Broad-flanged beams used alone or with one plate on each flange.

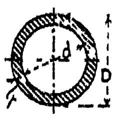


Size of beam.	Area	r, YY.	With two plates		
			Plates.	Area.	r, YY.
in.			in.		
8¼ × 8¼ × 46 lbs.	13·47	2·10	9 × [illegible]	22·47	2·31
10¼ × 10¼ × 61 ,,	18·69	2·60	12 × [illegible]	30·69	2·97
11[illegible] × 11[illegible] × 81 ,,	23·87	3·01	14 × [illegible]	41·37	3·48
14[illegible] × 11[illegible] × 101 ,,	29·68	2·96	16 × [illegible]	53·68	3·79
14[illegible] × 11[illegible] × 102 ,,	30·11	2·94	18 × [illegible]	57·11	4·16
15[illegible] × 11[illegible] × 110 ,,	32·32	2·95	18 × [illegible]	59·32	4·12
17[illegible] × 11[illegible] × 122 ,,	35·87	2·91	18 × [illegible]	67·37	4·14
19[illegible] × 11[illegible] × 135 ,,	39·58	2·87	18 × 1	75·58	4·14



Type 14.—Solid circular section of diameter D.

$$r = \frac{D}{4}.$$

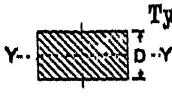


Type 15.—Hollow circular section.

External diameter $= D$.
Internal ,, $= d$.

$$r = \tfrac{1}{4}\sqrt{D^2 + d^2}.$$

Type 16.—Solid rectangular or square section.



$$r = 0{\cdot}289D.$$

Euler's Formula.—Assuming the conditions for an ideal round-ended column as enumerated at the commencement of this chapter, the following formula may be established by mathematical reasoning.

If P = ultimate or crippling load in tons; E = Modulus of elasticity of the material in tons per square inch; l = length of column in inches between centres of end bearings; and I = moment of inertia of cross-section in inch units,

$$P = \frac{\pi^2 EI}{l^2} = \frac{9 \cdot 87 EI}{l^2}$$

Substituting Ar^2 for I, where A = sectional area of column in square inches and r = least radius of gyration in inches,

$$P = \frac{9 \cdot 87 EA}{\left(\frac{l}{r}\right)^2}$$

The ratio $\frac{l}{r}$, previously noted, occurs in the denominator, and consequently the ultimate load P becomes smaller as $\frac{l}{r}$, or the slenderness of the column, increases. For any given material the value of E is sensibly constant, and the formula may be written

$$P = \text{constant} \times \frac{A}{\left(\frac{l}{r}\right)^2}$$

This formula forms the basis of most of the "practical" formulæ intended to give the ultimate load on columns. Being based on the assumption of an ideal column, it is not of much practical value, the ultimate load as given by it representing the extreme outside limit of load which a theoretically perfect column might withstand. By dividing the ultimate load P by any desired factor of safety, as 3, 4, 5, etc., the corresponding safe load would be obtained. It is, further, more convenient to express the safe load for a column in tons or pounds per square inch of sectional area, so that including the factor of safety and dividing P by the sectional area A, Euler's formula becomes $\frac{P}{FA} = p = \frac{\pi^2 E}{F \times \left(\frac{l}{r}\right)^2}$, where p = safe load in tons per square inch and F = factor of safety. Taking E as 13,400 tons per square inch for mild steel and a factor of safety F = 4, the formula reduces to

$$p = 33{,}064 \div \left(\frac{l}{r}\right)^2$$

It will be noticed that for low values of $\frac{l}{r}$, the formula would give abnormally high values for p. Thus for a column for which $\frac{l}{r}$ = say 40, p would = 20·6 tons per square inch, which is of course quite outside the *practical* range of load, since the elastic limit of the material would be exceeded. The formula may therefore only be used practically up to that value of $\frac{l}{r}$ for which the safe working load p does not exceed the safe crushing resistance of the material. If $6\frac{3}{4}$ tons per square inch be fixed as the highest permissible value for p for mild steel columns, the

corresponding value of $\frac{l}{r}$ is 70. The formula ceases then to have any practical significance for columns in which the ratio of length to least radius of gyration is less than 70. If increasing values of $\frac{l}{r}$ beyond 70 be inserted and the corresponding values of p be worked out, the results may be conveniently shown as in Fig. 106, by plotting values of $\frac{l}{r}$

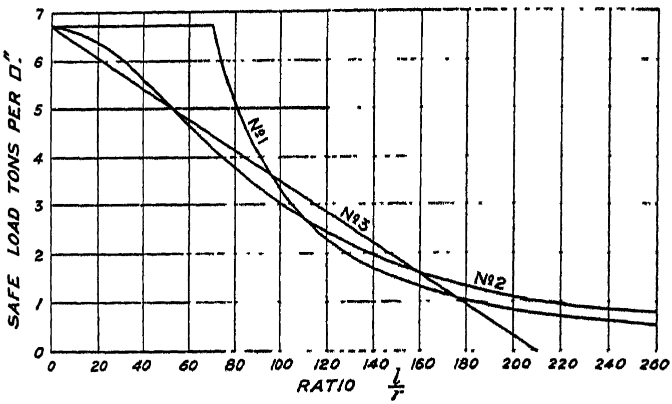


FIG. 106.

horizontally and the corresponding values of p vertically. The resulting curve is marked No. 1.

Rankine's Formula may be taken as typical of a large class of "practical" formulæ, many of which are based on the results of actual tests of columns. It may be expressed as

$$p = \frac{f}{1 + \text{constant} \times \left(\frac{l}{r}\right)^2}$$

where p = safe load in tons per square inch, f = crushing resistance in tons per square inch divided by the factor of safety, l and r being as before. The constant in the denominator is variously modified according to the conditions of end support. In this type of formula the safe compression f tons per square inch which may be put on a very short column is gradually reduced to p tons per square inch for longer columns as the ratio $\frac{l}{r}$ increases. Assuming 27 tons per square inch as the ultimate crushing resistance of very short columns of mild steel, and adopting a factor of safety of 4 as before, the formula becomes

$$p = \frac{6 \cdot 75}{1 + \frac{1}{8000}\left(\frac{l}{r}\right)^2}$$

$\frac{1}{8000}$ being the value of the constant for round-ended columns of mild steel. The resulting values of p obtained from this formula are shown by curve No. 2 in Fig. 106. The formulæ of Gordon, Claudel, Ritter, and Christie belong to this class.

Straight-line Formulæ.—These are a class of formulæ which have been devised to give approximate results to those above mentioned. They include the first power instead of the square of the term $\frac{l}{r}$. As an example of these, the following may be taken

$$p = 6\tfrac{3}{4}\left(1 - 0{\cdot}00475\frac{l}{r}\right)$$

where p = safe load in tons per square inch for round-ended columns, the factor of safety being 4 on the assumed ultimate strength of 27 tons for mild steel. The results of this formula are shown in Fig. 106, by the straight line No 3. The object of the straight-line formula is to obtain greater simplicity than is afforded by the various "curved" formulæ by avoiding the quadratic solutions which they necessitate. It should be noticed, however, that the safe loads for columns of varying $\frac{l}{r}$ cannot be correctly given by such formulæ. For the one in question, the load becomes zero for a column having the ratio $\frac{l}{r} = 210$, which is obviously incorrect. Fig. 106 shows that for ratios of $\frac{l}{r}$ between 100 and 180, which include a fairly large proportion of practical columns, the safe loads per square inch, whether calculated by Euler's, Rankine's, or the straight-line formula, do not greatly differ. In selecting a suitable column section, the length, total load and character of end supports are known beforehand. The *type* of cross-section desirable will depend mainly upon total load and connections to be made with the column. The method of selecting a suitable section of column by the aid of the above formulæ will then be as follows.

EXAMPLE 20.—*Required a suitable box section in mild steel, formed of two channels and two plates, to carry a central load of* 100 *tons, the length being* 25 *ft. and the ends considered rounded.*

1. *Using Euler's Formula.*—Try No. 3 section of Type 8.

$$r = 3{\cdot}44\,.\,l = 25 \times 12 = 300''.\ \frac{l}{r} = \frac{300}{3{\cdot}44} = 87$$
$$A = 25{\cdot}10 \text{ sq. in.}$$

From curve No. 1, the safe load per square inch for $\frac{l}{r} = 87$ is 4·3 tons. Hence total safe load = 25·10 × 4·3 = 107·9 tons. No. 2 section will be found too small, and No. 3 would be adopted.

2. *Using Rankine's Formula.*—The safe load per square inch for $\frac{l}{r}$ = 87 from No. 2 curve is 3·4 tons, and total safe load = 25·10 × 3·4 = 85·3 tons. This section is therefore too small. Try No. 5 section of

Type 8. $r = 4{\cdot}13$. $\therefore \frac{l}{r} = \frac{300}{4{\cdot}13} = 73$, and safe load per square inch is practically 4·0 tons. $A = 28{\cdot}85$ square inches, and total safe load $= 28{\cdot}85 \times 4{\cdot}0 = 115{\cdot}4$ tons. As section No. 4 would be considerably too light, No. 5 would be adopted. Note that this section has about 15 per cent. greater area than the one deduced by Euler's formula.

3. *Using the Straight-line Formula.*—Try section No. 3, Type 8. $\frac{l}{r} = 87$ as before, and safe load per square inch from No. 3 line, Fig. 106, $= 3{\cdot}9$ tons. Hence total safe load $= 25{\cdot}10 \times 3{\cdot}9 = 97{\cdot}9$ tons. That is, the same section as indicated by Euler's formula would be adopted but with an apparently less margin of safety.

The above-mentioned formulæ are open to the following objections. Euler's formula is based on the assumption of a theoretically perfect column, a case never realized in practice. The Rankine type of formulæ are based generally on the results of limited numbers of practical tests. Any formula to be of general practical value should show agreement, not only with a very large number of reliable tests of varying sections, but also tests covering a large range of ratios of l to r, in addition to several tests at each ratio of l to r. Few, if any, of the formulæ in general use satisfactorily approach these requirements. The straight-line formulæ can only be regarded as rough approximations, especially when employed over a wide range of ratios of l to r.

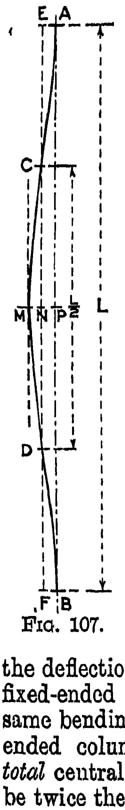


Fig. 107.

Fixed-ended Columns.—In Fig. 107, if ACDB represent the axis of a perfectly fixed-ended column of uniform section in a bent or deflected state, the portions AC and BD, after bending, remain tangent to the vertical line AB. The central portion CD is also tangent to a vertical line at its middle point M. C and D are points of contra-flexure, and at these points, therefore, no bending moment exists. In order to constrain the column to conform to these conditions there must be the same bending moment existing at A, M, and B. The points C and D must therefore be situated midway along AM and BM respectively, or, in other words, the central portion CD which bends in a single curve will be equal to half the total length AB of the fixed-ended column, and will behave similarly to a round-ended column. Further, the deflections AE, BF, and MN will all be equal. Stated generally, a fixed-ended column will carry the same load and be subject to the same bending moment, and consequently the same stress, as a round-ended column of half the length and the same cross section. The *total* central deflection MP, of the fixed-ended column AB, will, however, be twice the deflection MN of the equivalent round-ended column CD, under the same load.

The above statement must not *be confused with the idea that a fixed-ended column will carry twice the load of a round-ended column of equal length, which is, of course, quite erroneous.*

Hence, in selecting a section for a fixed-ended column of given

length, that section is employed which would be suitable for a round-ended column of *half* the length, after making reasonable allowance for the extent to which the presumably fixed-ended column fails to realize the conditions of absolute fixity.

Moncrieff's Formula.—The most complete and reliable investigation of the strength of columns is contained in a paper by Mr. J. M. Moncrieff, M.Inst.C.E., presented before the American Society of Civil Engineers in 1900.[1] The reader is referred to the original work for a complete account of the reasoning and deductions. The general outline of the method of inquiry followed is indicated in the following pages. Limited space prohibits giving more than a brief outline, but the reader is strongly recommended to study the original communication. The underlying principles upon which the reasoning is based are—

1. That a perfectly centred column of perfect material and straightness is probably never realized in practice; and—

2. That the various disturbing influences preventing such realization, each of which conduces to initial bending, are all capable, as regards their ultimate effect, of being represented or covered by an equivalent small eccentricity of loading.

In order to apply this principle to the case of practical columns under intentional and *apparently central* loads, the suitable value to be assigned to the "equivalent eccentricity" required to be carefully determined. This value was finally fixed after a careful analysis of the records of practically all the really reliable tests of columns made in Europe and America from 1840 down to date. In the course of this analysis, the results of 1789 tests of ultimate strength of columns of cast iron, wrought iron, steel, and timber were carefully considered, so that the formulæ deduced have the merit of being far more representative of practical strength than any previously proposed.

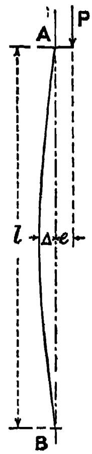


FIG. 108.

In Fig. 108, AB represents the axis of a round-ended column under the action of a load P applied at a small eccentricity e. $\Delta =$ The resulting central deflection of the column from the vertical AB. $l =$ Length of column It is easily established that

$$\Delta = \frac{Pl^2e}{8EI - \frac{5}{6}Pl^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where E = Modulus of elasticity of material and I = Moment of inertia of cross-section.

The bending moment at the centre of the column then

$$= P(e + \Delta) = \frac{f_b \times I}{y} = \frac{Ar^2f_b}{y},$$

where f_b is the stress per square inch caused by bending alone at a distance y from the neutral axis of the cross-section. A = Area of

[1] "The Practical Column under Central or Eccentric Loads." J. M. Moncrieff, *Transactions Am. Soc. C.E.*, vol. xlv. 1901.

cross-section, and r = Radius of gyration in the direction in which the column bends.

Rearranging $f_b = \pm \frac{P(e+\Delta)y}{Ar^2}$, the positive sign denoting compression at the concave side of the column, and the negative sign tension at the convex side. The *direct* compression on the column section, independent of the eccentricity of loading, $= f_a = +\frac{P}{A}$ = the average load per square inch on the sectional area of the column. Hence, the total stress per square inch at opposite edges of the section, due to both bending and direct compression,

$$= F = \pm f_b + f_a = \pm \frac{P(e+\Delta)y}{Ar^2} + \frac{P}{A}$$

Substituting the value of Δ in equation (1)

$$F = f_a\left\{1 \pm \frac{ye}{r^2}\left(\frac{48E + f_a\left(\frac{l}{r}\right)^2}{48E - 5f_a\left(\frac{l}{r}\right)^2}\right)\right\} \quad . \quad . \quad . \quad . \quad (2)$$

Using the + sign to determine the maximum compressive stress F_c and transposing equation (2),

$$\frac{l}{r} = \sqrt{\frac{48E}{5F_c + f_a\left(\frac{ye}{r^2} - 5\right)}\left[\frac{F_c}{f_a} - 1 - \frac{ye}{r^2}\right]} \quad . \quad . \quad (3)$$

Similarly the − sign will determine the minimum stress F_t at the opposite edge of the section, which may or may not be tensile, according as the tension due to bending exceeds, or does not exceed, the direct compression. The suffix t is only used conveniently, and does not necessarily imply that F_t is a tensile stress. Hence using the − sign in (2),

$$\frac{l}{r} = \sqrt{\frac{48E}{5F_t - f_a\left(\frac{ye}{r^2} + 5\right)}\left[\frac{F_t}{f_a} - 1 + \frac{ye}{r^2}\right]} \quad . \quad . \quad (4)$$

The formulæ (1), (2), (3), and (4) are all general expressions applicable to round-ended columns of any given material and form of section, and with any given value of eccentricity of loading probable in practical work. For fixed-ended columns the value of $\frac{l}{r}$ as given by formulæ (3) or (4) will be doubled. For flat-ended columns, $\frac{l}{r}$ will be the same as given for fixed-ends up to the point where tension begins to be developed at one edge of the end bearings. Since flat-ended columns are incapable of resisting tensile stress, F_t in equation (4) must

be made $= 0$. For the ratio $\frac{l}{r}$ at which tension begins to be developed the formula then becomes—

$$\frac{l}{r} = 2\sqrt{\frac{48\mathrm{E}\left(1 - \frac{ye}{r^2}\right)}{f_a\left(\frac{ye}{r^2} + 5\right)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Formulæ (3) and (4) give the relation between $\frac{l}{r}$, the elasticity E of the material, the maximum fibre stress F_c or F_t, and the *average* load per square inch, f_a, on the column section which will produce that maximum fibre stress. Formula (5) gives the relation between $\frac{l}{r}$ and the *average* load per square inch, f_a, consistent with no tension being set up in the material.

The use of equations (3) and (4) is as follows. Suppose that in a given column section it is decided that a certain value of maximum compressive stress F_c, or a certain value of minimum stress F_t, is not to be exceeded; these values being inserted in the formulæ together with the values of E and $\frac{ye}{r^2}$, corresponding with the material and the section of column and eccentricity of loading adopted, the result gives at once the value of $\frac{l}{r}$ corresponding to f_a, the average load per square inch. It will be seen the formulæ each contain the term $\frac{ye}{r^2}$. y and r depend *only* on the form of section. e is the "equivalent eccentricity" to be allowed for covering the defects of material, straightness of axis, etc., in the actual practical column By careful comparison of results given by the formula with those of the numerous tests previously mentioned, Mr. Moncrieff found that by making $\frac{ye}{r^2} = 0{\cdot}6$, the formula expressed very closely the strength of the weaker columns in the various series of tests, whilst by making $\frac{ye}{r^2} = 0{\cdot}15$, the strength of the stronger columns was fairly represented. Since the *practical* strength is actually represented by the weaker experimental results, it appeared advisable not to employ a lower value than 0 6 for the term $\frac{ye}{r^2}$ Adopting this value and inserting suitable values for E and F_c, and employing a factor of safety of 3 for dead loads, the results of the formulæ when applied to various materials are exhibited by the curves in Figs. 109, 110, 111, and 112.

The *average* dead loads indicated by these curves correspond with the following *maximum* stresses per square inch and Moduli of Elasticity for the various materials.

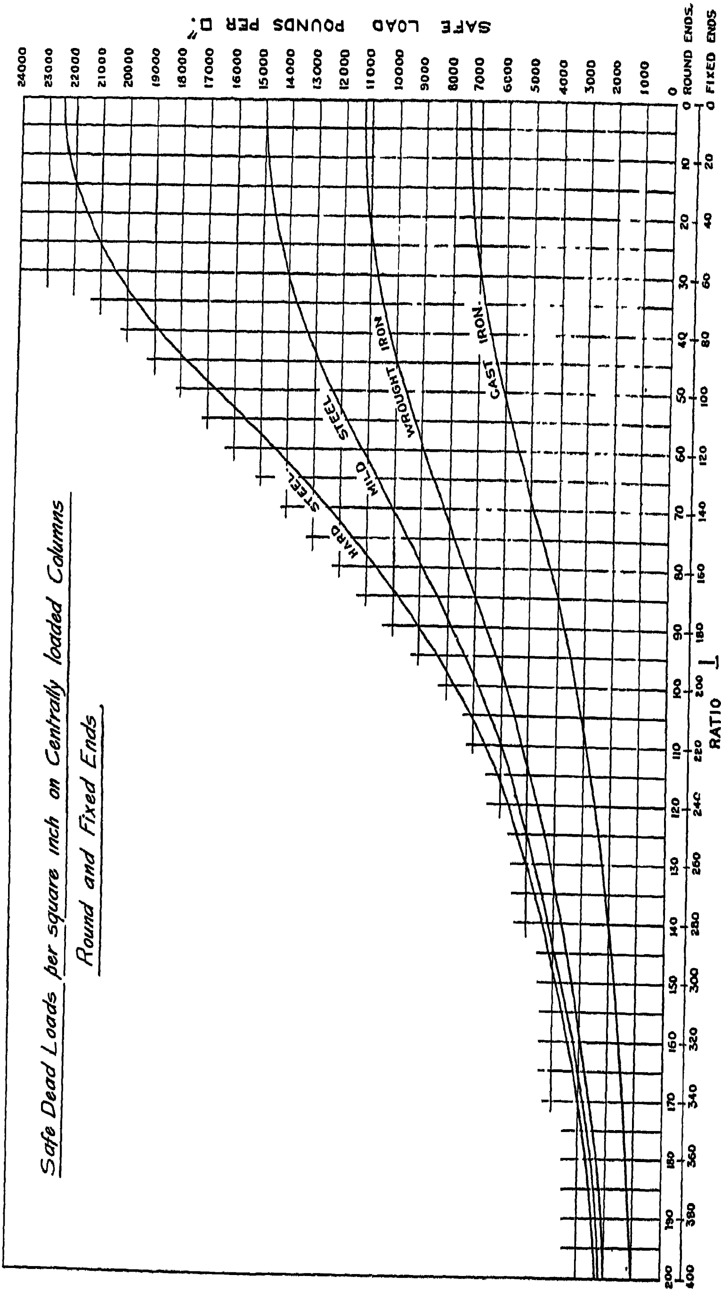

Safe Dead Loads per square inch on Centrally loaded Columns
Round and Fixed Ends
SAFE LOAD POUNDS PER □"
24000 23000 22000 21000 20000 19000 18000 17000 16000 15000 14000 13000 12000 11000 10000 9000 8000 7000 6000 5000 4000 3000 2000 1000 0
HARD STEEL
MILD STEEL
WROUGHT IRON
CAST IRON
ROUND ENDS
FIXED ENDS
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170 180 190 200
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400
RATIO $\frac{l}{r}$

FIG. 109.

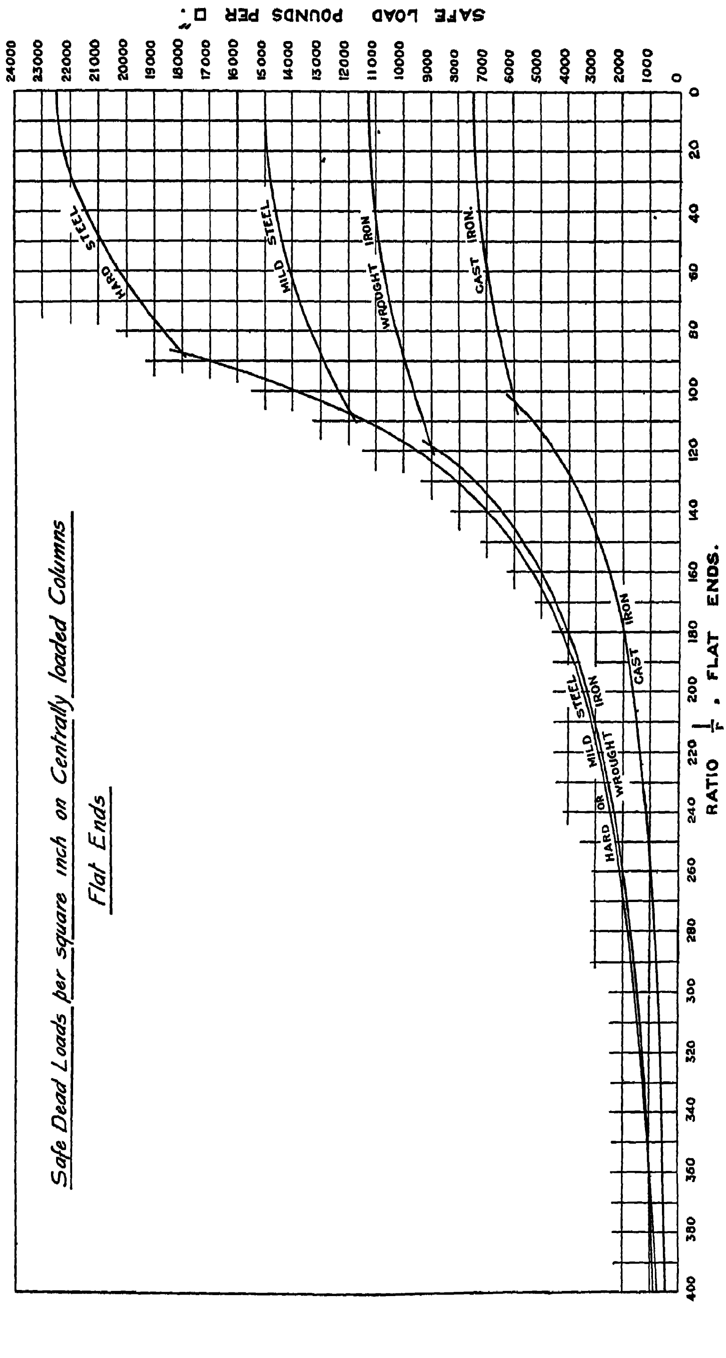

Safe Dead Loads per square inch on Centrally loaded Columns
Flat Ends
SAFE LOAD POUNDS PER □"
RATIO l/r . FLAT ENDS
HARD STEEL
MILD STEEL
WROUGHT IRON
CAST IRON.
HARD OR WROUGHT IRON
MILD STEEL
CAST IRON
0 1000 2000 3000 4000 5000 6000 7000 8000 9000 10000 11000 12000 13000 14000 15000 16000 17000 18000 19000 20000 21000 22000 23000 24000
0 20 40 60 80 100 120 140 160 180 200 220 240 260 280 300 320 340 360 380 400

Fig. 110.

Material.	F_c, lbs. per sq. in.	E
Common cast iron	12,000	14,000,000
Wrought iron, of tensile strength, 45,000 to 50,000 lbs. per square inch	18,000	28,000,000
Mild steel, of tensile strength, 60,000 to 70,000 lbs. per square inch	24,000	30,000,000
Hard steel, of tensile strength of about 100,000 lbs. per square inch	36,000	30,000,000
Yellow pine or pitch pine	2,000	2,200,000
White pine	1,800	1,400,000
French oak or Dantzic oak	2,000	1,200,000

The curves, Fig. 110, for the loads on flat-ended columns are identical with those for fixed-ended columns up to the point where

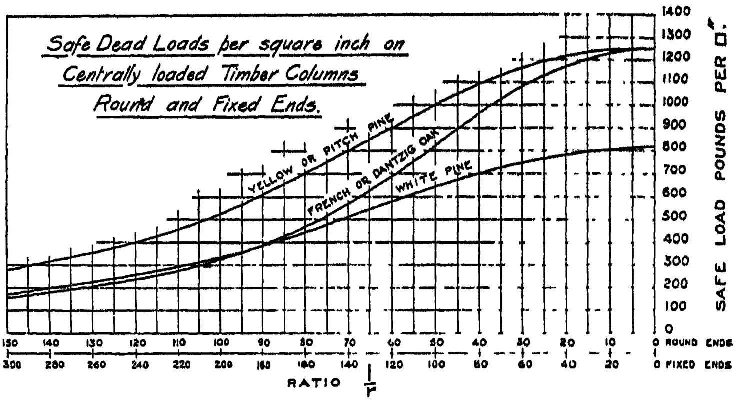


FIG. 111.

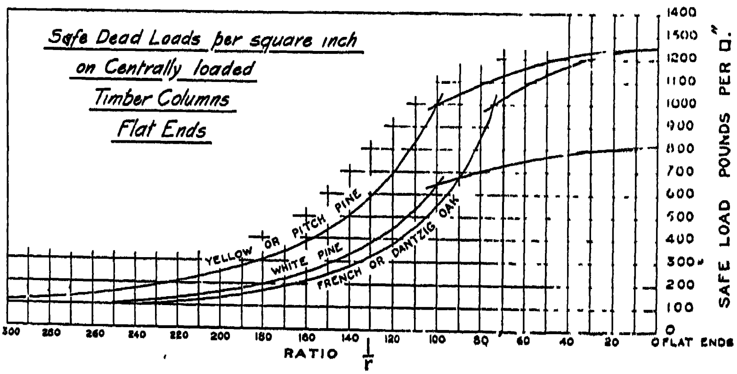


FIG. 112.

tension begins to be developed, which occurs in the neighbourhood of the ratio $\frac{l}{r} = 110$. Beyond this point the resistance of the flat-ended

column rapidly diminishes, as indicated by the sudden drop in the curves.

In the original paper the factor of safety of 3 is applied to the modulus of elasticity for reasons there fully explained. The use of the factor of safety in this manner has an inappreciable influence on the results of the formulæ when applied to short columns, while its effect gradually increases with the length, ultimately affording a factor of safety of 3 against failure by instability or buckling, in the case of very long columns, and the factor of safety against ultimate strength is fairly even between these extremes.

In making use of the curves plotted from the above formulæ, the particular amount of "equivalent eccentricity" for which they provide may be readily computed for any proposed column section. Thus taking section No. 3, Type 3, the least radius $r = 3{\cdot}68''$ and $y = 6\frac{1}{4}''$. Since $\frac{ye}{r^2} = 0{\cdot}6$, $\frac{6\frac{1}{4} \times e}{13{\cdot}54} = 0{\cdot}6$, and $e = 1{\cdot}3''$. The sectional area $= 23{\cdot}53$ sq. in. Taking a mild steel column of say 30 ft. $= 360$ in. length with fixed ends, $\frac{l}{r} = \frac{360}{3{\cdot}68} = 98$. From the curve for mild steel in Fig. 109, for fixed-ended columns, the safe dead load for this ratio $= 12{,}500$ lbs. per square inch. Hence the total safe dead lead $= \frac{23{\cdot}53 \times 12{,}500}{2240}$ $= 131$ tons. That is to say, in imposing an *intended* central dead load of 131 tons on this column, provision is made for a possible eccentricity of 1·3 in. to cover defects of elasticity, cross-section, initial curvature of axis and slight inaccuracies in erection.

Columns under intentionally Eccentric Loads.—In adapting the formulæ to cases where the load is intentionally applied at a known eccentricity, it is necessary to add the value $\frac{ye}{r^2} = 0{\cdot}6$ as determined for presumably centrally loaded columns, to the *known* value of $\frac{ye}{r^2}$ as obtained from the *intended* eccentricity e and the dimensions and form of sections which fix the values of y and r. Formulæ (3) and (4) then become—

$$\frac{l}{r} = \sqrt{\frac{48\text{E}}{5\text{F}_c + f_d\left\{\left(\frac{ye}{r^2} + 0{\cdot}6\right) - 5\right\}}\left[\frac{\text{F}_c}{f_d} - 1 - \left(\frac{ye}{r^2} + 0{\cdot}6\right)\right]}$$

and

$$\frac{l}{r} = \sqrt{\frac{48\text{E}}{5\text{F}_t - f_d\left\{\left(\frac{ye}{r^2} + 0{\cdot}6\right) + 5\right\}}\left[\frac{\text{F}_t}{f_d} - 1 + \left(\frac{ye}{r^2} + 0{\cdot}6\right)\right]}$$

where F_c and F_t are, as before, the maximum permissible working stresses in compression and tension respectively. Inserting the previous tabular values for E and F_c and employing a factor of safety of 3, the curves in Fig. 113 exhibit the relation between $\frac{l}{r}$ and the *average* working stress per square inch permissible for various degrees of eccentricity of loading, in the case of mild steel columns with round ends.

Since the maximum compressive stress developed in a column of symmetrical section always exceeds the maximum tensile stress, it is not necessary, in the case of mild steel, to use the formula containing F_t, as F_t may be taken, for this material, under working loads, as equal to F_c, the permissible compressive stress. In the case of cast iron it will, of course, be necessary to use both expressions, and to adopt whichever gives the lower value to the ratio $\frac{l}{r}$ for any given load f_d.

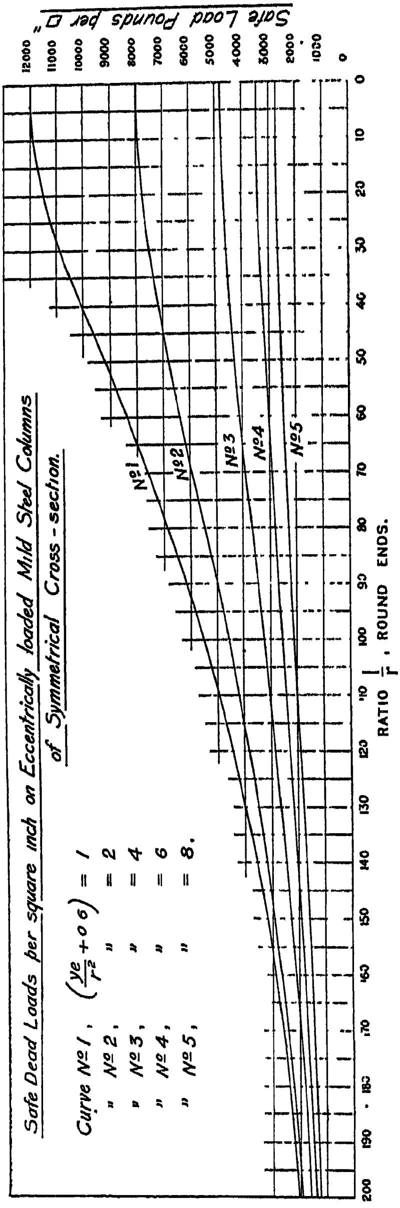


FIG. 113.

Practical Considerations in selecting a Type of Column. — Generally speaking, the simplest forms of cross-section are most desirable. The principal practical consideration is usually the number and kind of connections to be made with horizontal girders or joists, and whether such are to be attached on one or two only, or on all four faces of the column. In this respect Types 1, 2, 6, 8, 9, 12, and 13 are convenient forms. Those columns with the least amount of riveting are cheaper, and less likely to suffer distortion during construction.

Single joist sections are uneconomical as columns on account of their small radius of gyration as compared with sectional area, and consequently weight, but are widely used by reason of their cheapness and ease of making connections. Broad flange beams are preferable as columns to the British Standard beams, the additional width of flange giving them a considerably larger minimum

radius of gyration, weight for weight, and more latitude for riveted connections with girders. They are rolled in 36 sizes from $5\frac{1}{2}'' \times 5\frac{1}{2}'' \times 23$ lbs. per foot to $39\frac{3}{8}'' \times 11\frac{13}{16}'' \times 210$ lbs. per ft.[1] As a comparison, the B.S.B., $16'' \times 8'' \times 75$ lbs. has a minimum radius of gyration of 1·76″. The B.F.B., $11'' \times 11'' \times 76$ lbs. has a least radius of 2·81″. Both have practically the same weight and sectional area = 22 square inches. Used as a fixed-ended column 25 ft. high, the

$$\text{B.S.B. has } \frac{l}{r} = \frac{300}{1{\cdot}76} = 170, \text{ and safe load} = 8400 \text{ lbs. per sq. in.}$$

$$\text{the B.F.B. has } \frac{l}{r} = \frac{300}{2{\cdot}81} = 106, \quad ,, \quad = 12{,}000 \quad ,,$$

and the total safe loads would be 82·7 tons for the B.S.B. and 119·2 tons for the B.F.B., or 44 per cent. greater carrying power in favour of the B.F.B.

In section Types 2, 3, 5, 6, 7, 8, and 9, the dimensions may readily be modified, if desired, to give practically the same radius of gyration about either axis. This is desirable in cases where a column is not stiffened laterally in either direction by intermediate attachments.

The use of long columns in which the ratio $\frac{l}{r}$ is very high, should be avoided, since the working stress is very low, and the stiffness relatively small. It is often advisable to employ a heavier section than indicated in the case of very long columns in order to obtain necessary stiffness, although at some sacrifice of economy on the score of weight. Types 11, 12, 14, and 15 have the same radius of gyration in any direction.

Types 3, 7, 10, 11, and 12, formed by bracing together two joists or channels or four or eight angles by means of tie-plates at intervals, are inferior to columns with continuous webs or close lattice bracing, especially when required to carry eccentric loads or to withstand heavy lateral wind pressures as in the case of tall buildings. Type 10 is very generally used for struts of lattice bridge girders. Type 11 forms a light and economical section for very long struts in horizontal or inclined positions. These are subject to deflection due to their own weight, which unavoidably sets up the initial bending so detrimental to columns. It is therefore important to keep down the dead weight of such members. Crane jibs and very long bridge struts are usually of this type. Type 12, known as the "Gray" column, is much favoured in American practice. It is an economical section, easy to construct and equally convenient for making connections with girders on all four sides. The tie-plates should be closely spaced. This type is also used with continuous plates between the pairs of angles, the column being then much stronger.

Connections.—The usual connections required in the case of columns are—

1. Columns to built-up plate girders.
2. Columns to joists or beam sections.

[1] *Handbook No.* 18, *Broad Flange Beams*, R. A. Skelton & Co.

3. Junctions in long columns where the cross-section is gradually diminished from bottom to top.

4. Bases and caps.

Very great variety of arrangement of these details naturally exists on account of the large number of column sections available. In designing connections the following general principles should be attended to.

1. Simplicity of detail in order to minimize expense in shop work, and admit of rapid and easy assemblage of parts at site.

2. Riveted or bolted connections of beams or girders to columns must in every case provide adequate shearing and bearing resistance to the vertical load to be carried.

3. The joints should be arranged with a view to providing the greatest possible lateral rigidity for the columns as well as stiff end connections for the girders, since the degree of lateral stability obtained for intermediate points and heads of columns frequently decides whether they may be considered as approximating to the strength of fixed or round-ended columns. The capability of a structure to resist horizontal wind pressure or racking action under live loads depends entirely on the rigidity of the connections of vertical with horizontal members in cases where the character of the structure does not permit of efficient diagonal bracing.

4. Horizontal members should be attached to columns with the least possible amount of eccentricity in order to minimize the bending action on the columns. Simply supporting the load on wide brackets without at the same time firmly riveting or bolting to the face of the column is to be avoided, although a well-fitted bracket, where space allows, will largely contribute to lateral rigidity.

The following figures illustrate suitable types of connections between joists or girders and columns, such as commonly occur in practice. Bolts are indicated by black circles and rivets by open circles. On extensive works plant for putting in rivets *in situ* is frequently installed, in which case bolts will only be required in such positions as cannot be negotiated by the riveter. Generally, however, bolts are largely used to save the expense of riveting at site.

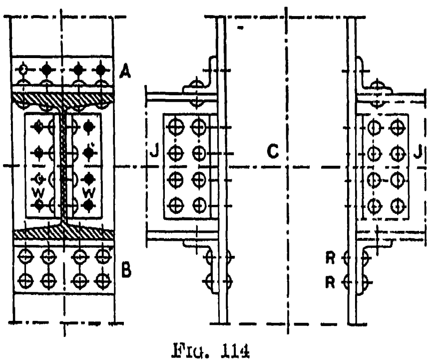


FIG. 114

Fig. 114 shows the usual connection of horizontal joists with a column of a single beam section. Top and bottom cleats A and B connect the flanges of the joists J with the faces of the column C, and a pair of web cleats W connect the web of the joist with the flange of the column section. The top cleat A is often omitted, but its use greatly stiffens the end of the joist.

Fig. 115 indicates a suitable arrangement for four broad flanged

joists meeting on a single B.F. joist column, together with a junction between a lower heavier column section A, and an upper lighter one,

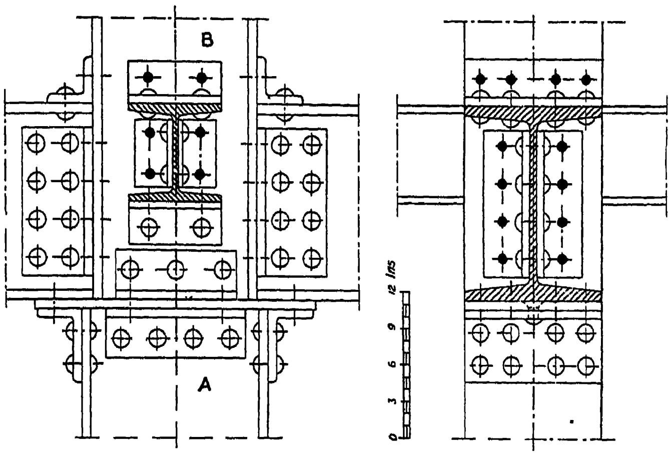


Fig. 115.

B. This type of joint is of common occurrence in high framed buildings where the sections of the columns are diminished every two or three storeys.

Fig. 116 illustrates a connection between heavy 3-beam compound girders and a column consisting of two beam sections and two plates. The width of the beam exceeds that of the column, and prohibits the use of web cleats. Brackets or "stools" G are riveted to the column faces, having sufficient rivets to resist the heavy end shear, and the girders are bolted down to the horizontal angles of the stools, and through to the flanges of the column by bolts passing through the upper flange cleats, which are previously riveted to the upper flanges of the girders. The vertical angles which hold up the lower flange cleats require packing plates at P, P.

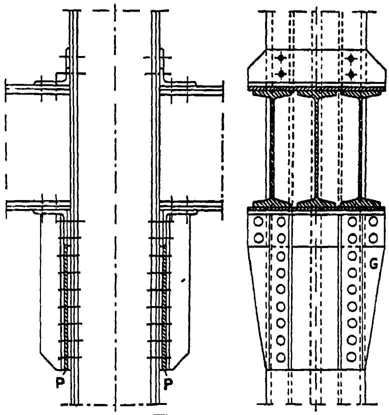


Fig 116.

In the case of columns running through several storeys of a building, it is desirable to preserve the continuity of the columns whenever possible, and to connect all girders or joists to the faces of the columns. Occasionally, however, this entails complexity of design, or one of the girders may require to be continuous. Figs. 117 and 118 show the detailed arrangement where a continuous plate girder A is carried over

the head of a lower column D, and two other slightly shallower girders B and C meet from opposite sides in a direction at right angles with

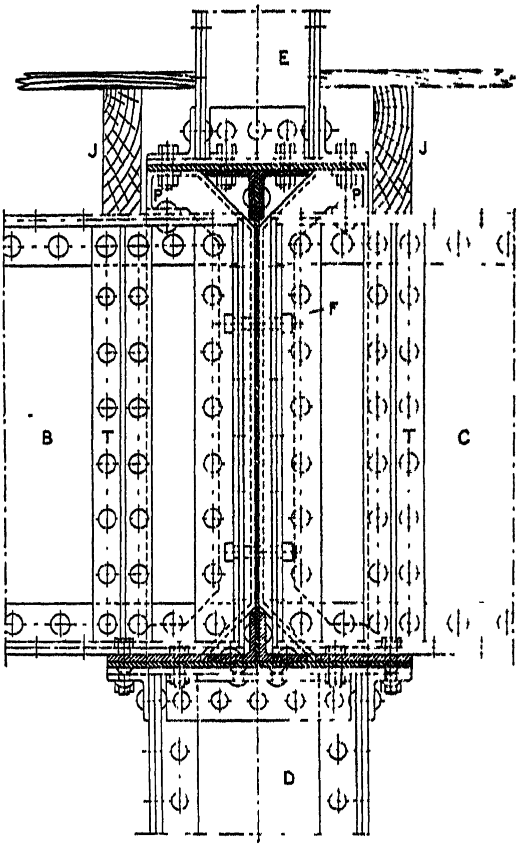


FIG. 117.

that of girder A. Fig. 117 shows a cross-section through girder A, and Fig. 118 a cross-section of B or C. The lower flanges of the girders are bolted down to the cap plate of column D, packings being inserted as indicated. The ends of girders B and C are left just clear of the web of girder A, but are bolted through it by four bolts, F. The continuation column E, of lighter box section, is planted on the upper flange of girder A, the under edges of the flange beneath the column foot being packed up on channels or bent plates P. Substantial gusset stiffeners S strengthen the web of girder A, and T-stiffeners T the webs of B and C, and assist in transmitting the load from the upper to the lower column. The timber floor joists J are carried on the upper flanges of girders B and C. This is, generally speaking, an inferior arrangement, since continuity of the columns, especially in tall buildings,

conduces to greater stiffness under the vertical loads, and more efficient resistance to oscillation or racking due to lateral wind pressure.

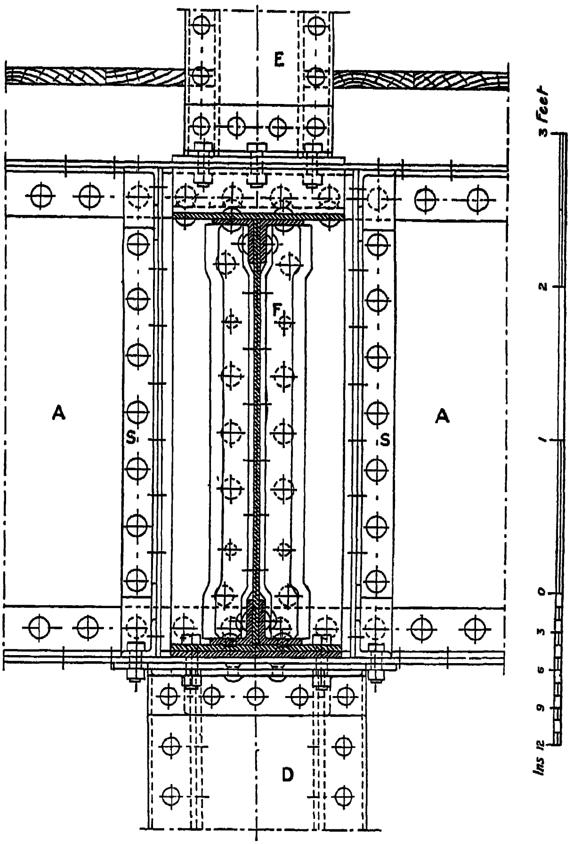


FIG. 118.

Fig. 119 illustrates a common case of column construction, where two rail girders G carry the ends of travelling crane girders in adjacent bays of a shop. The general arrangement is shown at A. The lower part of the column, up to the level of the seatings of girders G, consists of three joist sections with two plates each, laced together with diagonal angle braces. Brackets B are built out to support the girder bearing plates shown in the plan. Immediately beneath the girders the three joists are tied together by deep gusset plates P, forming the back plate for the brackets B. Above the level of the girders, the central joist, reinforced by two channels, is carried up to support the adjacent spans of the roof. In the side elevation E, the rail girders are removed. Double and treble columns of this type frequently have the component joists tied together with short rectangular plates. Diagonal bracing is,

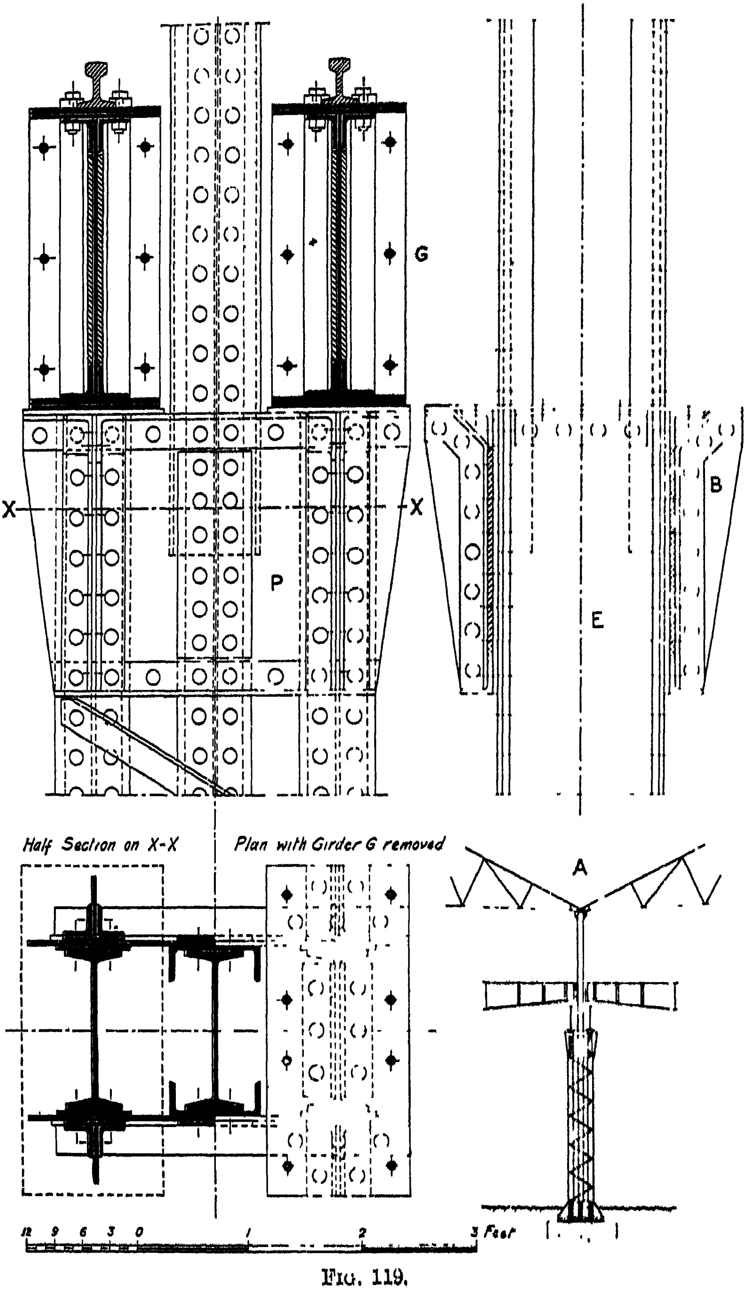

G
X
X
P
B
E
Half Section on X-X
Plan with Girder G removed
A
12 9 6 3 0 1 2 3 Feet

FIG. 119.

however, more satisfactory, as ensuring a more uniform distribution of the total load between the individual joists.

Caps and Bases.—Columns are fitted with cap and base plates attached to the faces by angles. Before riveting on the cap and base plates, the ends of the column, in all good-class work, are planed up square with the axis to ensure a fair bearing on the end plates. The plates and bar sections composing the column are, previously to riveting, sawn to dead lengths, but the distortion consequent on riveting usually leaves the ends uneven, thus necessitating the planing. The cap and

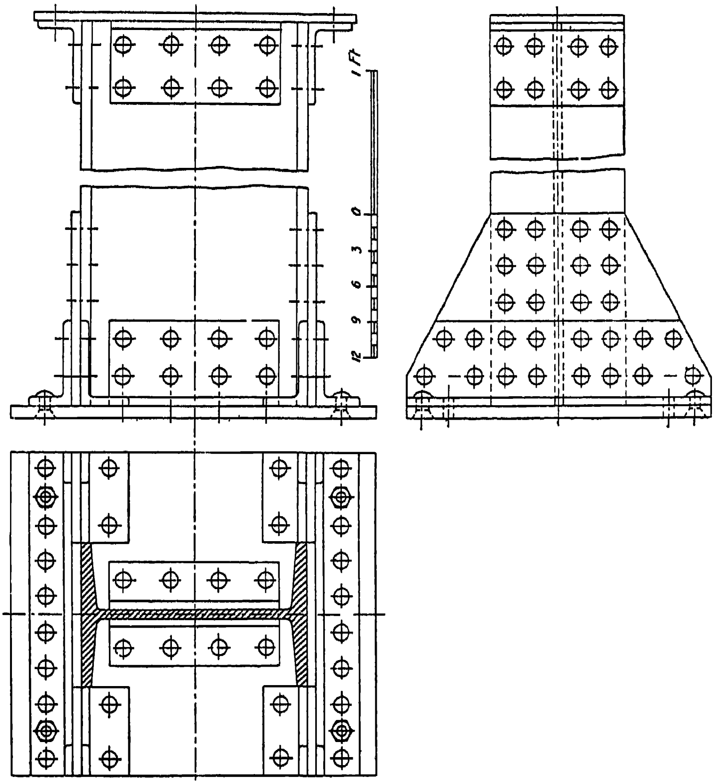


FIG 120.

base plates are usually riveted with rivets having countersunk heads to the outside, in order to leave flush surfaces for bearing on the foundation block at the lower end, and against the lower flanges of the girders supported at the upper end. Fig. 120 shows a typical cap and base for a 20″ × 12″ broad flange column. Base plates vary in thickness from $\frac{5}{8}$ in. to $1\frac{1}{4}$ in., and for columns carrying exceptional loads are sometimes $1\frac{1}{2}$ in. to 2 in. Four, six, or more holes are left in the base plate, at accessible points, through which pass the foundation bolts. These latter are embedded in the concrete foundation block with the

screwed ends projecting ready for lowering the column into place. The bolts are carefully placed in the correct positions by the aid of a wooden template having holes drilled at the requisite spacings.

Columns are frequently mounted on steel or iron castings interposed between the base plate and foundation block. This construction may

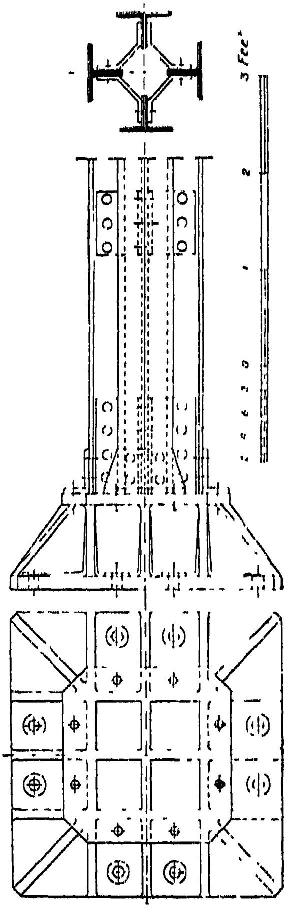


FIG. 121.

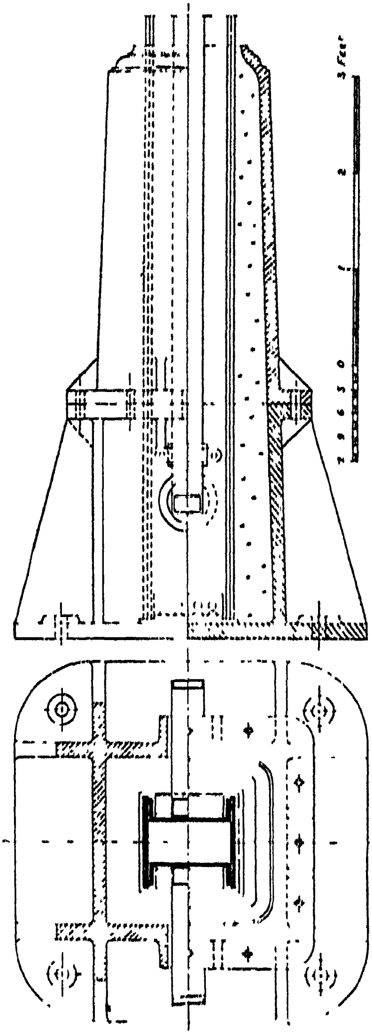


FIG. 122.

be adopted in cases where it is inconvenient to use a large base plate with gussets or brackets for distributing the load from the column to the foundation. Such built-up brackets frequently entail difficult and expensive riveting, and the use of a casting effects the necessary dis-

tribution of load over a larger area in a cheaper manner. Fig. 121 shows a 15″ × 15″ Gray column mounted on a steel casting having a 36 inch square base. The bearing area of the casting on the foundation

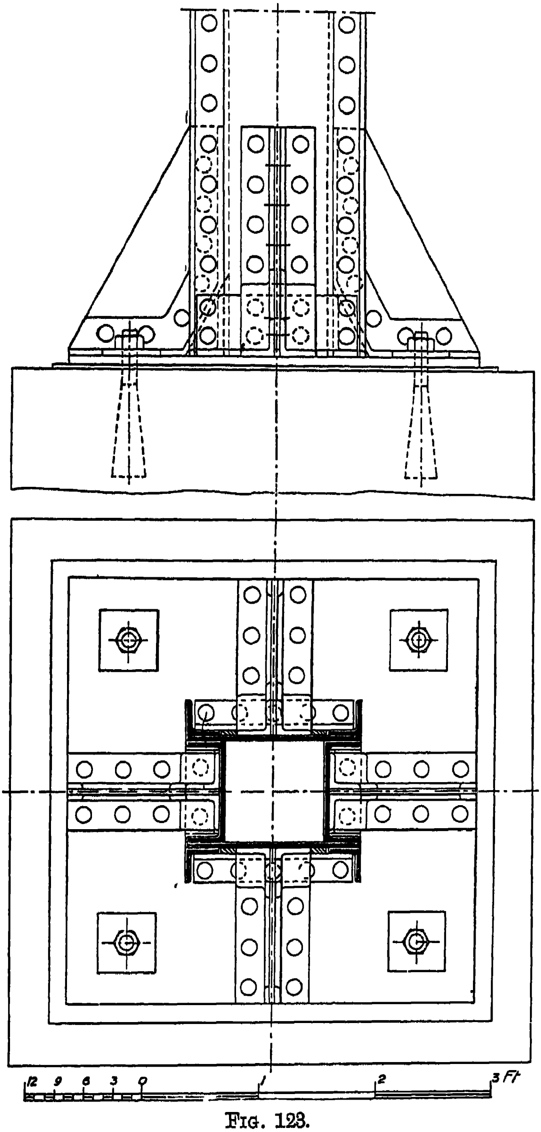


FIG. 123.

block is thus 9 square feet, whilst the bearing area of the concrete block on the actual ground would be still further increased. Fig. 122 shows the detail of a steel box-section column embedded at its lower

end in concrete filled into a hollow casting, which is bolted down to a suitable foundation. This is a type of base frequently employed for columns supporting warehouses where the basement is open to heavy vehicular traffic. The base castings project 3 or 4 feet above the pavement, and protect the columns from damage. In this example, the drainage from the roof is led through down-pipes screwed to the faces of the column. Fig. 123 is an example of a column of channel

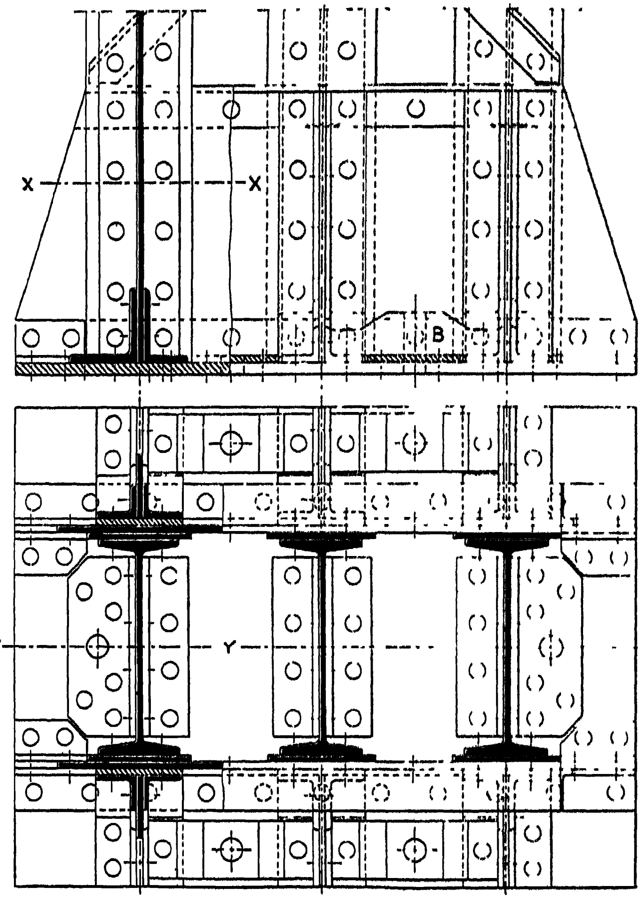


FIG. 124.

box section reinforced with four angles. The column is 18 in. square, and is provided with a 3 ft. 6 in. square base plate, 1 in. thick. Four deep brackets, consisting of $\frac{5}{8}$ in. gusset plates, and $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angles, assist in distributing the load to the base plate In arranging the brackets at the base of a column, no useful end is served by theoretical calculations as to the probable distribution of pressure effected by any proposed system of bracketing. The essential points

to bear in mind are, to provide a reasonably thick base plate in proportion to the load to be carried; to place brackets in well-distributed positions around the base, and to increase their number in accordance with the size of column, having regard to convenience in riveting. Small columns will generally be provided with two gusset plates, as in Fig. 120. Box sections generally admit of four brackets, one on each face, whilst double and treble compound columns may conveniently be

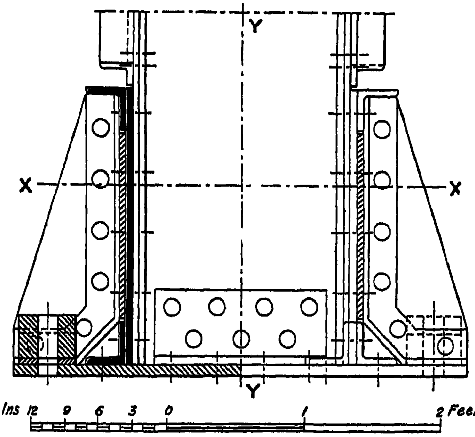


FIG. 125.

provided with eight, ten, or twelve brackets. Figs. 124 and 125 give an example of a built-up base for a treble-joist column. Here, two deep gusset plates are riveted to the flanges of the joists and bevelled off at the ends to form four brackets, whilst other three brackets are placed at right angles to the main gusset plates on either side. The pressure transmitted to the base plate will naturally be more intense immediately beneath the brackets, but by suitably increasing their number and adopting a rational thickness of base plate, the eventual distribution of pressure on the foundation will be sensibly uniform It is desirable in all heavy column construction that the foundation bolts should pass through the horizontal tables of the angles connecting the brackets with the base plates, so that the nuts may bear on a considerable thickness of material. Where this cannot be conveniently arranged, as in Fig. 124, wrought-iron or steel bridge blocks B should grip the horizontal angles attached to the brackets, and the foundation bolts be passed through the blocks. Brackets should have a good depth in proportion to their offset from the column face. It is a common practice to provide as many rivets through the column faces and vertical angles of the brackets as will give a shearing or bearing resistance at least equal to the total load on the column. This rule provides for the transmission of the load through the brackets to the base plate, independently of the bearing of the column end on the base plate, which in rough work is often far from satisfactory. Thus, in a column carrying 120 tons, assuming the resistance of $\frac{7}{8}$ inch rivets in single shear as 3 tons, the minimum number through the

vertical bracket angles would be 40. If the rivets bear on $\frac{1}{2}$ inch thick angles, the bearing resistance of each rivet at 8 tons per square inch $= 8 \times \frac{1}{2} \times \frac{7}{8} = 3\frac{1}{2}$ tons, and the minimum number for bearing $= 120 \div 3\frac{1}{2} = 35$.

Column Foundations.—Small columns carrying light loads are still occasionally attached to stone foundation blocks, but the majority are bolted down to concrete blocks having a basal area sufficiently large to suitably distribute the pressure on the soil. The base plate should rest on a sheet of 6 or 8 lbs. lead laid on the top of the foundation block. The heads of the foundation bolts are embedded near the bottom of the block, and bear against flat or angle bars threaded on to the bolts and laid longitudinally and transversely in the concrete. The proportions of such foundation blocks are determined from the character of the loads on the column and the safe pressure which may be put on the soil, and detailed examples will be considered later on.

Grillage Foundations.—Foundations for columns on poor bearing ground require the pressure distributing over a large area. In such cases, a "spread" or "grillage" foundation is most suitable. As seen in Fig. 127, the base casting rests on a layer of parallel beams or girders projecting considerably beyond the edge of the casting. These rest again on a second layer or "grill" of beams projecting beyond the limits of the first grill, and these again may rest upon a third and fourth layer if necessary. With each successive grill, the bearing area is largely increased, until the intensity of pressure on the soil is reduced to the desired value. The beams are tied together with bolts and separators, the latter acting as stiffeners for the webs, and giving them the necessary rigidity for resisting the vertical shearing force. The projection of the beams takes the place of several footings in an ordinary foundation, and the advantages of grillage foundations are that the successive offsets may be much longer than in the case of masonry or concrete footings, and the necessary bearing area is obtained at a considerably less depth than for a foundation block of the ordinary type. The stack of beams is eventually rammed up with and encased in concrete, which serves the double purpose of aiding uniform distribution of pressure amongst the separate beams in each grill, and protecting the steelwork from corrosion. In very large grillage foundations, the uppermost grill frequently consists of built-up plate or box girders, whilst the size of the beams diminishes in the various grills from top to bottom. In Fig. 126 AB represents a single layer of beams beneath a casting C carrying a loaded column. Let p tons per foot run = the upward reaction of the ground against the under side of

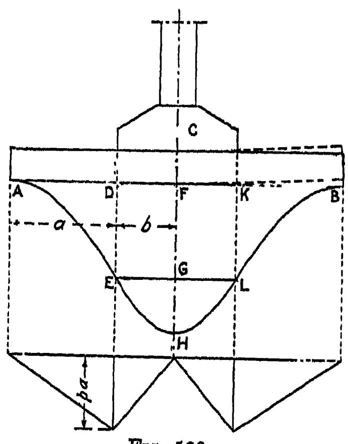


FIG. 126.

each beam. The portion AD obviously acts as a cantilever, subject to a uniformly distributed upward pressure of p tons per foot run, and tends to bend as shown by the dotted lines on the right-hand side. Therefore bending moment at $D = \frac{pa^2}{2}$ foot-tons $= DE$, and the semi-parabola AE forms the B.M. diagram from A to D.

It is frequently stated in text-books and hand-books that DE is the maximum bending moment on the grillage beams. Mr. Max am Ende, M.Inst.C.E., has shown, however, that the maximum bending moment occurs at the centre of the beam, and $= \frac{pa^2}{2} + \frac{pab}{2}$ foot-tons, where b is the distance from the centre of the beam to the edge of the base casting, or base plate, if no casting be employed.[1] This may be demonstrated as follows:—

Total upward pressure against $AB = p \times 2(a + b)$ tons $=$ total downward pressure exerted over the length $2b$ feet of the casting per pitch width of beams. The central portion DK of the beam is, therefore, subject to a *downward* pressure

$$= \frac{2p(a + b)}{2b} = \frac{p(a + b)}{b} \text{ tons per foot run,}$$

and an *upward* pressure of p tons per foot run, or a resultant downward pressure

$$= \frac{p(a + b)}{b} - p = \frac{pa}{b} \text{ tons per foot run.}$$

The span $DK = 2b$, hence B.M. due to the resultant downward load

$$= \frac{pa}{b} \times \frac{(2b)^2}{8} = \frac{pab}{2} \text{ foot-tons.}$$

The parabola EHL having a central height $GH = \frac{pab}{2}$ represents the moments over the span DK, due to the unbalanced load of $\frac{pa}{b}$ tons per foot, and since there is already a B.M. at D and $K = \frac{pa^2}{2}$, the total moment at $F = DE + GH = FH = \frac{pa^2}{2} + \frac{pab}{2}$.

It is sometimes urged that this moment will not be developed unless the casting bends appreciably, and, no doubt, the extent to which the moment FH may be approached in an actual foundation will depend on the relative rigidity of the casting and the girders beneath it. With a heavy steel casting, the yielding will be relatively small, but since the beams in the first grill are often much deeper than the casting, their rigidity is also considerable, and it is only reasonable, in designing the beams, to provide for the maximum moment FH, even although the actual moment may be somewhat less. Moreover, in the absence of a rigid casting, the relatively flexible base-plate of the column will readily

[1] *Mins. Proceedings Inst. C.E.*, vol. cxxviii. p. 36.

follow the deflection of the stiffer beams beneath it. The shearing force diagram is shown below, the shear increasing from zero at the ends of the beam to pa tons below D and K, and decreasing again to zero at the centre.

The following example illustrates the method of designing a grillage foundation for a column.

EXAMPLE 21.—*A column carries a total load of 240 tons, and is mounted on a casting having a base 4 feet square. The pressure on the ground is not to exceed $1\frac{1}{2}$ tons per square foot. Required a suitable grillage foundation.*

Minimum ground area required $= \dfrac{240}{1\frac{1}{2}} = 160$ square feet. This may be obtained by a rectangle 13 feet × 12 feet 6 inches. The heavier beams will be in the upper grill and may be 13 feet long. Five beams may be conveniently arranged beneath the four feet casting, in order to leave suitable room between the flanges for packing in concrete. Hence upward pressure per foot run against each beam of the upper grill $= \frac{1}{5}$ of $\frac{240}{13} = \frac{48}{13}$ tons; $a = 4$ feet 6 inches, $b = 2$ feet.

$\therefore$ B.M. at centre of each beam $= \frac{48}{13}\left(\dfrac{4\cdot5 \times 4\cdot5}{2} + \dfrac{4\cdot5 \times 2}{2}\right) = 54$ foot-tons $= 54 \times 12 = 648$ inch-tons.

Employing a working stress of 9 tons per square inch, the required modulus of section $= \frac{648}{9} = 72$.

The B.S.B., 14″ × 6″ × 57 lbs., has a modulus of 76·2. Hence the first grill may consist of five 14″ × 6″ × 57 lbs. beams spaced at 10 in. centres (Fig. 127).

Assuming 16 beams in the second grill, the upward pressure per foot run against each beam $= \frac{1}{16}$ of $\dfrac{240}{12\cdot5} = \frac{6}{5}$ tons; $a = 4$ feet 3 inches, $b = 2$ feet.

B.M. at centre of each beam $= \frac{6}{5}\left(\dfrac{4\cdot25 \times 4\cdot25}{2} + \dfrac{4\cdot25 \times 2}{2}\right) = \frac{255}{16}$ foot-tons $= \frac{255}{16} \times 12 = 191\cdot25$ inch-tons, and section modulus required $= \dfrac{191\cdot25}{9} = 21\cdot25$.

The B.S.B., 8″ × 5″ × 28 lbs., has a modulus of 22·3. Hence the second grill may consist of sixteen 8″ × 5″ × 28 lbs. beams spaced at 10 in. centres. Four rows of plate separators may be placed between the upper beams, and tube separators between the lower beams as indicated in Fig. 127. A depth of 16 inches of concrete is shown below the lower grill, the beams and casting being embedded as in the figure.

Maximum shear at edge of casting for each beam in the upper grill $= \frac{48}{13} \times 4\cdot5 = 16\cdot6$ tons. Sectional area of web $= 12'' \times \frac{1}{2}'' = 6$ sq. inches. Mean shear stress on web $= \dfrac{16\cdot6}{6} = 2\cdot8$ tons per sq. inch.

Maximum shear, 4 feet 3 inches from end of each beam in the lower grill $= \frac{6}{5} \times 4\cdot25 = 5\cdot1$ tons. Sectional area of web $= 6\cdot5 \times 0\cdot35 = 2\cdot27$ sq. inches. Mean shear stress on web $= \dfrac{5\cdot1}{2\cdot27} = 2\cdot25$ tons per

sq. inch. These stresses are well within the safe limit of working shear if the beams are well supported by the separators and concrete.

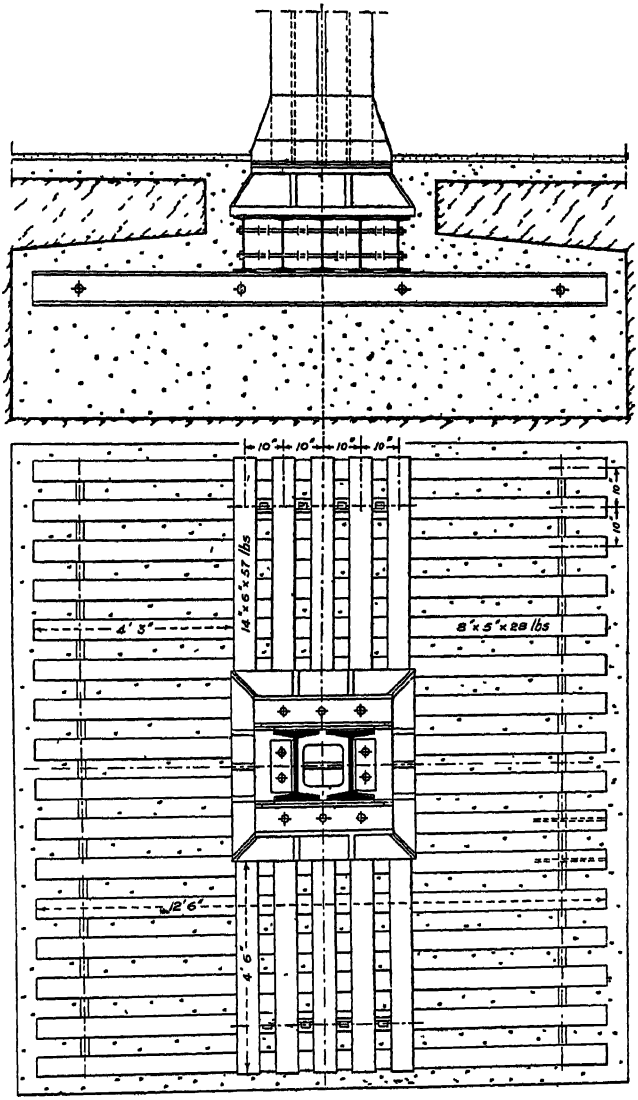


FIG. 127.

Practical Applications.—Some applications of the calculations to practical examples of columns and struts will now be given. It must be emphasized at the outset that the principal difficulty in designing

columns is to decide whether they may be treated as fixed-ended or round-ended, or to what extent they may be considered as approximating to one or other of these conditions of end support. In many cases this is a matter for personal judgment only, and probably no two persons would arrive at the same conclusion in a debatable case. As previously noted, *no* column is ever as stiff as the theoretically fixed-ended column, and in assuming any practical column to be actually fixed-ended, its strength must thereby be somewhat over-estimated.

In many cases, notably those in which pin-ended connections are employed, no doubt exists as to the columns being round-ended, whilst in cases where considerable doubt exists as to the character of the end support, the safest procedure is to treat the column as round-ended.

EXAMPLE 22.—*A mild-steel bridge-strut,* 18 *ft. long, consisting of four angles with lattice bracing similar to Type* 10, *is required to resist a maximum compression of* 55 *tons, the ends being fitted with pin bearings. Deduce a suitable section.*

Taking section No. 8, Type 10, with four 5″ × 3″ × ½″ angles. $r = 2\cdot53''$. $l = 18 \times 12 = 216''$. $A = 15$ square inches.

$$\frac{l}{r} = \frac{216}{2\cdot53} = 85$$

From the curve in Fig. 109, safe load for ratio $\frac{l}{r} = 85$, for round-ended struts = 8400 lbs. per square inch.

$$\therefore \text{Total safe load} = \frac{15 \times 8400}{2240} = 56\tfrac{1}{4} \text{ tons.}$$

Fig. 128 indicates the general design. The width W will depend on the interior breadth of the boom to which the strut is attached, and

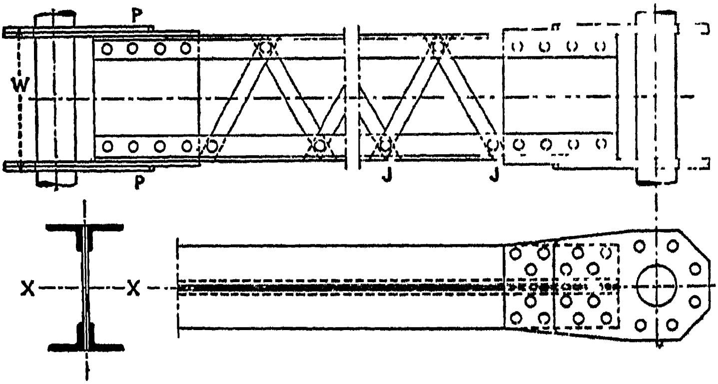


FIG. 128.

does not affect its resistance to bending in the plane X–X. The connecting plates P, P, must be attached to the angles by a sufficient number of rivets to provide the necessary resistance to shearing and bearing, and will further be reinforced by additional plates to give the necessary bearing area on the pins. The lacing bars usually consist of

flat bars from 2 in. to $2\frac{1}{4}$ in. wide by upwards of $\frac{3}{8}$ in. thick, and must be sufficiently close to prevent local failure of the angles between two junctions as J, J. This is very unlikely to occur with the usual proportions of lacing in general use. The method of calculation for local failure will be given in subsequent examples.

If the ends of the strut were securely riveted between very stiff booms the strut would approximate to the condition of having fixed ends. Section No. 1, Type 10, would then be suitable.

$$r = 1{\cdot}68'', \ \frac{l}{r} = \frac{216}{1{\cdot}68} = 130, \text{ and safe load } = 10{,}650 \text{ lbs. per sq. in.}$$

$$A = 12 \text{ sq. in. and total safe load} = \frac{10650 \times 12}{2240} = 57 \text{ tons.}$$

EXAMPLE 23.—*A strut of the type shown in Fig.* 129 *is* 7 *ft.* 6 *in. long, and is required to resist a compression of* 6 *tons, with pin-connected ends. Make a suitable design for the strut in mild steel.*

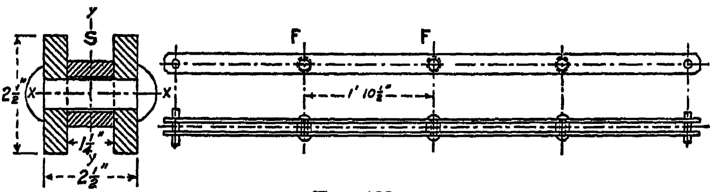


FIG. 129.

Struts of this type, consisting of two flats with distance ferules at F, F, are frequently used in roof trusses. By varying the length of the ferules, the two bars may be separated by any desired distance to give a suitable radius of gyration at the central section. They must, however, be tied together at sufficiently frequent intervals to prevent risk of local failure of one bar over the length FF. The dimensions adopted are shown on the enlarged cross-section at S.

1. Considering local failure of one bar for the unsupported length FF. Compression on one bar $= \frac{6}{2} = 3{\cdot}0$ tons. Sectional area $= 2\frac{1}{2}'' \times \frac{5}{8}'' = \frac{25}{16}$ sq. in. Pressure per square inch $= \dfrac{3{\cdot}0}{1{\cdot}56} = 1{\cdot}92$ tons, $= 4300$ lbs. $r = \frac{5}{8}'' \times 0{\cdot}289$ (for rectangular section) $= 0{\cdot}18''$. The length FF must be treated as round-ended, since the ferules are incapable of fixing the ends in a strut of this type.

The ratio $\dfrac{l}{r}$ for a safe stress of 4300 lbs. per square inch $= 136$, Fig. 109, for mild steel with round ends.

$$\therefore \frac{l}{0{\cdot}18} = 136, \text{ and } l = 136 \times 0{\cdot}18 = 24{\cdot}5'' = 2'\ 0\tfrac{1}{2}'',$$

say four equally spaced rivets at $1'\ 10\frac{1}{2}''$.

2. Considering failure of the strut as a whole with regard to bending in the plane y–y, and assuming $\frac{5}{8}''$ rivets, $A = 2{\cdot}34$.

$$r = 0{\cdot}827'', \; l = 90'', \text{ and } \frac{l}{r} = \frac{90}{0{\cdot}827} = 109$$

$$\text{Safe load per square inch for } \frac{l}{r} = 109 = 6000 \text{ lbs.}$$

$$\text{Total safe load} = \frac{6000 \times 2{\cdot}84}{2240} = 6{\cdot}27 \text{ tons, or practically 6 tons.}$$

3. Considering failure as a whole with regard to bending in the plane x–x.

Moment of inertia about y–y $= \frac{1}{12} \times (\frac{5}{2} - \frac{5}{8}) \times (2{\cdot}5^3 - 1{\cdot}25^3) = 2{\cdot}14$.

$$\text{Sectional area} = 2{\cdot}84 \text{ sq. in.} \quad \therefore r = \sqrt{\frac{2{\cdot}14}{2{\cdot}84}} = 0{\cdot}95''$$

$$\frac{l}{r} = \frac{90}{0{\cdot}95} = 95, \text{ and safe stress per sq. in.} = 7300 \text{ lbs.}$$

$$\therefore \text{Total safe load} = 2{\cdot}84 \times \tfrac{7300}{2240} = 7{\cdot}6 \text{ tons.}$$

The strut is therefore slightly weaker as regards its resistance to bending in the plane y–y.

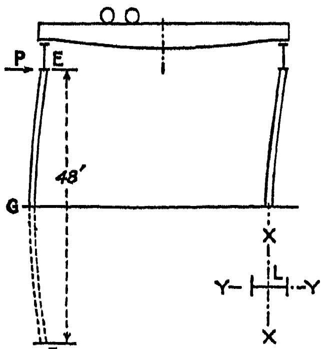


FIG. 130.

EXAMPLE 24. *A mild-steel gantry column for supporting the rail girder of an overhead travelling crane is 24 ft. high. The maximum load on it is 45 tons, and the columns are efficiently braced together lengthwise of the gantry.*

In Fig. 130, as regards bending transversely, the columns will be equivalent to a round-ended column EF of 48 ft. length, or twice the length of the actual column EG. For best resisting bending in the transverse plane, the columns will be placed as at L in plan, so that the greatest radius of gyration comes into play. Taking the broad-flanged beam section $11'' \times 11'' \times 76$ lbs., the greatest radius of gyration $= 4{\cdot}73''$. Sectional area $= 22{\cdot}26$ sq. in., and $\frac{l}{r} = \frac{48 \times 12}{4{\cdot}73}$ $= 127$.

The safe load for mild-steel round-ended columns for this ratio $= 4800$ lbs. per sq. inch.

$$\therefore \text{Total safe load} = \frac{4800 \times 22{\cdot}26}{2240} = 47{\cdot}7 \text{ tons, or a little in excess}$$

of the load to be carried. With regard to bending in the plane X–X, the columns will realize a high degree of end fixation, and the section adopted will have a large excess of strength considered as a fixed-ended column 24 ft. long.

It should be noticed that a complete examination of this case would

necessitate a consideration of the wind pressure P acting on the head and face of the column, which will create additional bending moment at the basal section. The area exposed to the wind by the ends of crane girders and face of rail girders would need to be fairly accurately known in order to take account of this action. As this case is similar to a subsequent example, it will be treated more fully later on. For the present, assuming 80 square feet as the effective area exposed to the wind and a maximum horizontal wind pressure during the working of the crane of 25 lbs. per square foot, the total horizontal pressure at heads of windward and leeward columns $= 80 \times 25 = 2000$ lbs. This is resisted by two columns, giving 1000 lbs. acting horizontally at the head of each column, resulting in a bending moment at the foot of the column $= \dfrac{1000 \times 24 \times 12}{2240} = 129$ inch-tons. To this must be added the B.M. due to wind pressure on face of column. Area of face = say 24 square feet. Total pressure on column $= 24 \times 25 = 600$ lbs., acting halfway up the column.

$$\therefore \text{B.M.} = \frac{600 \times 12 \times 12}{2240} = 39 \text{ inch-tons.}$$

Total B.M. due to wind-pressure $= 129 + 39 = 168$ inch-tons. The section modulus of the $11'' \times 11''$ beam about X–X = 90·3. Hence bending stress due to wind pressure alone $= \dfrac{168}{90 \cdot 3} = 1 \cdot 9$ tons per square inch. This calculation will give an indication of the probable amount of stress likely to be caused by wind pressure under ordinary working conditions, and the $11'' \times 11''$ beam section, as deduced when the wind pressure was neglected, will evidently be too light. Further disturbing action comes on the columns when the load is being cross-traversed. The additional B.M. resulting therefrom cannot be more than roughly approximated, but making a rational allowance for this, together with the wind pressure, a beam section $14'' \times 12'' \times 101$ lbs. would be a reasonable section to employ.

EXAMPLE 25.—*Deduce a suitable section for a mild-steel column 20 ft. high, supporting a vertical load of 5 tons and a horizontal pressure of 2 tons due to wind, acting at its upper end.*

These conditions of loading occur in the case of columns supporting roofs. The action of the loads is similar to that in the last example, excepting that the vertical load in the case of a roof is relatively small, whilst the bending action due to the wind is considerable, and usually constitutes the principal cause of stress in the column. This case will be considered more fully than the last.

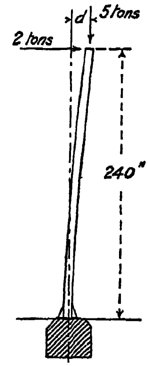


FIG. 131.

1. Calculate the horizontal deflection d, Fig. 131, due to the pressure of two tons applied at the head of the column. If the lower end be firmly fixed to an adequate foundation, the column is acting under similar conditions to those obtaining in a cantilever fixed at one end and loaded at the other.

Deflection $d = \frac{1}{3} \cdot \frac{Wl^3}{EI}$, where $l = 240''$, $W = 2$ tons, and $E = 13{,}400$.

The deflection cannot be calculated until I is known, and since this depends on the cross-section adopted for the column, a preliminary value must be assumed for I, which will probably require correction before finally deciding on the cross-section. Taking the British Standard beam section 14″ × 6″ × 57 lbs., from the section book, A = 16·77 square inches. I = 533, and the section modulus = 76·15.

$$\text{Deflection } d = \frac{1}{3} \times \frac{2 \times (240)^3}{13{,}400 \times 533} = 1{\cdot}29''$$

B.M. at foot of column, due to horl. pressure = 2 × 240 = 480 in.-tons.
" " vertl. " = 5 × 1·29 = 6·45 "

Total B.M. = 486·45 "

and stress due to bending at foot of column $= \frac{486{\cdot}45}{76{\cdot}15} = \pm\, 6{\cdot}39$ tons per square inch, that is, 6·39 tons per square inch compression on concave or leeward face of section and 6·39 tons per square inch tension on convex or windward face of section.

$$\text{Direct compression} = \frac{5}{16{\cdot}77} = 0{\cdot}3 \text{ ton per square inch.}$$

∴ Maximum compressive stress at concave side = 6·39 + 0·3 = 6·69
and " tensile " convex " = − 6·39 + 0·3 = − 6·09

tons per square inch. As these stresses are well within the safe limit for mild steel, implying a factor of safety of just over 4, the 14″ × 6″ section may be adopted.

Note.—The above calculation neglects the small *additional* deflection caused by the *vertical* load of 5 tons, once the column is deflected away from the vertical. In the great majority of cases, the additional B.M. due to this deflection is very small compared with that due to the horizontal pressure, and may be safely neglected. It should be taken into account if the vertical load is very heavy, which is seldom the case in roof construction. The B.M. of 6·45 inch-tons, due to the deflection of 1·29 in. is also very small compared with 480 inch-tons, and might in this case also have been neglected without appreciably affecting the result. The calculation further assumes exact central loading. If a reasonable amount of "accidental" or "equivalent" eccentricity of loading be assumed in accordance with the theory previously stated, it would, in this case

$$= e = \frac{0{\cdot}6 \times r^2}{y} = \frac{0{\cdot}6 \times (5{\cdot}6)^2}{7''} = 2{\cdot}69'',$$

and this again would have little influence on the total bending moment, on account of the small vertical load. The inference to be drawn from examples such as this, is that slight eccentricity of loading and deflection within reasonable limits have little bearing on the practical design of columns under light vertical loading, whilst the relatively large

horizontal load acting at the great leverage equal to the length of the column practically converts the case into one of beam design. It is advisable to calculate the deflection for any proposed section, in order to make sure it does not exceed a reasonable limit, since a relatively large deflection alternately to right and left would exercise a deteriorating effect on the stability of the foundation and on the connections at the head of the column.

EXAMPLE 26.—*Required a strut section consisting of two angles back to back, ½ in. apart, to resist a compression of* 18 *tons. Length* 10 *ft. Ends considered fixed.*

This type of strut, Fig. 132, is largely used in roof trusses and light lattice girders. It is not an economical form since the load is necessarily applied eccentrically. The rivets connecting the strut with a junction plate or neighbouring member, pass through the centre CC of the vertical legs V, V, of the angles, whilst the axis XX passing through the centre of gravity of the cross-section is distant e' from CC.

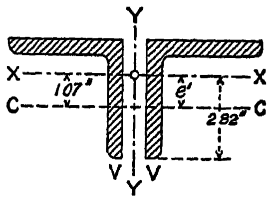


FIG. 132.

Assuming two angles 4″ × 4″ × ½″, the least radius is about the axis XX, which is situated 1·18 in. from the upper edge of the section, and bending takes place most easily in the plane YY. The centre line of rivets CC is 2·25 in. from upper edge and the eccentricity e' is therefore = 2·25″ − 1·18″ = 1·07″.

Least radius of gyration about X–X = 1·227 in.

The maximum intensity of compression will occur at the edges V, V, of the vertical legs. These are distant 2·82 in. from the axis XX. Hence,

$$\frac{ye'}{r^2} = \frac{2\cdot82 \times 1\cdot07}{1\cdot227 \times 1\cdot227} = \frac{3\cdot01}{1\cdot5} = 2,$$

and
$$\therefore \frac{ye'}{r^2} + 0\cdot6 = 2\cdot6$$

The strut being considered fixed-ended and 10 ft. long, the equivalent round-ended strut will be 5 ft. long and $\frac{l}{r} = \frac{60''}{1\cdot227} = 49$, say 50.

Referring to the curves of safe loads for eccentrically loaded mild steel struts, Fig. 113, for various values of $\frac{ye'}{r^2} + 0\cdot6$, the safe load for ratio $\frac{l}{r} = 50$ and $\frac{ye'}{r^2} + 0\cdot6 = 2\cdot6$, is 6000 lbs. per square inch. (This value has been interpolated between curves Nos 2 and 3.)

Sectional area of two 4″ × 4″ × ½″ angles = 7·5 sq. in.

$$\therefore \text{Safe load} = \frac{7\cdot5 \times 6000}{2240} = 20\cdot09 \text{ tons,}$$

or 2 tons more than the specified load. This section will therefore be satisfactory.

Note.—Care should be taken to ascertain about which axis the radius of gyration is least. For struts formed of two *equal* angles, the least radius is *always* about X–X. If two *unequal* angles be used with the longer legs back to back the least radius may be about X–X or Y–Y, depending on the size of angles used. The angles are riveted together with $\frac{3}{8}$ in. or $\frac{1}{2}$ in. thick buttons or washers between, at sufficiently close intervals to prevent local failure of one angle.

EXAMPLE 27.—*A lattice box strut of Type 11, is required for a horizontal wind brace between the main girders of a bridge. The length is 30 feet and maximum compression 15 tons. Obtain a suitable section, treating the ends as rounded.*

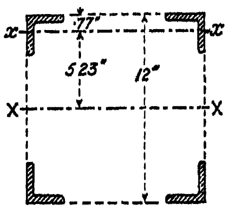


FIG. 133.

In this example, the strut being in a horizontal position, three actions are to be considered.

1. The bending stress due to the dead weight of the strut.

2. The bending stress due to the deflection and end thrust.

3. The direct compression caused by the end thrust of 15 tons. Assuming the section in Fig. 133, consisting of four $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{3}{8}''$ angles, laced on all four sides with $1\frac{3}{4}'' \times \frac{5}{16}''$ lacing bars, the approximate weight of the strut for a length of 30 ft. is about 1050 lbs. This, acting as a distributed load, will create a bending moment

$$\frac{Wl}{8} = \frac{1050 \times 30 \times 12}{2240 \times 8} = 21\cdot1 \text{ inch-tons.}$$

From the section book, the moment of inertia of *one* angle about x–x = 0·989, and sectional area = 1·733 sq. in.

$$\therefore \left.\begin{array}{l}\text{Mt. of inertia of one angle}\\ \text{about X–X}\end{array}\right\} = 0\cdot989 + 1\cdot733 \times (5\cdot23)^2 = 48\cdot1$$

and I_x for four angles = $48\cdot1 \times 4 = 193\cdot6$, say 194. Total sectional area = $1\cdot733 \times 4 = 6\cdot93$ sq. in.

$$\therefore (\text{Radius of gyration})^2 = \frac{194}{6\cdot93} = 27\cdot9$$

Making allowance for an "equivalent eccentricity" of loading

$$= \frac{0\cdot6 r^2}{y} = \frac{0\cdot6 \times 27\cdot9}{6''} = 2\cdot79'',$$

the deflection caused by the thrust of 15 tons acting at this eccentricity

$$= \frac{Pl^2 e}{8EI - \frac{5}{6}Pl^2} = \frac{15 \times 360^2 \times 2\cdot79}{8 \times 13400 \times 194 - \frac{5}{6} \times 15 \times 360^2} = 0\cdot29''$$

The deflection caused by the dead weight of the strut $= \frac{5}{384} \times \frac{Wl^3}{EI}$,

which, substituting the known values for W, l, E and I, = 0·12 in. If this be added to the equivalent eccentricity of 2·79 in. it gives a total

eccentricity of 2·91 in. and the resulting deflection due to end thrust is 0·3 in. instead of 0·29 in. as above calculated. The effect of this deflection of 0·12 in. (due to dead weight) is so slight in cases of light struts, it may be neglected. Total eccentricity = equivalent eccentricity + deflection due to end thrust = 2·79 + 0·30 = 3·09 in.

B.M. at centre of strut due to 15 tons end thrust = 15 × 3·09 = 46·35
" " due to dead weight = 21·10
and total B.M. = 67·45 inch-tons.

Moment of resistance of section $= \frac{f\mathrm{I}}{y} = \frac{f \times 194}{6''} = 67{\cdot}45$; whence $f = \pm\ 2{\cdot}1$ tons per sq. inch.

$$\text{Direct compression} = \frac{15}{6{\cdot}93} = +\ 2{\cdot}2 \text{ tons per square inch.}$$

∴ Maximum stress at upper edge of section = + 2·1 + 2·2 = 4 3
and minimum " lower " = − 2·1 + 2·2 = 0·1
tons per square inch, compression in both cases.

Note.—If the tensile stress caused by bending is found to more than annul the direct compression, the calculation should be revised, deducting the rivet holes in calculating the I of the section. The maximum stress of 4·3 tons per square inch is rather low, but making allowance for the fluctuating action of wind load, a lighter section is not commendable. It should be noted also that corrosion is more rapid in light lattice work of this description, and the working stress should be lower than in heavier parts of the same structure, in order to secure approximately equal length of life.

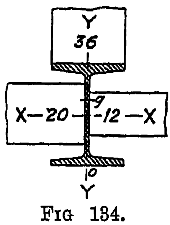


FIG 134.

EXAMPLE 28.—*Three girders are connected with a column* 20 *feet high, as shown in plan in Fig.* 134. *The girder connected with the flange imposes a load of* 36 *tons, and those connected with the web, loads of* 20 *and* 12 *tons. The column carries, in addition, a central load of* 40 *tons. Required a suitable section.*

This illustrates a very general case of eccentric loading. The eccentricity in the plane Y–Y is relatively large.

Taking moments about o, and assuming a 12″ × 12″ beam section, 36 × 12″ + (12 + 20 + 40) × 6″ = 108 × go, whence go = 8″, or the centre of gravity of the total load is situated in a plane passing through g, 2 in. from the axis X–X. The eccentricity in the plane X–X is very slight, as will be found by taking moments about Y–Y, and for practical purposes, the c g. of the loads may be assumed at g on YY. The intentional eccentricity $e' = 2$ in. Radius of gyration of section about X–X = 5 07 in. Hence—

$$\frac{ye'}{r^2} + 0{\cdot}6 = \frac{6 \times 2}{25{\cdot}7} + 0{\cdot}6 = 1\ 07$$

Treating the column as fixed-ended, the length of the equivalent round-ended column = 10 ft. or 120 in.

Ratio $\frac{l}{r} = \frac{120}{5{\cdot}07} = 24.$ Referring to Fig. 113, the safe load per

square inch for ratio 21, from curve No. 1, is 11,200 lbs. As the value of $\frac{ye'}{r^2} + 0{\cdot}6$ is here a little higher than 1, the safe load will be a little under 11,200 lbs., say 11,000 lbs.

$$\therefore \text{Safe load on column} = \frac{23{\cdot}6 \text{ (sectl. area)} \times 11000}{2240} = 116 \text{ tons.}$$

The total load is actually 108 tons.

Foundation Blocks for Columns.—It is very essential that foundation blocks for columns may not undergo any appreciable subsidence, as the stresses in the members of the structure carried by the columns would thereby be seriously affected. The subsoil must therefore provide a firm bearing surface, and the safe pressure on the soil not be exceeded. On gravel and hard clay foundations the safe pressure may be 4 to 5 or 6 tons per square foot. Grillage foundations are preferable to concrete blocks where the safe pressure is below $1\frac{1}{2}$ tons per square foot. In the case of columns carrying central loads only, the foundation block simply requires to be of sufficient area to reduce the bearing on the soil to the required safe limit, and of sufficient depth to resist shearing of the block along a vertical plane beneath the edge of the base plate. In the case of columns subject to side loads, the centre of pressure may fall very near one edge of the foundation block, especially where the vertical load on the column is small. In Fig. 135 a column 20 feet high is bolted to a concrete base 4 feet square and 3 feet deep. The column carries a vertical load of 12 tons, including its own weight, and is subject to a horizontal wind pressure of 1 ton at the top, and $\frac{1}{4}$ ton at the centre. The resultant horizontal pressure = $1\frac{1}{4}$ tons acting at a point 21 feet above the foundation. The weight of the foundation block at 140 lbs. per cubic ft. = 3 tons, and total vertical load on foundation = 15 tons.

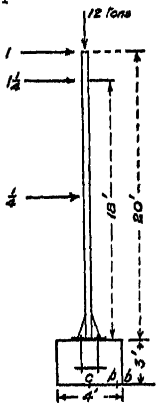


FIG. 135.

Hence centre of pressure p is situated $\frac{1\frac{1}{4}}{15}$ of $21' = 1'\ 9''$ from c, or 3 in. from edge of base. The resulting maximum intensity of pressure on foundation at $b = \frac{2 \times 15}{4 \times 4}\left(2 - \frac{3}{16}\right) = 3{\cdot}4$ tons per sq. ft. (see page 388). Provided the foundation will resist this pressure without appreciably yielding, the column will not tilt. If, however, this pressure exceeds what may be safely put upon the soil, the bearing area of the base block must be increased.

Suppose a column of the same height to carry an inclusive vertical load of 50 tons, and to be subject to the same horizontal pressures. Using the same size of base block, the centre of pressure is $\frac{1\frac{1}{4}}{53}$ of $21' = 0{\cdot}49'$, say 6 in. from c, or 18 in. from b, and intensity of pressure on foundation at $b = \frac{2 \times 53}{4 \times 4}\left(2 - 1\frac{1}{8}\right) = 5{\cdot}8$ tons per sq. ft.

On a hard foundation this would not be an excessive pressure, and it will be seen from this example that a lightly loaded column may require a foundation block quite as large as a much more heavily loaded column, when subject to lateral pressure. In practice there is usually 1 to 3 feet of earth over the top of the block, and a little additional stability is derived from its weight.

Foundation Bolts for Columns.—In cases where the centre of pressure due to the resultant of all the vertical and horizontal loads above the base plate falls within the base of the bolt holes, there will be no uplift on the foundation bolts. Frequently, however, with columns subject to lateral wind loading, the centre of pressure falls beyond the bolts on the leeward side of the column, and the windward bolts must have a sufficient section to safely resist the resulting uplift. Considering, Fig. 136, height of column above base

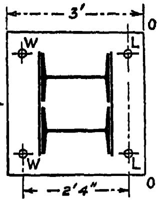


Fig. 137.

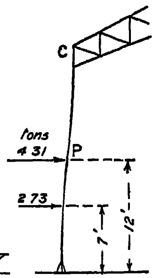


Fig 136a.

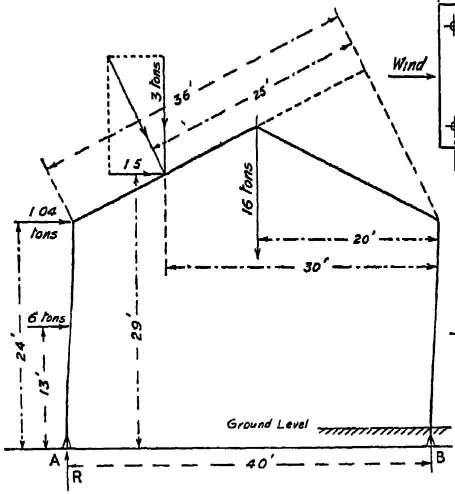


Fig. 136.

plate = 24 feet, and above ground-level 22 feet. Longitudinal spacing of columns 20 feet. Roof span, 40 feet, with rise 10 feet. Suppose the columns to carry the side of the building and to be exposed to a maximum horizontal wind pressure of 30 lbs. per square foot. Then, total wind pressure on side per 20 ft. length = $20 \times 22 \times 30$ lbs. = 5·9, say 6 tons, acting at 13 feet above base plate. Assume the effective horizontal wind pressure against the roof as 1·5 tons acting at 29 feet above base plate, weight of roof per 20 ft. length, 7 tons, and weight of each column 2 tons. Lastly, take the weight of roof girder, framing, and corrugated sheeting for one side of building per 20 ft. run as $2\frac{1}{2}$ tons.

If the connection of the roof with the columns be very stiff and rigid, the lateral wind pressure tends to tilt the structure as a whole, with the result that the pressure on the windward columns is reduced,

and that on the leeward columns increased. Let R = upward reaction beneath column A. Take moments about B—

$$R \times 40' + 6 \times 13' + 1\cdot5 \times 29' = 16 \text{ (total wt. tons)} \times 20' + 8 \text{ (vertl. compt. of wind)} \times 30'$$

$$\therefore R = 7\cdot02, \text{ say } 7 \text{ tons.}$$

With a firm attachment at C, the column will bend as shown in Fig. 136A, with reversal of curvature at P, situated at one half the height of the column above the base plate, or at a somewhat less height varying with the depth of the roof truss connection. As the exact position of P is doubtful of location, assume it in the highest possible position, *i.e.* 12 feet above base. The total horizontal pressure applied above this level = ($\frac{25}{36}$ of 1·5) horizontal component of wind reaction at C + ($\frac{12}{22}$ of 6) proportion of wind pressure on face of building, applied above P = 4·31 tons. This pressure is transferred to P, where it acts as a concentrated load applied at 12 feet above the base. The wind load on the face below level of P = $\frac{10}{22}$ of 6 = 2·73 tons applied at 7 feet above the base. The dimensions of the base plate and spacing of bolts are shown in Fig. 137. As the strip of plate between the leeward bolts L, L, and outer edge OO, is relatively weak, it appears reasonable to take moments about LL in considering the uplift on the windward bolts W, W. Let T = tension in *each* windward bolt, then, taking moments about LL,

$$2T \times 2\tfrac{1}{3}' + 7 \text{ tons} \times 1\tfrac{1}{6}' = 4\cdot31 \times 12' + 2\cdot73 \times 7',$$

whence T = 13·4 tons. Employing a working stress of 8 tons per sq. inch, sectional area = $\frac{13\cdot4}{8}$ = 1·67 sq. in., and diameter of bolt having this net section at bottom of thread = $1\frac{3}{4}$ in.

With the usual type of connection of V-roofs to columns, the degree of rigidity assumed above will seldom be realized, and the roof will tend to rack over as in Fig. 102, A. In such a case the lateral pressure on the roof will be applied at the head of the column instead of at P, and the pressure on the face of the structure will *all* be applied half-way up the covering. In cases where a fair amount of rigidity may be attained by the use of knee braces, etc., the point P will be situated somewhere between the head and centre of the column, but its exact location is practically impossible.

Compound Columns.—Compound columns consisting of two or three beam sections connected by tie-plates or bracing are largely used for supporting crane tracks and roof in works buildings. With tie-plates as habitually employed, the distribution of load amongst the individual beam sections will be very imperfect, and each beam will practically carry the load resting immediately upon it. Thus, in the column in Fig. 95, E, the two outer beams support much heavier loads than the central one. Tie-plates, as here shown, may only be conveniently riveted to the outside halves of the flanges of the outer beams as well as to both halves of the central beam flanges, and will therefore be of little value in transferring a portion of the outer loads W_2 and W_3 to the central beam. If stiff diagonal angle or channel

bracing be employed instead of tie-plates, the load distribution will be more equal, although still incapable of exact determination. The tie-plate compound column is uneconomical and much inferior to a compound column with continuous plates.

Live Loads on Columns.—No hard-and-fast rule can be followed in the treatment of live loads. These are so variable in character and effect that each case must be considered on its own merits. The most suitable procedure will be to increase the moving load by a suitable percentage, dependent on the character of the load, and treat the resulting increased load as so much equivalent dead load. In extreme cases, where columns are liable to sudden shocks, it will be necessary to increase the live load causing such shocks by nearly, or quite, 100 per cent. Where the live load is more gradual in rate of application a lower percentage will serve, but its estimate must be the result of careful judgment. In designing columns for supporting crane girders, the crane load should invariably be doubled.

Maximum Rivet Pitch in built-up Steel Columns.—If the pitch of rivets connecting the outer plates with the main members of built-up or compound columns be too great, there will be risk of local failure of individual plates by buckling, as shown in Fig. 138. In a column composed of separate elements, such as two or more plates connected by angles or channels with other rolled section bars, the strength of the column will be represented by that of its weakest component. The outer plates of built-up columns possessing a smaller radius of gyration than the other component sections, are the most liable to fail by local buckling, whilst from their position in the cross-section (farthest from the neutral axis) they are further subject to the maximum intensity of stress due to both bending and direct compression. The tendency to buckle of the local unsupported lengths of plate, such as ab, will be most marked in the case of short columns very heavily loaded, or longer columns subject to relatively heavy lateral loading. In both these cases the maximum allowable compression of, say, 9 to 10 tons per square inch may be approached, and the vertical pitch l of the rivets must be such that the ratio $\frac{l}{r}$ does not exceed the value corresponding with this working stress; r being the radius of gyration for the thinnest plates employed. Again, a length of plate, such as ab, can be very little stronger than a round-ended column, since the only fixation tending to constrain it to act as a fixed-ended column is that derived from the grip of the outer edges of the rivet heads, whilst the plate is considerably weakened across the section of the rivet-holes, especially where the rivets may be four or six abreast.

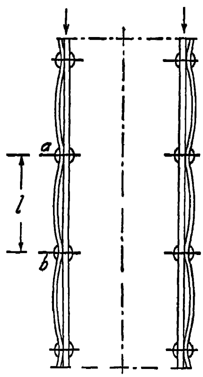


FIG. 138.

The ratio $\frac{l}{r}$ for a safe buckling stress of 24,000 lbs. per square inch for mild steel columns with rounded ends is 65 (from Euler's formula),

and r for $\frac{3}{8}$ in. plates $= 0{\cdot}108$ in., for $\frac{1}{2}$ in. plates $= 0{\cdot}144$ in., and for $\frac{5}{8}$ in. plates $= 0{\cdot}18$ in.

Hence, maximum safe pitch of rivets in outer plates—

$$\begin{aligned} \tfrac{3}{8}'' \text{ thick} &= 65 \times 0{\cdot}108'' = 7{\cdot}02'' \text{ say } 7'' \\ \tfrac{1}{2}'' \quad ,, \quad &= 65 \times 0{\cdot}144'' = 9{\cdot}36'' \;,,\; 9\tfrac{3}{8}'' \\ \tfrac{5}{8}'' \quad ,, \quad &= 65 \times 0{\cdot}180'' = 11{\cdot}70'' \;,,\; 11\tfrac{5}{8}'' \end{aligned}$$

Plates $\frac{5}{8}$ in. thick will seldom be employed in compound columns. In order to keep well within any possible risk of local failure, a maximum pitch of 6 in. may be adopted for $\frac{5}{8}$ in. plates, 5 in. for $\frac{1}{2}$ in. plates, and 4 in. for $\frac{3}{8}$ in. plates, since other influences, such as the transverse pitch of the rivets, also enter into consideration. That these values will be amply safe is also borne out by the results of tests of columns in which failure has taken place by local buckling of the plates. In practice, 4 inches is very commonly adopted as the rivet pitch for any ordinary thickness of plate.

Proportions of Lacing in Laced Columns. Columns of Two Beam Sections with Lacing. Fig. 138A (A).

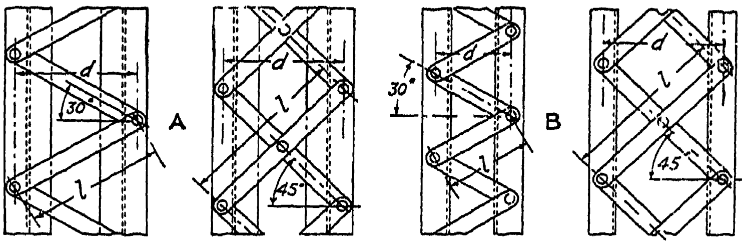


FIG. 138A.

Width of beam flange .	inches	8	$7\frac{1}{2}$	7	6	5	4
Width of lacing bar	,,	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	$2\frac{1}{4}$	2
Diameter of rivets	inch	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{8}$

Minimum thickness of lacing bars for single lacing inclined 30° $= \frac{1}{40}l$.
Minimum thickness of lacing bars for double lacing inclined 45° $= \frac{1}{60}l$.
Single lacing is suitable for values of d not exceeding 15 inches.

Columns of Two Channel Sections with Lacing. Fig. 138A (B).

Depth of channel. . . .	inches	17	15	12	10	9	8	7
Width of lacing bar . .	,,	$2\frac{3}{4}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{2}$	$2\frac{1}{4}$	$2\frac{1}{4}$	2
Diameter of rivets . . .	inch	$\frac{7}{8}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{5}{8}$

Minimum thickness of lacing bars for single lacing inclined 30° $= \frac{1}{40}l$.
Minimum thickness of lacing bars for double lacing inclined 45° $= \frac{1}{60}l$.
Single lacing suitable for values of d not exceeding 15 inches. No lacing bars should be used less than $\frac{3}{8}$ inch thick.

CHAPTER VI.

PLATE GIRDERS.

PLATE girders consist of a combination of plates and angles riveted together in the manner shown in Fig. 139. The web plate is necessarily continuous from end to end of the girder, but may vary in thickness. The flanges are usually composed of a pair of flange angles continuous for the full length of the girder, and one or more plates,

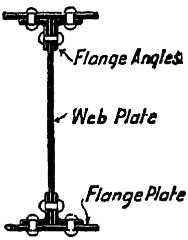


FIG 139.

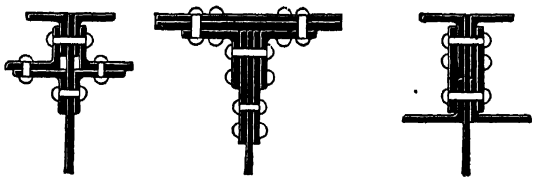


FIG. 140.

not necessarily extending the full length of the girder. Different forms of flanges are occasionally used, especially in America, and examples of such are given in Fig. 140.

Economic Limiting Span.—The principal difference between plate and lattice girders lies in the construction of the web. The continuous webs of plate girders require no complicated and expensive connections with the flanges, but for very large girders the depth of web necessitates special stiffeners, or very thick web plates, which add considerably to the weight and cost of the girders. These considerations limit the span for which plate girders may be economically used to about 100 feet, above which some form of lattice girder would be more economical.

Depth.—The stresses in the flanges, and consequently the flange area, vary inversely as the depth of the girder, but the sectional area of the web plate increases rapidly with an increasing depth of beam. The present practice, which has been found to give economical proportions of flanges and web, is to make the depth of plate girders equal to $\frac{1}{10}$ to $\frac{1}{14}$ of the span, according to the purpose for which they are to be used.

Breadth of Flange.—If girders be unsupported laterally, a sufficient width of flange must be adopted to resist any side pressure that may be imposed on the girder. No general rule can be given for the flange

width, on account of the very varying lateral forces to which the girders may be subjected; each case must be considered for its special conditions of loading. To prevent local buckling, flange plates should not project more than 2 to 3 inches beyond the flange angles, or they should be stiffened by knee web stiffeners at frequent intervals.

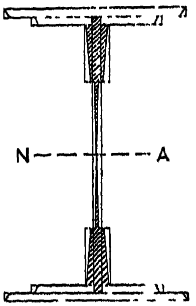


FIG. 141.

Estimated Weight.—The weight of plate girders depends upon the system of loading, the unit stresses adopted, and other factors, so that no accurate general formula can be deduced, but the weight as obtained from the formula given in Chapter II. may be used as a first approximation.

Sectional Area of Flanges.—In Fig. 141 the modulus figure for a plate girder is shown, and demonstrates the small part played by the web and the vertical legs of the angles in resisting the bending stresses. Except for very light girders, the resistance to bending of the web and vertical legs of the angles is neglected, and the flange plates and horizontal legs of the angles made sufficiently strong to resist the whole of the bending stresses.

Let D = distance between centres of gravity of flanges.
f = maximum intensity of stress in the flanges.
A = total area of each flange.

Since the thickness of the flange is very small compared with its distance from the neutral axis, the stress in the flanges may be considered as equally distributed over the whole of the flange section. The flange area may then be considered as concentrated at its centre of gravity, and the *effective depth* of the girder equal to the distance between the centres of gravity of the flanges.

The moment of inertia of the flanges

$$= 2\left\{I_x + A\left(\frac{D}{2}\right)^2\right\}$$

I_x is so small as to be negligible, and the moment of inertia may be considered $= \frac{AD^2}{2}$.

$$\therefore \text{M.R.} = f\frac{\frac{AD^2}{2}}{\frac{D}{2}}$$

$$= fAD$$

The required net area of each flange may therefore be obtained from

$$\text{B.M.} = fAD$$

or

$$A = \frac{\text{B.M.}}{fD}$$

D must be measured in the same units of length as the bending moment.

EXAMPLE 29.—*To find a suitable section of flange at the middle of the span for a plate girder 60 feet span, 5 feet deep, carrying a distributed load of 2 tons per foot run, the stress not to exceed 8 tons per square inch.*

The bending moment at the middle of the span

$$= \frac{60 \times 2 \times 60}{8}$$
$$= 900 \text{ ft.-tons}$$

The net area of flange required

$$= \frac{900}{5 \times 8} = 22{\cdot}5 \text{ sq. in.}$$

Suppose a flange width of 18 inches and $4'' \times 4'' \times \frac{1}{2}''$ angles be adopted. The area of the horizontal legs of the angles, allowing for $\frac{7}{8}$ in. rivets,

$$= (8 - 2 \times \tfrac{15}{16}) \times \tfrac{1}{2}$$
$$= \text{(say) } 3 \text{ square inches.}$$

The area of plate will therefore

$$= 22{\cdot}5 - 3 = 19{\cdot}5 \text{ square inches.}$$

Thickness of plate $\quad = \dfrac{19{\cdot}5}{18 - 2 \times \frac{15}{16}} = 1{\cdot}2$ in.

This thickness may be made up by using two $\frac{5}{8}$ in. plates, or two $\frac{3}{8}$ in. plates, and one $\frac{1}{2}$ in. plate, as in Fig. 142.

Thickness of Web Plate.—It is usual when designing plate girders to assume that the web resists the whole of the vertical shearing force, if the flanges be parallel. The action of the stresses in the web is a matter of great controversy, and various methods have been suggested

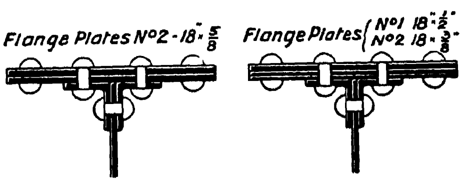


FIG. 142.

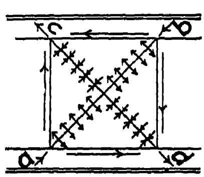


FIG. 143.

for calculating the required thickness of the web. The shear in the web is accompanied by tensile and compressive stresses acting at right angles to each other, and in parallel girders, at 45° to the vertical, Fig. 143. The web plate along *ab* is therefore in compression, and may fail in a similar manner to a long strut.

Let t = thickness of plate.
l = length along *ab*.

The least radius of gyration

$$r = \sqrt{\frac{t^3}{12} \times t} = \frac{t}{\sqrt{12}}$$

The ratio
$$\frac{l}{r} = \frac{l\sqrt{12}}{t}$$

The ends may be assumed fixed, and the safe intensity of stress obtained from the curve in Fig. 109. The actual intensity of stress along the line *ab* is equal to the intensity on a vertical section of the web due to the vertical shear at the section, *i.e.* the vertical shear divided by the sectional area of the web. If the stiffeners of the girder be placed closer together than the depth of the girder, the length l will be reduced to the distance between the stiffeners measured along a line at 45° with the vertical.

Another, and perhaps the most widely adopted, method of designing the web is to limit the intensity of shear stress on the vertical plane to a fixed safe value. The thickness of the web will by this method $= \frac{S}{f_s D}$,

where S = vertical shear at the section ;
D = depth of web ;
f_s = safe intensity of shear stress.

f_s may vary for mild steel between 2·0 and 2·5 tons per square inch.

To allow for corrosion the minimum thickness of web plate should be $\frac{3}{8}$ in. Theoretically, the thickness of web plate should increase towards the supports, but the maximum thickness is usually employed throughout the length to overcome the difficulty of flange connection. In cases where the web changes thickness, packings must be placed under the flange angles.

Stiffeners.—To strengthen the web against buckling stiffeners are riveted to the girder at intervals along its length. The usual forms of stiffeners are shown in Fig. 144. *a* is a single angle stiffener used to a large extent in America, but seldom employed in this country. A tee stiffener as at *c* is the more usual English type, and has the advantage, when used at a web joint, of being riveted directly to both portions of the web. At *d* is shown a gusset stiffener, used where the flange has a large overhang beyond the flange angles. It serves to support the plates of the tension flange and reduces the buckling tendency of the plates in the compression flange. It is also very effective in stiffening the girder laterally. The stiffeners *a*, *b*, and *c* are carried over the vertical legs of the flange angles either by placing

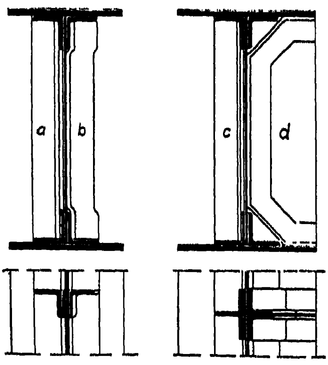


Fig. 144.

packing plates equal in thickness to the flange angles between the web and stiffeners or by joggling the ends of the stiffeners an amount equal to the thickness of the flange angles. The former method adds material to the girder, but, except in cases where the flange angles are very thick, is less costly than the second method.

Although formerly the spacing was much greater, it is now customary to space the stiffeners at distances not exceeding the depth of the girder. The modification of the stresses in the web due to the introduction of stiffeners is at present very uncertain, but there is no doubt that stiffeners spaced at distances greater than the depth of the girder, do, to a certain extent, stiffen the girder; the closer spacing, however, conforms better with the theory of the principal stresses in the web. In deep girders the joints in the web plates will necessitate covers and stiffeners at intervals of less than the depth, and in no case should the spacing exceed 5 feet.

Pitch of Rivets in Flange Angles.—It has already been proved that the intensity of horizontal shear along any horizontal plane of a beam is equal to $\frac{AYS}{wI}$, and the total stress for one foot length of the beam, supposing the intensity to remain constant, $= \frac{12AYS}{I}$. The rivets through the vertical legs of the angles must resist the horizontal shear stress between the flanges and the web. The moment of inertia of the flanges may be assumed $= 2AR^2$

where A = the area of each flange;
R = distance of the centre of gravity of the flanges from the neutral axis
= half of the depth of the girder
$= \frac{D}{2} = Y$, in the above formula.

Substituting these values for I and Y—

$$\frac{12AYS}{I} = 12\left(\frac{A \times \frac{D}{2} \times S}{2A \times \left(\frac{D}{2}\right)^2}\right)$$

$$= \frac{12S}{D}$$

the depth D of the girder is measured in inches. If D be measured in feet, the horizontal shear per foot length $= \frac{S}{D}$, that is = the average vertical shear per foot depth.

Let R = resistance of one rivet.

Then the number of rivets required per foot length $= \frac{S}{DR}$.

R will be the resistance of one rivet in double shear, or the bearing resistance of one rivet in the web plate or angles, whichever is the smaller. It is obvious that the pitch of the rivets through the flange

plates would, by calculation, be greater than the pitch of the rivets through the web, but for practical reasons the same pitch is adopted.

Lateral Bracing.—Bracing is introduced between the main girders of bridges to provide the necessary stiffness against lateral wind pressure. The flooring members will generally sufficiently stiffen one

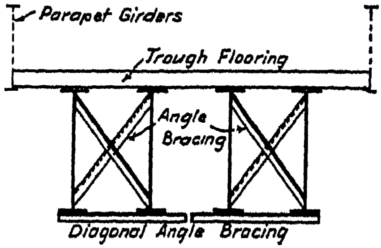


FIG. 145.

pair of girder flanges and the remaining flanges may be stiffened by diagonal angle bracing as shown in Fig. 145, by overhead bracing as in Fig. 161, or by specially wide stiffeners riveted to the deck beams and webs of the girders as in Fig. 160.

EXAMPLE 30.—**Design of plate girders for 25-ton crane, 60 feet span.**

Let it be supposed that the specification requires the following working stresses and particulars to be adopted.

Working stresses not to exceed—

Tension	7	tons per	square inch
Compression . . .	7	„	„
Vertical shear in web	$2\frac{1}{4}$	„	„
Rivet shear . . .	5	„	„
Bearing pressure .	8	„	„

Double shear to be taken equal to $1\frac{3}{4}$ times single shear.

Flanges.—The tension flange to be calculated on the net section, *i.e.* the gross section of the plates and horizontal tables of the angles minus the area of the greatest number of rivet holes in any transverse section, and the area of any other rivets that may occur within $2\frac{1}{2}$ inches of such section. The diameter of the rivet holes to be taken as $\frac{1}{16}$ inch larger than the nominal diameter of the rivets. Where the flanges are parallel, the compression flange to have the same gross area as the tension flange.

Web.—The minimum thickness of web to be $\frac{3}{8}$ inch. The shearing resistance of the web to be calculated on the gross area of the web at any vertical section.

Depth.—The depth of the girders at the centre of the span to be not less than $\frac{1}{12}$ the span.

Let Fig. 146 represent the effective depths of the main girders. The loads supported by the girders consist of—

(1) Load to be lifted
(2) Dead load of crab
(3) „ „ girders.

The load raised (25 tons), when lifted suddenly, will exert a pressure on the girders far exceeding its actual dead weight, and in an extreme case equal to double its dead weight. It is therefore necessary to increase the dead load lifted to the probable force it would, at any time, exert on the girders. In the present case, 100 per cent. will be added for this dynamic action, making the equivalent dead load on the crab equal to 50 tons. The weights of crabs vary, and the weight would, in an actual design, be obtained after the crab had been designed. Seven tons being a probable weight for such a crab, will be adopted. The axles of the crab will be assumed to be at 5 feet centres. The total load on each wheel will then be $\frac{50 + 7}{4} = 14\frac{1}{4}$ tons. An estimate of the dead weight of the girders may be obtained from the formula given in Chapter II. Dead weight of girder $= \frac{WL}{530}$,

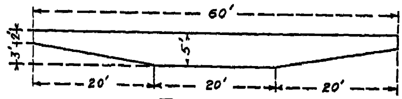


FIG. 146.

where L = span of girder in feet,

W = an equivalent distributed load producing a maximum bending moment equal to the maximum bending moment produced by the live loads.

In this case the wheel loads may be assumed concentrated at the middle of the span, when the maximum bending moment would—

$$= \frac{WL}{4} = \frac{2 \times 14\frac{1}{4} \times 60}{4}$$
$$= 427 \cdot 5 \text{ ft.-tons.}$$

The equivalent distributed load to produce a similar bending moment—

$$= \frac{WL}{8} = \frac{W \times 60}{8} = 427 \cdot 5$$
$$\therefore W = 57 \text{ tons}$$

The approximate weight of each girder will therefore

$$= \frac{WL}{530} = \frac{57 \times 60}{530}$$
$$= 6 \cdot 4 \text{ (say) } 6\frac{1}{2} \text{ tons.}$$

The bending moment diagrams may now be constructed by the methods illustrated in Chapter III.

In Fig. 147 *a* is the curve of bending moments for the assumed distributed weight of the girder, *b* is the curve of maximum bending moments produced by the axle loads, *c* is the curve of total bending moments obtained by adding together the ordinates of *a* and *b*.

The horizontal flange stress at any section is obtained by dividing the bending moment at the section, as scaled off from the curve *c*, by

the effective depth of the girder at the section. The horizontal stress thus obtained will be the direct stress for the horizontal portions of the

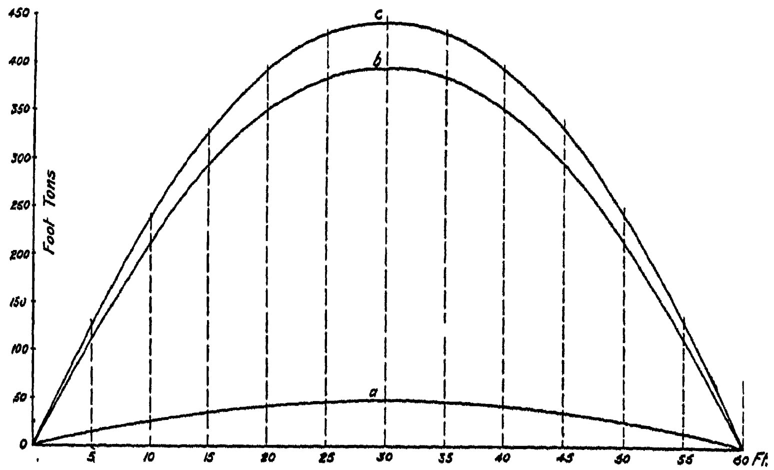


Fig. 147.

flanges. The direct stress in the inclined portions of the lower flange will be to the horizontal stress as the inclined length is to the horizontal length. If ab, Fig. 148, represent the horizontal stress at any section of the flange, and θ be the angle of inclination of the flange, then the inclined stress will be represented by the length ac,

$$i.e. \text{ the inclined stress} = \text{horizontal stress} \times \frac{ac}{ab}$$
$$= \text{horizontal stress} \times \sec. \theta$$

The vertical component bc of the inclined stress is part of the vertical shear at the section resisted by the flange. The web in such a case will only be called upon to resist the maximum vertical shear at the section minus the shear bc resisted by the flange. The horizontal

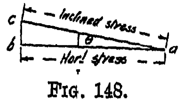


Fig. 148.

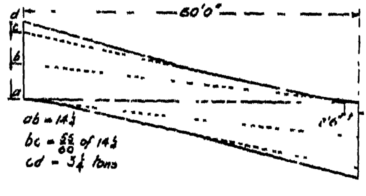


Fig. 149.

and inclined stresses in the flanges at sections 5 feet apart are tabulated in the following table, columns 4 and 5. Column 7 of the same table shows the vertical shearing force resisted by the inclined portion of the lower flange at such sections.

The maximum vertical shearing force diagram, Fig. 149, has been

constructed by the method in Chapter III., and the vertical shearing forces at the different sections are tabulated in column 6 of the following table.

Distance from end	Effective depth	Maximum B.M.	Horizontal flange stress	Inclined flange stress	Maximum vertical shear	Vertical shear taken by flange.	Vertical shear taken by web.
ft.	ft. in	ft.-tons.	tons.	tons.	tons.	tons	tons.
0	2 0	0	0	—	30·56	—	30 56
5	2 9	125	45·5	45·9	27·54	6·82	20·72
10	3 6	241	68·8	69·5	24·65	10·32	14·33
15	4 3	330	77·6	78·4	21·57	11·64	9·93
20	5 0	393	78·6	{79·4	18·85	11·79	7·06
				{ —	18 85	—	18·85
25	5 0	430	86·0	—	15·96	—	15·96
30	5 0	441	88 2	—	13 06	—	13·06

The two sets of figures given at the 20-feet section are for the stresses immediately to the left and right of the section.

A diagram of the stresses in the flanges may now be drawn from the data given in columns 4 and 5.

Fig. 150 is a diagram for both flanges ; the left-hand half represents

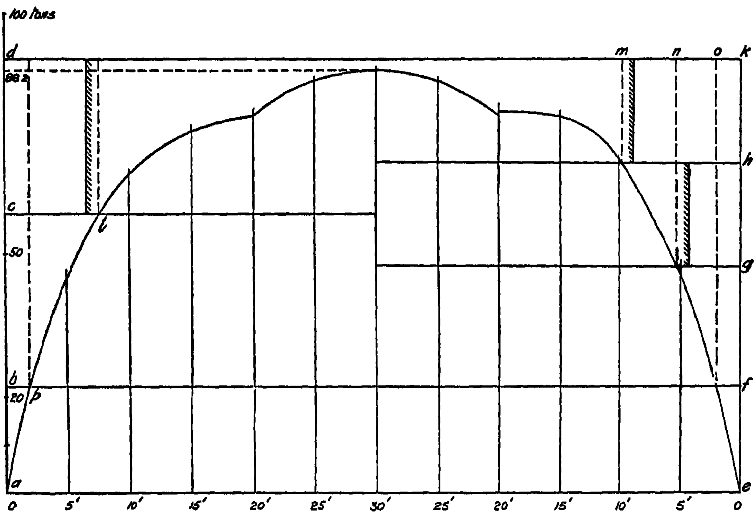


FIG. 150.

the stresses in the upper flange, and the right-hand half the stresses in the lower flange.

At the centre the required flange area $= \dfrac{88·2}{7} = 12·6$ sq. inches. Suppose a width of 10 inches be selected for the flange, and $4'' \times 4'' \times \frac{1}{2}''$ angles, and $\frac{7}{8}''$ rivets be adopted. The net area of the horizontal tables of the angles $= 2(4 - \frac{15}{16})\frac{1}{2} = 3·125$ sq. inches.

The net area of the flange plates must therefore $= 12{\cdot}6 - 3{\cdot}125 = 9{\cdot}475$ sq. inches.

The net width of plates $= 10 - 2 \times \frac{15}{16} = 8\frac{1}{8}$ in.

The total thickness of plates therefore—

$$= \frac{9{\cdot}475}{8{\cdot}125} = 1{\cdot}16 \text{ in.}$$

This thickness may be made up by using either one $\frac{9}{16}$ in. and one $\frac{5}{8}$ in. plate, or two $\frac{3}{8}$ in. and one $\frac{7}{16}$ in. plate. For the upper flange the former sections will be adopted, and for the lower the latter. The area of the flanges is slightly in excess of the theoretical requirements, as the exact area cannot be obtained with practical sections. The plates should be arranged in order of thickness, the thickest being placed next the angles, and the thinnest on the outside.

The stresses in the different plates and angles are as follows :—

2 angles	$= 3{\cdot}125 \times 7$	$= 21{\cdot}875$	tons.
$\frac{5}{8}$ in. plate	$= 8{\cdot}125 \times \frac{5}{8} \times 7$	$= 35{\cdot}54$	„
$\frac{7}{16}$ „	$= 8{\cdot}125 \times \frac{7}{16} \times 7$	$= 24{\cdot}88$	„
$\frac{9}{16}$ „	$= 8{\cdot}125 \times \frac{9}{16} \times 7$	$= 32{\cdot}00$	„
$\frac{3}{8}$ „	$= 8{\cdot}125 \times \frac{3}{8} \times 7$	$= 21{\cdot}32$	„

As the flange stress decreases towards the ends of the girder, a proportionate reduction of flange area could be made. It would, however, be impracticable to have a gradual change of section, or many small changes, and so the same sectional area is maintained until the stress has diminished to such an extent that the outermost plate may be discontinued. The positions where the changes of section may take place are readily obtained from the flange stress diagram, Fig. 150.

Set out ab = the stress in the angles
bc = „ „ $\frac{5}{8}$ in. plate
cd = „ „ $\frac{9}{16}$ „

Through b, c and d draw horizontal lines cutting the curve at p and l.

At the section l the total stress in the flange $= ac$, which is equal to the resistance of the angles and the $\frac{5}{8}$ in. plate. Between l and the end of the girder the $\frac{9}{16}$ in. plate is unnecessary, and may be discontinued. In practice the plate is continued for about one foot beyond the section at l. At the section through p the $\frac{5}{8}$ in. plate could be stopped, but it would be inadvisable as the lateral resistance of the girder would be seriously reduced. The lengths of the $\frac{3}{8}$ in. plates on the lower flange are found in a similar manner on the right-hand portion of Fig. 150. For parallel girders, where the effective depth is constant, the bending moment diagram is also, to a different vertical scale, the flange stress diagram, and may be used for such to obtain the lengths of the flange plates.

The lengths of the flange plates will be—

	in.	in.	ft.	in.	
top flange	10 ×	$\frac{5}{8}$ ×	60	0	
	10 ×	$\frac{9}{16}$ ×	47	8	
bottom flange	10 ×	$\frac{7}{16}$ ×	60	0	(horizontal projection)
	10 ×	$\frac{3}{8}$ ×	51	3	„ „
	10 ×	$\frac{3}{8}$ ×	42	4	„ „

Plates exceeding 40 feet in length are charged extra per foot of

length over 40 feet, and it is more economical to employ cover plates when the length is much greater than the ordinary rolling limit. In the present case the plates 60 feet and 51 feet 3 inches would be jointed, the extra on the other plates exceeding 40 feet in length not being equal to the cost of covers. Joints in the two flanges, or in the flange angles, should not be in the same vertical section. A convenient position for the joint in the $\frac{5}{8}$ inch top flange plate would be about 5 feet on the left of the centre.

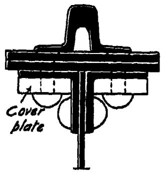


FIG. 151.

The cover plates should have a strength equal to that of the jointed plate, and must be connected to the flange by rivets having an equal resistance to that of the cover. The section of the top flange is shown in Fig. 151, and it will be seen that the only available position for the covers is under the top tables of the angles. The width of covers will be $3\frac{1}{2}$ inches, and, deducting the rivet area, the section $= 2(3\frac{1}{2} - \frac{15}{16}) \times$ thickness.

The maximum stress in the $\frac{5}{8}$ inch flange plate = 35·54 tons. Therefore the thickness of covers $= \dfrac{35{\cdot}54}{7 \times 2(3\frac{1}{2} - \frac{15}{16})} = 0{\cdot}99$ inch, or, say, 1 inch.

The shearing resistance of a $\frac{7}{8}$ inch rivet in single shear = 3 tons.

The number of rivets required at each side of the joint will therefore $= \dfrac{35{\cdot}54}{3} = 12$.

The bearing resistance of the rivets in the $\frac{5}{8}''$ plate $= 12 \times \frac{7}{8}'' \times \frac{5}{8}'' \times 8 = 52{\cdot}5$ tons, or far above the requirements.

The pitch of the rivets = 4 inches (calculated later).

The length of the covers will therefore be 4 feet.

Suppose the $\frac{7}{16}$ inch plate in the bottom flange to be jointed 5 feet to the right of the centre. A $\frac{7}{16}$ inch plate riveted to the under side of the flange will be equal to the strength of the plate jointed.

The maximum stress in the plate = 24·88 tons

The number of rivets in single shear required $= \dfrac{24{\cdot}88}{3} = 9$.

As the rivets are in pairs, ten rivets must be used at each side of the joint. The bearing resistance is again in excess of requirements. The length of the cover plate to each side of the joint will be 20 inches. The same plate may be utilised as a cover for the joint in the $\frac{3}{8}$ inch

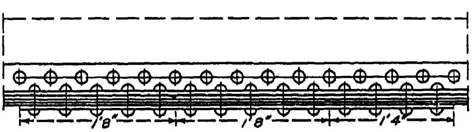


FIG. 152.

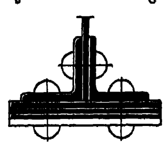


FIG. 153.

plate, by making such joint 20 inches from the joint in the $\frac{7}{16}$ inch plate (Figs. 152 and 153), and continuing the cover plate the necessary additional length.

Stress in the $\frac{3}{8}$ inch plate = 21·32 tons.

Number of rivets required at each side of joint $= \dfrac{21{\cdot}32}{3} = 8$ for rivets in single shear.

Bearing resistance of rivets $= 8(\frac{7}{8} \times \frac{3}{8}) \times 8 = 21$ tons.

This is slightly below the stress in the plate, but, as the bearing pressure would only be $\frac{21 \cdot 32 \times 8}{21} = 8 \cdot 12$ tons per square inch on the rivet, the small excess intensity may be allowed. The additional length of cover plate to the right of the joint will be 16 inches.

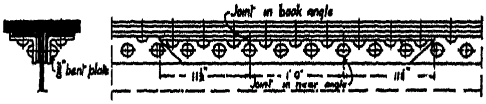



FIG. 154.

The joints in the flange angles are covered by bent plates or wrappers as in Fig. 154.

The stress in each angle $= 10 \cdot 937$ tons.

$$\text{The thickness of the covers} = \frac{10 \cdot 937}{7(6\frac{1}{2} - 2 \times \frac{15}{16})} = 0 \cdot 35 \text{ in.}$$

$\frac{3}{8}$ inch bent plates would be used.

$$\text{Number of rivets in single shear required} = \frac{10 \cdot 937}{3} = 4$$

$$\text{,, \quad ,, \quad bearing \quad ,,} = \frac{10 \cdot 937}{\frac{7}{8} \times \frac{3}{8} \times 8} = 5$$

The joints and arrangement of rivets are shown in Fig. 154, and the positions of the joints on the girders are shown on Fig. 157, in the elevation.

Web Plate.—The maximum vertical shear on the web occurs at the ends of the girder, where it is equal to 30·56 tons. The section of the web will be = 24 in. × thickness of web.

Since the allowable shearing stress $= 2\frac{1}{4}$ tons per square inch the net area of web required $= \frac{30 \cdot 56}{2\frac{1}{4}} = 13 \cdot 5$ square inches. The thickness will therefore $= \frac{13 \cdot 5}{24} = 0 \cdot 56$ inch. The nearest practical size being $\frac{9}{16}$ in. such thickness would be adopted. The required thickness of web decreases very rapidly from the end to a section 20 feet from the end, owing to the depth increasing whilst the shear decreases. At this section the required thickness would be less than the minimum thickness specified. The shape of the web plate renders it convenient to have the web joints at these sections, and the central 20 feet portion of the web may be reduced to $\frac{3}{8}$ in. thickness. To allow of the flange angles being kept straight $\frac{3}{32}$ in. packings must be placed between them and the $\frac{3}{8}$ in. web.

If the covers and rivets at the web joints were designed to resist the actual vertical shear at the joints, the thickness of covers and the number of rivets required would be reduced to unpractical sizes. The covers, therefore, will be of the minimum thickness, $\frac{3}{8}$ in. with a double row of rivets at either side of the joint. Packings $\frac{3}{32}$ in. thick are required under the inner halves of the covers owing to the change of thickness of the web plate.

Pitch of Rivets.—The horizontal shear per foot length of girder is given by the expression $\frac{S}{D}$. The minimum pitch will occur where the shear is the greatest, *i.e.* at the ends of the girders. The number of

rivets required through the vertical tables of the angles per foot length, at the ends of the girder

$$= \frac{S}{DR} = \frac{30{\cdot}56}{2 \times 5{\cdot}25} = \text{(say) 3 for shear}$$

and

$$= \frac{30{\cdot}56}{2 \times \frac{7}{8} \times \frac{9}{16} \times 8} = \text{(say) 4 for bearing in web.}$$

A pitch of 3 in. is therefore necessary at the ends. As the pitch is inversely proportional to $\frac{S}{D}$ it would increase towards the middle of the span, being at 5 feet from the end equal to 4·7 in. and at the centre equal to 12 in. For practical reasons the pitch is kept constant throughout the length of the girder or has a practical minimum number of changes. In the present case it would be advisable to have a pitch of 3 in. for a distance of 5 feet from either end, and the remaining portion pitched at 4 in. From Fig. 150 it will be seen that the rate of increase of stress in the inclined portions of the lower flange is, for all practical purposes, equal to the rate of increase of the horizontal stress, and therefore the pitch of the rivets in the inclined flange will be made the same as for the upper flange.

Stiffeners.—6″ × 3″ × $\frac{3}{8}$″ tees will be used as stiffeners and spaced at intervals of 5 feet. Packings ½ in. thick will be placed between them and the web to save joggling the tees over the flange angles.

Rail.—A 70 lbs. bridge rail riveted to the flange at 1 ft. 3 in. and 1 ft. 4 in. pitches will be used for the crab to travel on.

To check the Assumed Weight of Girder.

No	Description	Section.	Length.	Total length	Weight per ft	Weight.
		in. in	ft in	ft	lbs.	lbs
1	Flange plate	10 × $\frac{5}{8}$	60 0	60	21 25	1,275
1	,, ,,	10 × $\frac{9}{16}$	47 8	47 67	19 13	912
1	,, ,,	10 × $\frac{7}{16}$	60 0	60	14 88	892
1	,, ,,	10 × $\frac{3}{8}$	51 3	51 25	12·75	653
1	,, ,,	10 × $\frac{3}{8}$	42 4	42 3	12 75	540
2	,, covers	3½ × 1	4 0	8	11 9	95
1	,, ,,	10 × $\frac{7}{16}$	4 8	4 67	14 88	69
2	Flange angles	4 × 4 × ½	60 0	120 0	12 75	1,530
2	,, ,,	4 × 4 × ½	60 3	120 5	12 75	1,536
4	,, ,, covers	3½ × 3½ × $\frac{3}{8}$	3 0	12	8·45	102
4	,, packings	4 × $\frac{1}{32}$	20 0	80	1 13	90
1	Web plate	60 × $\frac{3}{8}$	20 0	20	76 5	1,530
2	,, ,,	(60 to 24) × $\frac{9}{16}$	20 0	40	80 33	3,213
4	,, covers	12 × $\frac{3}{8}$	4 4	17 3	15 3	265
4	,, packings	6 × $\frac{3}{32}$	4 4	17 3	1·92	33
10	,, stiffeners	6 × 3 × $\frac{3}{8}$	4 11	49·17	11·0	585
4	,, ,,	,, ,,	4 2	16·67	,,	183
4	,, ,,	,, ,,	3 5	13 67	,,	150
4	,, ,,	,, ,,	2 8	10·67	,,	117
10	,, ,, packings	6 × ½	4 4	43·33	10·2	442
4	,, ,, ,,	,, ,,	3 7	14·33	,,	146
4	,, ,, ,,	,, ,,	2 10	11·33	,,	116
4	,, ,, ,,	,, ,,	2 1	8·33	,,	85
						14,559
	Rivets, say 5 per cent.					728
	Total weight					15,287
	= 6·83 tons.					

The assumed weight was therefore $6{\cdot}88 - 6{\cdot}5 = 0{\cdot}38$ ton too small; but such a small difference in weight would not produce stresses warranting any change of the designed sections.

End Girders or Cradles.—The ends of the main girders are supported on lateral girders, to which are fixed the wheels for the longitudinal motion of the crane. The general arrangement of connections is shown in Fig. 157. The main girders are carried through to the web CD of the end girder, the web AB being in three parts, each connected to the main girders by vertical angles. The maximum loading of the end girders will occur when the crab, fully loaded, is at the end of its travel. It may be assumed that the whole of the weight of the crab and load is then carried by the adjacent end girder. The load from each main girder will be half the weight of one main girder, plus half the weight of the crab plus half the pressure due to the load lifted

$$= \tfrac{1}{2}(6{\cdot}83 + 7 + 50) = 31{\cdot}91 \text{ tons (say) } 32 \text{ tons.}$$

Fig. 155 is a diagram of loading, bending moments, and shear forces. The weakest section of the end girder will be where the flange plates stop, 18 in. from the centre line of the main girders.

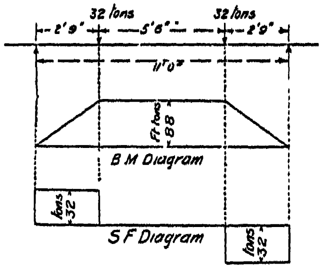


FIG. 155.

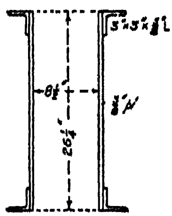


FIG. 156.

Bending moment at that section $= 32(2'\ 9'' - 1'\ 6'') = 40$ ft.-tons.

Suppose the section, Fig. 156, be assumed for the end girder. The modulus of section $= 182{\cdot}3$ inches^3.

$$\text{The maximum stress} = \frac{40 \times 12}{182{\cdot}3} = 2{\cdot}6 \text{ tons per square inch.}$$

This is considerably less than the allowable stress, but the sections being the minimum no reduction can be made.

The maximum shear intensity will occur at the section through the wheel axles. The shear on each web $= 16$ tons. The average shear intensity $= \dfrac{16}{(26{\cdot}25 - 4{\cdot}75)\frac{3}{8}} =$ (say) 2 tons per square inch, 4·75 in. being the diameter of the axle hole.

A $\frac{3}{8}$ in. stiffening plate is riveted to the web, to which the wheel bearings are attached.

Connections.—The load from the main girders will be equally divided between the two webs of the end girders. The rivets at y will be subject to a vertical shear of 16 tons.

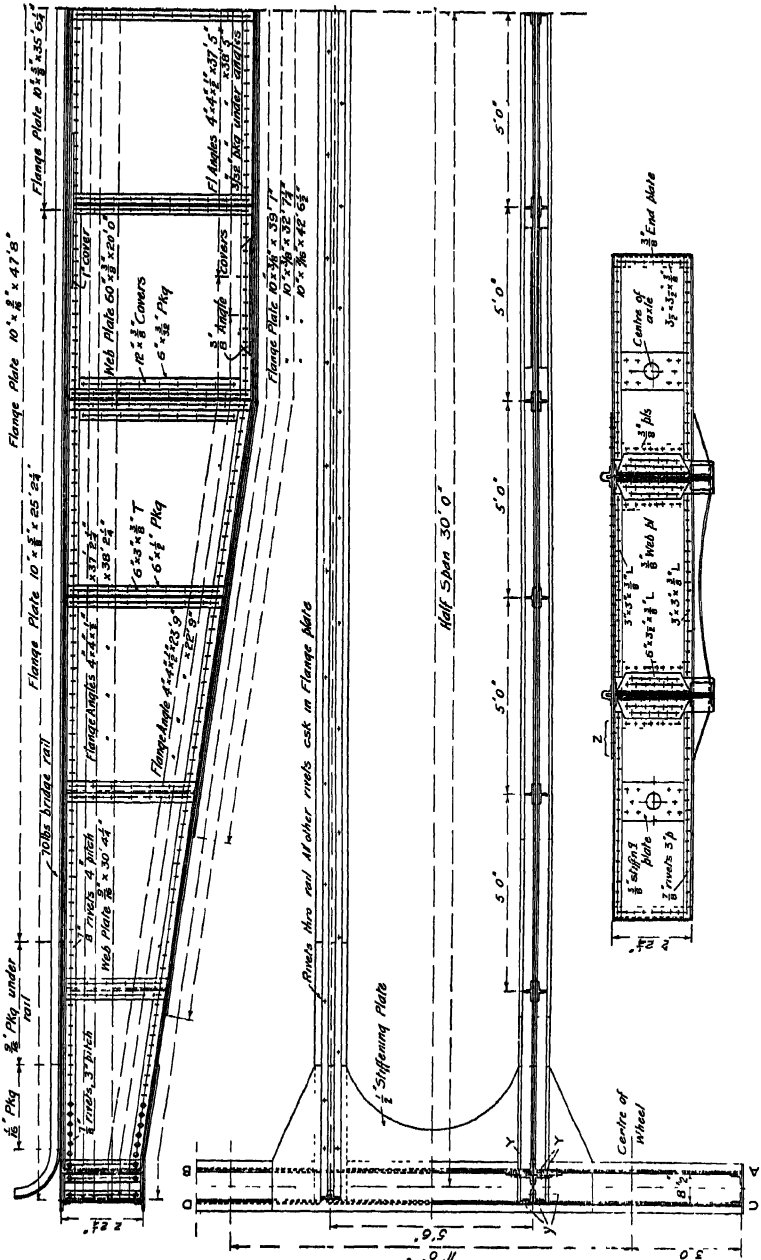

Half Span 30' 0"
70 lbs bridge rail
Rivets thro rail All other rivets csk in Flange plate
1/2" Stiffening Plate
Centre of Wheel
Centre of axle
3/8" End plate
3/8" Web Pl
5' 0"
5' 6"
11' 0"
3' 0"

FIG. 157.

number of rivets required

$$\text{in double shear} = \frac{16}{5\cdot25} = 4$$

$$\text{in bearing} = \frac{16}{\frac{7}{8} \times \frac{9}{16} \times 8} = (\text{say})\ 4$$

The rivets Y′ (Figs. 157 and 158) will be subject to a horizontal shearing force, due to the bending moment to be resisted by the con-

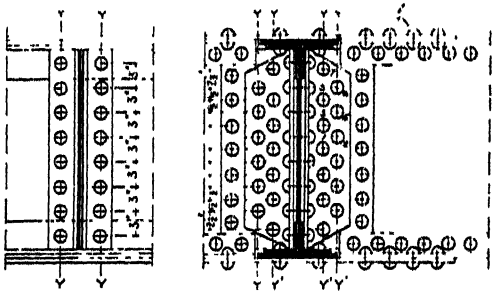


FIG. 158.

nection, in addition to the vertical shear. The rivets at Z will be put into shear by the bending action, and the couple formed by the shearing resistance of these rivets at the upper and lower flanges of the end girder will act in opposition to the bending moment.

Resistance of rivets at Z to shear = 4 × 3 = 12 tons.
Moment of resistance = 12 × 26·25 = 315 in.-tons.
The bending moment at the connection = 16 × 33″ = 528 in.-tons.
The bending moment to be resisted by the rivets at Y′

$$= 528 - 315 = 213 \text{ in.-tons.}$$

Let f_s = the horizontal shear stress per sq. in. on the outside rivet,
a = sectional area of the rivets.
Then the moment of resistance of the whole system of rivets

$$= \frac{2af_s}{y}(y^2 + y_1^2 + y_2^2 + \text{etc.}). \quad (\text{See Chap. IV.})$$

$$= \frac{2 \times 0\cdot6 \times f_s}{10} \times 2\{(\tfrac{3}{2})^2 + 3^2 + (4\tfrac{1}{2})^2 + 6^2 + (7\tfrac{1}{2})^2 + 10^2\}$$

$$= 53\cdot7 f_s$$

$$\therefore f_s = \frac{213}{53\cdot7} = 3\cdot96 \text{ tons per square inch.}$$

Total horizontal shearing force on the outside rivets

$$= 3\cdot96 \times 1\cdot2 = 4\cdot75 \text{ tons.}$$

Each rivet will be subject to a vertical shearing stress of $\frac{16}{13} = 1\cdot23$

tons, due to the vertical shearing force on the section. The actual shearing force will be the resultant of the horizontal and vertical forces

$$= \sqrt{4{\cdot}75^2 + 1{\cdot}23^2} = 4{\cdot}9 \text{ tons.}$$

The resistance of one $\frac{7}{8}$ in. rivet in double shear = 5·95 tons. Therefore the shearing resistance of the system is sufficient to transmit the bending moment.

The force of 4·9 tons on the outside rivet would produce a bearing stress of $\dfrac{4{\cdot}9}{\frac{7''}{8} \times \frac{3''}{8}}$ = 14·9 tons per square inch, which far exceeds the safe bearing pressure. The bearing area may be increased by riveting an extra $\frac{3}{8}''$ plate to the web. The bearing pressure would then be reduced to 7·45 tons per square inch.

The horizontal intensity of shear stress on rivet (6)

$$= \frac{7{\cdot}5}{10} \times 3{\cdot}96 = 2{\cdot}97 \text{ tons per square inch}$$

Horizontal shear on (6)

$$= 2{\cdot}97 \times 1{\cdot}2 = 3{\cdot}56 \text{ tons.}$$

Resultant shear on (6)

$$= \sqrt{3{\cdot}56^2 + 1{\cdot}23^2} = 3{\cdot}77 \text{ tons.}$$

Bearing stress on the web without the bearing plate

$$= \frac{3{\cdot}77}{\frac{7}{8} \times \frac{3}{8}} = 11{\cdot}49 \text{ tons per square inch}$$

Bearing stress on the web and the bearing plate = 5·75 tons per square inch.

The bearing stresses for the remaining rivets on the web and bearing plate will be found to be—

Rivet	(5) = 4·72	tons	per	square	inch.
,,	(4) = 3·29	,,	,,	,,	
,,	(3) = 2·6	,,	,,	,,	
,,	(2) = 2·1	,,	,,	,,	
,,	(1) = 1·87	,,	,,	,,	

If p_b be the bearing stress of any rivet on the web and bearing plate, the stress transmitted to the bearing plate = $\frac{7}{8}'' \times \frac{3}{8}'' \times p_b$. The bearing plate must be riveted to the web by a system of rivets whose resistance is equal to the pressure transmitted to the bearing plate.

Total pressure transmitted to the bearing plate

$$= \tfrac{7}{8} \times \tfrac{3}{8} \times \{2(7{\cdot}45 + 5{\cdot}75 + 4{\cdot}72 + 3{\cdot}29 + 2{\cdot}6 + 2{\cdot}1) + 1{\cdot}87\}$$
$$= 17{\cdot}3 \text{ tons.}$$

Bearing resistance of one rivet

$$= \tfrac{3}{8} \times \tfrac{7}{8} \times 8 = 2{\cdot}62 \text{ tons.}$$

The number of rivets required

$$= \frac{17 \cdot 3}{2 \cdot 62} = 7$$

The lower half of the group of rivets at Y will be in tension due to the bending moment of 213 in.-tons.

The moment of resistance of the system

$$= \frac{f_t \times 0 \cdot 6 \times 2 \times 2}{10 \cdot 5} \{(\tfrac{3}{2})^2 + (\tfrac{9}{2})^2 + (\tfrac{15}{2})^2 + (\tfrac{21}{2})^2\}$$

$$= 172 \cdot 8 f_t$$

$$\therefore f_t = \frac{213}{172 \cdot 8} = 1 \cdot 23 \text{ tons per square inch.}$$

The maximum tension in the rivets is therefore well below the working stress.

Example 31.—**Design of Plate Girder Railway Bridge.**—Span 80 feet. The bridge to carry a double track of rails, ballast, and plate flooring carried by longitudinal rail bearers and cross-girders supported by two main girders, Figs. 160 and 160A. The maximum live load to consist of locomotives covering the span, the heaviest axle load being 19 tons, and the driving axles 8 feet apart. Working stresses to be fixed by the Range Formula, Table 21, Chapter II.

Longitudinal Rail Bearers or Stringers.—Assume the cross-girders to be spaced at 8-feet intervals. Then the maximum stress in the flanges of the rail bearers will occur when the 19-ton axle load is at the centre of the span of 8 feet.

$$\text{Load on each wheel} = 9 \cdot 5 \text{ tons.}$$

$$\text{B.M. at centre} = \frac{9 \cdot 5 \times 8}{4} = 19 \text{ foot-tons.}$$

The distributed load on each stringer due to track, ballast, flooring, and own weight, amounts to nearly 2·5 tons (see below), and the additional B.M. due to this load

$$= \frac{2 \cdot 5 \times 8}{8} = 2 \cdot 5 \text{ foot-tons.}$$

$$\text{Total B.M.} = 19 + 2 \cdot 5 = 21 \cdot 5 \text{ foot-tons.}$$

It should be noted this is an outside estimate of the bending moment, since the concentrated wheel load is to some extent distributed over the stringer through the agency of the rail, sleepers, and ballast. The extent of this distribution cannot however be accurately calculated, as it depends on the relative stiffness of the rail and stringer, positions of adjacent axle loads, and other factors.

Assume the depth of stringer as 1 ft. 6 in.

$$\text{Total flange stress} = \frac{21 \cdot 5}{1 \cdot 5} = 14 \cdot 33 \text{ tons.}$$

$$\text{Flange stress due to dead load} = \frac{2 \cdot 5}{1 \cdot 5} = 1 \cdot 67 \text{ tons.}$$

$$\text{Percentage of dead load stress} = \frac{1 \cdot 67}{14 \cdot 33} \times 100 = 11 \cdot 6$$

The working stress from Table 24 for 11·6 per cent. of dead load stress = 5 tons per square inch. The flange area required $= \frac{14\cdot33}{5} = 2\cdot87$ sq. in.

Using $4'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ angles and $\frac{7}{8}$ in. rivets,

Area of horizontal tables of angles $= (8 - 2 \times \frac{15}{16}) \times \frac{1}{2} = 3\cdot06$ sq. in.

From Table 24, the coefficient for estimating the dead load equivalent to 89 per cent. of moving load is 1·9.

Hence, equivalent maximum shear when the axle load is entering or leaving the span $= (9\cdot5 \times 1\cdot9) + 1\cdot25 = 19\cdot3$ tons.

Adopting a web plate $\frac{1}{2}$ in. thick, the average shear stress on the web $= \frac{19\cdot3}{18 \times 0\cdot5} = 2\cdot15$ tons per square inch.

A somewhat thinner web would be strong enough, but it is desirable to make the webs of stringers and cross-girders of about equal thickness, in order to ensure the same length of life against corrosion.

Weight of one stringer.

2 Flange angles $4'' \times 3\frac{1}{2}'' \times \frac{1}{2}'' \times 8'\ 0''$	= 190 lbs.
2 „ „ $4'' \times 3\frac{1}{2}'' \times \frac{1}{2}'' \times 13'\ 0''$	= 310 „
1 Web plate $18'' \times \frac{1}{2}'' \times 8'\ 0''$	= 245 „
2 Tee stiffeners $5'' \times 3'' \times \frac{3}{8}'' \times 1'\ 6''$	= 29 „
2 „ packings $5'' \times \frac{1}{2}'' \times 10''$	= 15 „
	789 „
Rivets, say 3 per cent.	= 24 „
Total	= 813 „

Cross-Girders.—The span of the cross-girders equals 27 feet; the distance between the centres of main girders.

Dead load on each cross-girder—

Weight of ballast @ $\frac{1}{3}$ ton per foot run	8,960 lbs.
„ rails = $4 \times \frac{8}{3}$ @ 86 lbs. per yard . .	917 „
„ sleepers = 6 @ 125 lbs. each . . .	750 „
„ chairs, etc., 12 @ 50 lbs. each . . .	600 „
Weight of permanent way .	11,227 „
„ asphalte, $24' \times 8' \times 1\frac{1}{2}''$	3,600 „
„ floor plating = $24' \times 8' \times \frac{7}{16}''$ . . .	3,360 „
„ fender plates, say	300 „
Assume weight of cross-girder = $2\frac{1}{4}$ tons . . .	5,040 „
Total distributed load on cross-girder	23,527 „
	= (say) 10·5 tons.

A portion of the above weight is actually applied by the stringers as concentrated load at the junctions with the cross-girders, but the error due to considering it as distributed load is very small, since the live load is relatively large.

The weight of the stringers imposes four concentrated dead loads = 813 lbs. each.

Live load on cross-girders—

Four concentrated loads of 9·5 tons each.

The loading on the cross-girder is indicated in Fig. 159.

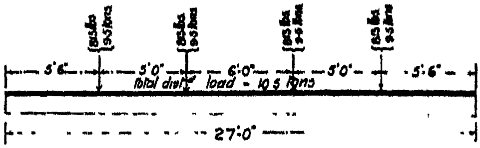


FIG. 159.

Bending moment at centre due to—

$$\text{Distributed dead load} = \frac{10·5 \times 27}{8} = 35·44 \text{ ft.-tons.}$$

$$\text{Concentrated dead loads} = \frac{813 \times 2 \times 10·5 - 813 \times 5}{2240} = 5·8 \text{ ft.-tons.}$$

$$\text{Concentrated live loads} = 9·5 \times 2 \times 10·5 - 9·5 \times 5 = 152 \text{ ft.-tons.}$$

Stress in flanges, assuming an effective depth of 2 ft. 6 in., due to—

$$\text{Dead load} = \frac{35·44 + 5·8}{2·5} = 16·5 \text{ tons.}$$

$$\text{Live load} = \frac{152}{2·5} = 60·8 \text{ tons.}$$

$$\text{Percentage of dead load stress} = \frac{16·5}{60·8 + 16·5} \times 100 = 21·3 \text{ per cent.}$$

Working stress from Table 24 = 5·9 tons per square inch

$$\text{The area of flange required} = \frac{60·8 + 16·5}{5·9} = 13·10 \text{ sq. in.}$$

Assume a flange width of 14", 4" × 4" × ½" angles and ⅞" rivets.

$$\text{Area of horizontal tables of angles} = (8 - 2 \times \tfrac{15}{16})\tfrac{1}{2} = 3·06 \text{ sq. in.}$$

$$\text{Thickness of plates} = \frac{13·10 - 3·06}{14 - (2 \times \frac{15}{16})} = 0·83 \text{ in.}$$

Say two plates, ½" and ⅜" thick for the lower flange. For the upper flange, one plate 15" × ½" and one 9" × ⅜" will be used, leaving a 3" margin on each side to which to rivet the floor plates.

The coefficient to obtain the equivalent of the live load in terms of the dead load is, from Table 24, = 1·67.

The maximum shear is therefore—

$$\begin{aligned} \text{from live load} &= 19 \times 1·67 = 31·73 \text{ tons.} \\ \text{from distributed load} &= 5·25 \text{ ,,} \\ \text{from concentrated dead loads} &= \frac{813 \times 2}{2240} = 0·72 \text{ ,,} \\ \text{Total shear at ends} &= 37·70 \text{ ,,} \end{aligned}$$

Assuming the thickness of web to be $\frac{1}{2}''$, the average intensity of shear stress in the web $= \frac{37\cdot70}{30 \times \frac{1}{2}} = 2\cdot51$ tons per square inch.

Designing the web as a fixed-ended strut of length equal to the distance between the lines of rivets through the flange angles, measured along a line inclined at 45°—

$$\text{length of strut} = (30 - 4\tfrac{1}{2})\sqrt{2} = 36\cdot06 \text{ in.}$$

$$\text{least radius of gyration} = \frac{1}{2\sqrt{12}}$$

$$\frac{l}{r} = \frac{36\cdot06}{\frac{1}{2\sqrt{12}}} = 249\cdot5$$

Safe intensity for this ratio, taken from the curve of safe loads on fixed-ended struts (Fig. 109), is 5000 lbs., or 2·23 tons per square inch. Hence $\frac{1}{2}$ inch thickness for the web is insufficient.

The average intensity of shear stress on a $\frac{9}{16}$ inch plate = 2·24 tons per square inch. The safe intensity = 2·67 tons per square inch. A $\frac{9}{16}$ inch plate will therefore be adopted.

Main Girders.—The dead and live loads may be assumed as uniformly distributed on the main girders.

Dead weight per 8-foot length—

Permanent way	11,227 lbs.
Asphalte	3,600 „
Floor plating	3,360 „
Fender plates	300 „
4 stringers	3,252 „
1 cross-girder (from calculated weight as designed)	5,000 „
	26,739 „
Weight per foot run	3,342 „
Dead load per foot run per girder	1,671 „
	0·75 ton.

The equivalent distributed live load for a span of 80 feet is here taken at 2 tons per foot run.

The estimated dead weight of each girder

$$= \frac{WL}{510} = \frac{(0\cdot75 + 2)80 \times 80}{510}$$
$$= 34\cdot5 \text{ tons.}$$

Bending moment at centre due to

$$\text{dead load} = \frac{(0\cdot75 \times 80)80 + 34\cdot5 \times 80}{8}$$
$$= 945 \text{ ft.-tons.}$$

$$\text{live load} = \frac{2 \times 80 \times 80}{8}$$
$$= 1600 \text{ ft.-tons.}$$

Assuming the effective depth of the girder = $8\frac{1}{2}$ ft.
The flange stresses due to

$$\text{dead load} = \frac{945}{8{\cdot}5} = 111{\cdot}2 \text{ tons.}$$

$$\text{live load} = \frac{1600}{8{\cdot}5} = 188{\cdot}2 \text{ tons.}$$

$$\text{Percentage dead load stress} = \frac{111{\cdot}2}{111{\cdot}2 + 188{\cdot}2} \times 100 = 37{\cdot}1 \text{ per cent.}$$

Working stress from Table 24 = 7 tons per square inch.

$$\text{Flange area required} = \frac{299{\cdot}4}{7} = 42{\cdot}77 \text{ sq. inch.}$$

Assume a breadth of flange of 24″, $6'' \times 6'' \times \frac{1}{2}''$ angles, and $\frac{7}{8}''$ rivets.
Area of horizontal tables of angles $= (12 - 4 \times \frac{15}{16})\frac{1}{2} = 4{\cdot}125$ sq. in.
Thickness of plates required in tension flange

$$= \frac{42{\cdot}77 - 4{\cdot}125}{24 - 4 \times \frac{15}{16}} = 1{\cdot}81 \text{ in.}$$

Say three $\frac{1}{2}$ in. plates and one $\frac{3}{8}$ in. plate, or three $\frac{5}{8}$ in. plates. For the compression flange the rivet holes need not be deducted.

Area of horizontal tables of angles $= 12 \times \frac{1}{2} = 6$ sq. in.

$$\text{Thickness of plates} = \frac{42{\cdot}77 - 6}{24} = 1{\cdot}53 \text{ in.}$$

One $\frac{5}{8}$ in. and two $\frac{1}{2}$ in. plates may be used.

Thickness of Web.—Assume a web joint at each 8 feet section along the girder, with double angle and plate stiffeners at the joints and intermediate tee stiffeners. The horizontal length of unsupported web will be 2 feet.

The length of the web column $= 24'' \times \sqrt{2} = 34$ in.
The shear at the ends of the girders

$$\begin{aligned} \text{due to dead load} &= 0{\cdot}75 \times 40 + 17{\cdot}25 = 47{\cdot}25 \text{ tons.} \\ \text{,, live load} &= 2 \times 1{\cdot}44 \times 40 \quad = 115{\cdot}2 \quad \text{,,} \\ &\qquad\qquad \text{Total} = 162{\cdot}45 \quad \text{,,} \end{aligned}$$

Assume a $\frac{9}{16}$ in. web plate.
Average intensity of shear in web

$$= \frac{162{\cdot}45}{102 \times \frac{9}{16}} = 2{\cdot}83 \text{ tons per square inch.}$$

Least radius of gyration of plate

$$= \frac{\frac{9}{16}}{\sqrt{12}} = 0{\cdot}162$$

$$\frac{l}{r} = \frac{34}{0{\cdot}162} = 210$$

Safe stress for this ratio from Fig. 109 = 6400 lbs. = 2·85 tons per square inch.

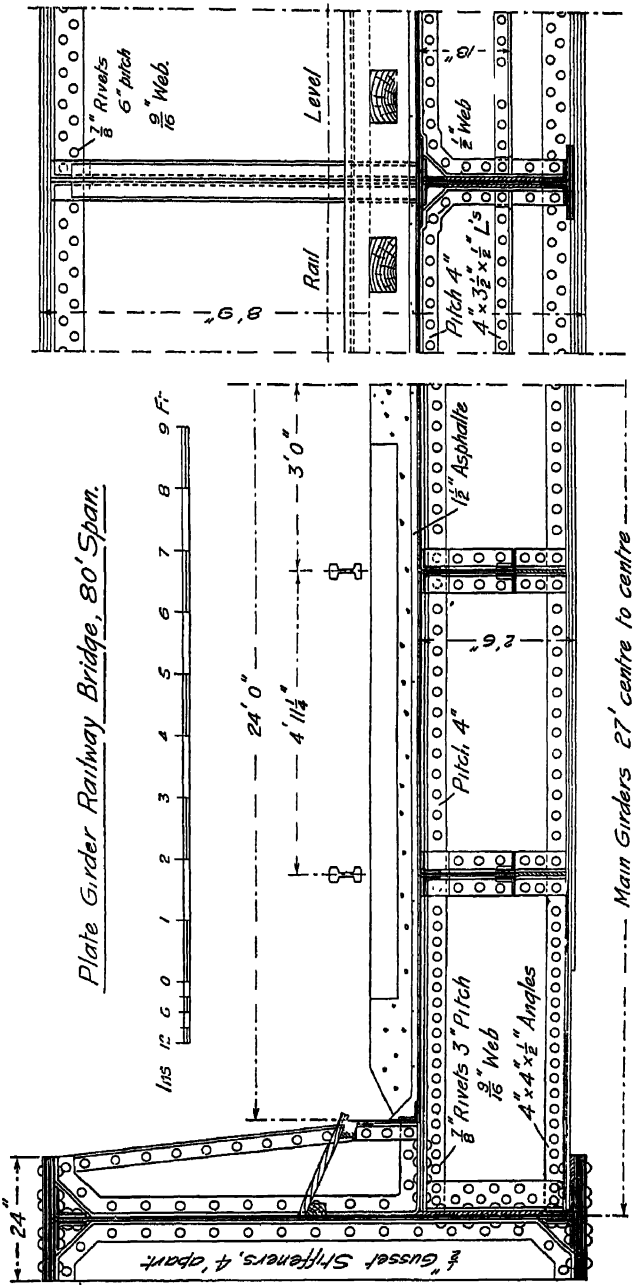


Plate Girder Railway Bridge, 80' Span.
$\frac{7}{8}$" Rivets 6" pitch
$\frac{9}{16}$" Web.
Level
Rail
8' 6"
$\frac{1}{2}$" Web
10"
Pitch 4"
4"×3$\frac{1}{2}$"×$\frac{1}{2}$" L's
$1\frac{1}{2}$" Asphalte
3' 0"
24' 0"
4' 11$\frac{1}{4}$"
2' 6"
Pitch 4"
Main Girders 27' centre to centre
$\frac{7}{8}$" Rivets 3" Pitch
$\frac{9}{16}$" Web
4"×4"×$\frac{1}{2}$" Angles
24"
$\frac{1}{2}$" Gusset Stiffeners, 4' apart.

FIG. 160.

In a girder of this size the web thickness might be reduced towards the middle of the span with an appreciable saving in weight.

Pitch of Rivets in Main Angles.—Maximum shear per foot of depth of web $= \dfrac{162{\cdot}45}{8{\cdot}5} = 19{\cdot}1$ tons. Number of rivets required per foot run of angles, at 5 tons per rivet in double shear $= \dfrac{19{\cdot}1}{5} = 4$. A 3 in. single pitch or 6 in. double pitch is suitable, the latter being adopted. The bearing stress on the web plate, with this pitch, is 9·7 tons per square inch. A 5 in. double pitch would reduce the bearing stress to 8 tons per square inch.

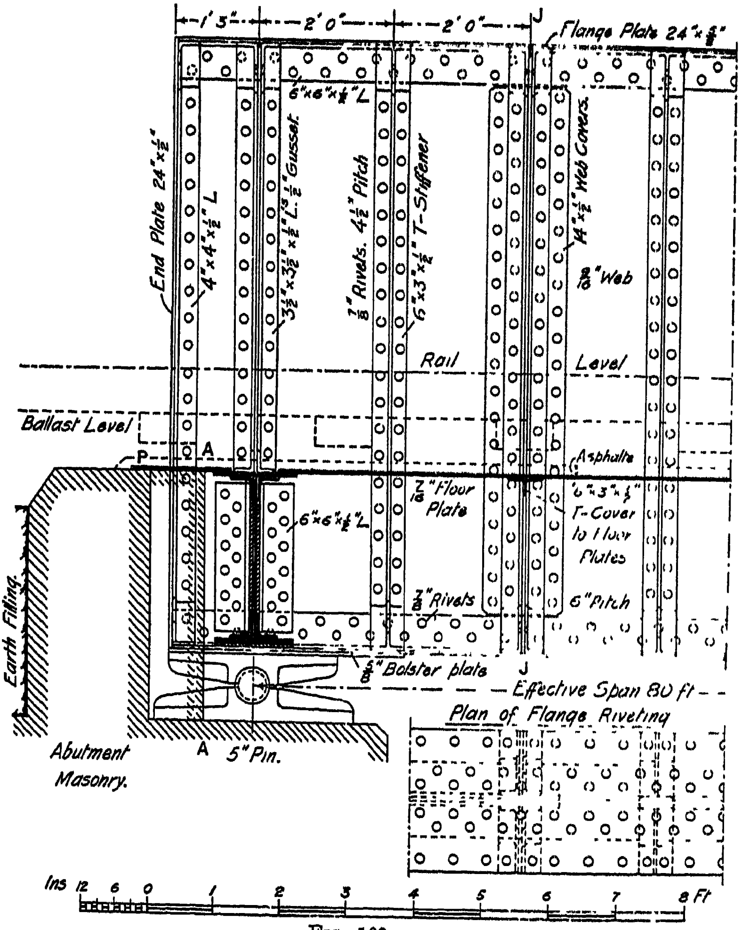


FIG. 160A.

Web Joint.—The first vertical joint in the web occurs at J–J, Fig. 160A, 4 feet from the bearing pin.

$$\text{Shear at J–J} = \tfrac{36}{40} \times 162{\cdot}45 = 146{\cdot}2 \text{ tons.}$$

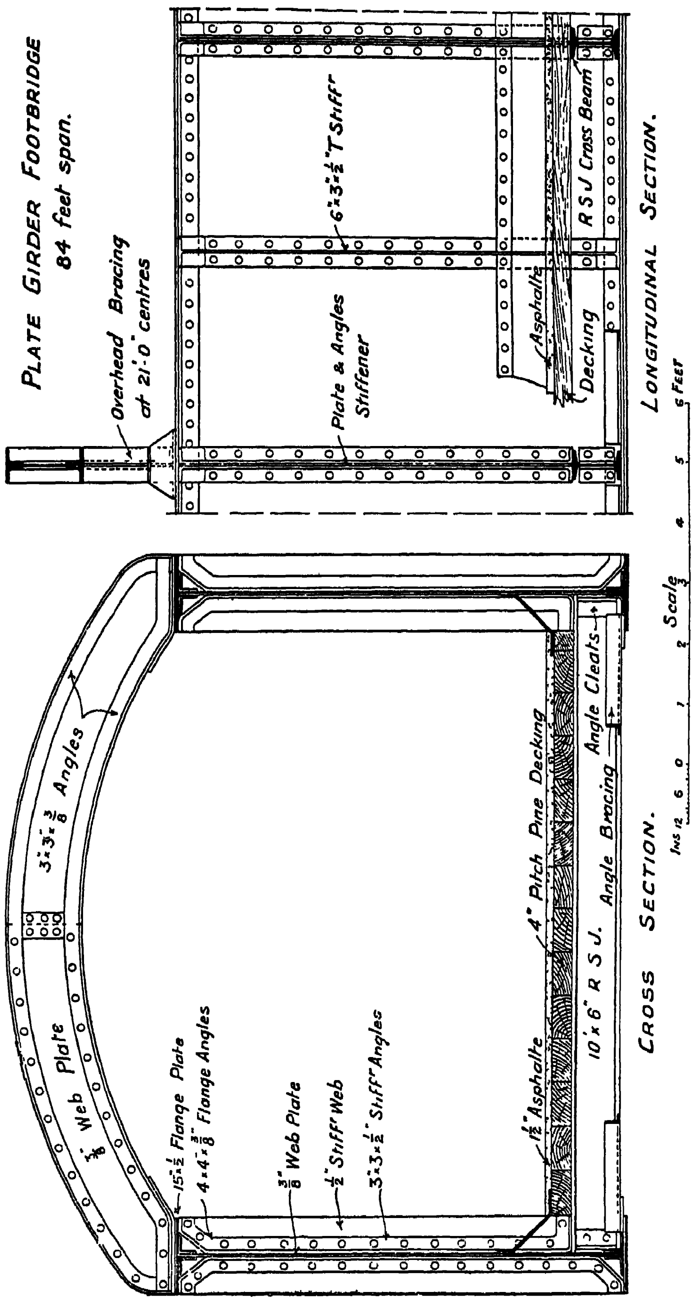


PLATE GIRDER FOOTBRIDGE
84 feet span.
Overhead Bracing
at 21'-0" centres
6"x3"x1/2" T Stiffr
Plate & Angles
Stiffener
Asphalte
Decking
RSJ Cross Beam
LONGITUDINAL SECTION.
3"x3"x3/8" Angles
3/8" Web Plate
15"x1/2" Flange Plate
4"x4"x3/8" Flange Angles
3/8" Web Plate
1/2" Stiffr Web
3"x3"x1/2" Stiffr Angles
1 1/2" Asphalte
4" Pitch Pine Decking
10"x6" R S J.
Angle Bracing
Angle Cleats
CROSS SECTION.
Scale
INS 12 6 0 1 2 3 4 5 6 FEET

FIG. 161.

Number of rivets in double shear required on each side of joint $= \frac{146\cdot2}{5} = 30$.

Bearing resistance of one $\frac{7}{8}$ inch rivet in $\frac{9}{16}$ inch plate, at 8 tons per square inch

$$= \frac{7}{8} \times \frac{9}{16} \times 8 = 3\cdot94 \text{ tons}.$$

Number of rivets required to limit bearing on web to 8 tons per square inch

$$= \frac{146\cdot2}{3\cdot94} = 38.$$

Pitching the rivets in the web covers at $4\frac{1}{2}$ in. provides 39 on each side of the joint. Web cover plates 14″ × $\frac{3}{8}$″ may be used.

The lengths of flange plates will be readily deduced by the method of the previous example.

The detailed arrangement is shown in Figs. 160 and 160A. Fig. 160 is a half cross section of the bridge and part longitudinal section showing the connection of stringers to cross-girders. Fig. 160A shows the detail at end of girder. The over-all length is 82 feet 6 in. and pin bearings are placed 80 feet apart for carrying the structure. An expansion roller bearing similar to that in Fig. 213 would be employed under one end. The face of the abutment between the main girders is built up close to the end cross-girder at A–A, and the ballast carried over the gap by a short length of floor plate P. The fender plates are omitted in Fig. 160A.

Where the end cross-girder cannot be placed close to the end of the main girders, additional rail bearers carry the floor and track between the end cross-girder and the abutment, their outer ends resting on bed stones built into the abutment.

Fig. 161 shows the cross section and part longitudinal section of a plate girder footbridge for a span of 84 feet. The decking is composed of rolled steel joist cross beams at 6 feet centres resting on the lower flanges of the girders and cleated to the girder webs. Pitch-pine timbers 4 inches thick are bolted to the top flanges of the joists and support a $1\frac{1}{2}$-inch layer of asphalte. Plated overhead bracing at 21 feet centres and diagonal angle bracing under the decking stiffen the bridge laterally.

CHAPTER VII.

LATTICE GIRDERS.

Types of Lattice Girders.—Girders having open-work webs consisting of ties and struts are classed generally as lattice girders, in contradistinction to plate girders with continuous webs. They are constructed in various forms and often referred to by distinctive names, according to the arrangement of the lattice bars and uniformity or variation of depth. Fig. 162 shows the more usual types employed.

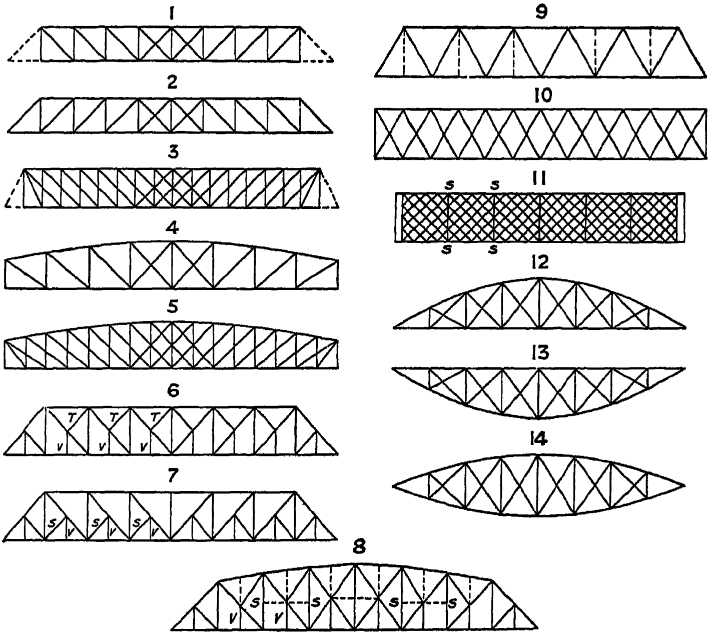


FIG. 162.

No. 1 is the Linville or N-girder, called in America, the Pratt truss. The vertical members are struts and the diagonal ones ties. No. 2, the Howe truss, has the diagonals reversed, so that they become struts, whilst the verticals are in tension. The Howe truss is seldom constructed entirely in steel but is more suitable for composite girders

having the sloping struts and upper boom of timber and the vertical ties and lower boom of steel.[1] The N-truss is preferable, since the shorter members are in compression. No. 3 is the double N, also known as the Whipple truss, formed by inserting additional verticals and diagonals midway between those of No. 1. The advantages resulting from this arrangement are the reduced length of the segments of the upper *boom* or *chord*, thereby reducing the tendency to buckling and enabling it to be made of lighter section; lighter compressive stresses in the vertical struts and shorter spacing of floor beams or cross-girders, thereby requiring shorter, and consequently lighter, longitudinal girders or bearers beneath the road or railway. Types 1 and 3 are frequently built with sloping ends, as indicated by the dotted lines. Nos. 4 and 5 are respectively singly and doubly braced hog-backed lattice girders, often referred to as lattice-bow girders. Generally speaking, they are used for larger spans than the parallel girders with resulting economy in weight of material, principally due to the shorter length of the vertical struts near the ends of the girder where the compressive stresses are greatest. The curved upper boom also relieves the verticals of a proportion of the compressive stress, by resisting part of the shear, whilst in the parallel types of girder, the horizontal upper boom resists the direct stress only and takes practically no part in resisting the vertical shear. Nos. 6 and 7 are two forms of the Baltimore truss, an almost exclusively American type. The normal outline is that of an N-girder, with additional members V, V, known as *sub-verticals*, inserted midway along the main panels. These serve as suspenders for intermediate floor or cross-beams and so achieve the same result as the double system of bracing in No. 3. The stresses in the sub-verticals may be transferred to the main panel points either by ties T, T, as in No 6, or by struts S, S, as in No. 7. No. 8, known as the Pennsylvania truss, is also of American origin and aims at combining the advantages of the Baltimore and double-N trusses by employing sub-verticals V, V, together with that of the lattice-bow type by possessing a greater depth at the centre than near the ends. In large span trusses, additional members shown by dotted lines are frequently added for the purpose of stiffening the main struts S, S, near the centre of their length. These dotted members, however, have no part in resisting the primary stresses in the truss.

Nos. 9 and 10 are respectively the single and double Warren girders, both ties and struts being inclined, usually at angles varying between 60° and 45° with the horizontal. Vertical members as shown by the dotted lines are occasionally inserted in No. 9 for the support of intermediate loads or for stiffening the segments of the upper boom. No. 11, known as the multiple lattice type, although formerly largely employed for main girders of long span, is now almost confined to parapet girders and main girders of foot-bridges. It possesses several systems of bracing, the bars of which are riveted to each other at the intersection points, the intention being to increase the rigidity. The effect, however, is to render it impossible to estimate at all accurately the stresses in the various systems. When constructed with flat bars only at close spacing, vertical stiffeners S, S, of T or channel section

[1] *Mins. Proceedings Inst. C. E.*, vol. cxxviii. p. 222, Plate 5.

are necessary, and the girder approximates closely in character to a plate girder, but involves more workmanship (see Fig. 198). Nos. 12 and 13 are respectively upright and inverted bowstring girders. In girders of this type, the stress in the booms is nearly uniform throughout, whilst the stresses in the web bracing is also much more uniform than is the case in parallel girders. The web members may therefore be made of equal section with very little sacrifice of economy. Bowstring girders are usually built with crossed diagonals in every panel, in which case the diagonals are designed for resisting tension only. If built with a single diagonal in each panel, the diagonals must be capable of resisting both tension and compression if the girder be required to carry a travelling load. No. 14 is a modification of the bowstring girder known as the Pauli or lenticular truss. Relatively few important spans have been bridged on this principle. It possesses the same general advantages as the bowstring type.

The essential points of difference between lattice and plate girders are as follow. In the lattice girder, the various members are arranged so that each is subject to stress only in the direction of its length, and the arrangement of the joints should be such that the members are further subject to direct tension or compression only. The direct bending stress, which in a plate girder is resisted mainly by the flanges, acts as direct compression or tension in the booms or chords of a lattice girder. The compression flange of a plate girder being attached to the web at short intervals of a few inches only, as determined by the rivet pitch, is less liable to buckle than the segments of the compression boom of a lattice girder, which are unsupported for the whole panel width. The stresses set up by the shearing force in a plate web are transferred to the flanges along innumerable lines in the web, whereas in a lattice girder these stresses are localized and constrained to act as direct tensile or compressive forces along a few well-defined lines, represented by the axes of the ties and struts. The load may be applied to a plate girder at any number of points along its length, but in a lattice girder it should only be applied as a number of concentrated loads at the panel points or intersections of the bracing bars with the booms. Any application of load *between* the panel points causes local bending of the boom segments, which must be of correspondingly stronger section to resist such bending in addition to the direct compression coming upon them. Curved members are economically inadmissible in lattice structures, since the direct stress acting through their ends sets up more or less bending moment on the central section, the amount depending on the sharpness of the curve given to the member. The upper booms of hog-backed lattice girders are often curved in outline to avoid the more complex joints and plate profiles which result from a polygonal outline. The curvature in such cases is, however, small, and the increment of stress due to bending correspondingly small also, whilst the girder has a neater appearance than if of polygonal outline. The common practice of introducing curved members in lattice structures, especially in roof trusses, presumably for aesthetic reasons, is, however, bad construction, and cannot be too strongly condemned.

Spans of Lattice Girders.—Considerable divergence is met with as

regards the length of span for which any particular type of truss is most suitable. Plate girders are rarely used beyond 100 ft. span, and about 80 ft. may be said to be their practical economic limit. The various types of parallel lattice girders with single system of bracing are generally employed for spans of from 80 to 150 ft., although in roof construction, light Warren and N-girders of from 20 to 50 ft. span are in very general use. Parallel and lattice bow girders with double systems of bracing are necessary for larger spans from about 150 to 350 ft. and upwards. Larger spans are the Kuilenberg bridge over the river Lek in Holland of 492 ft. span, the Ohio river bridge of 519 ft. span, the Covington and Cincinnati bridge over the Ohio river having one span of 550 ft. and two of 490 ft., and the Ohio river bridge at Metropolis, of 720 ft.[1] Bowstring girders have been widely employed for spans of from 80 to 300 ft., whilst the lenticular trusses of the Saltash bridge have a span of 455 ft.

Stresses in Braced Girders.—For the purpose of computing the stresses in braced girders, they may conveniently be divided into two classes—parallel girders, and girders of varying depth. The stresses in any parallel braced girder may be readily written down from a consideration of the shearing force for the loading and span in question. Thus, in Fig. 163, let AK represent an N-girder of eight panels carry-

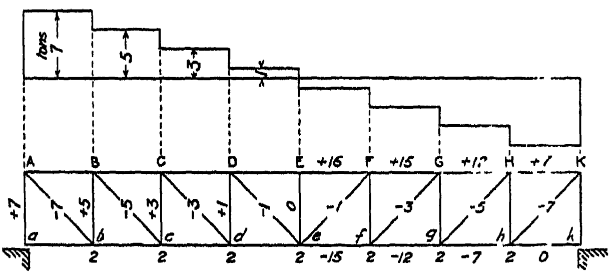


FIG. 163.

ing a load of 2 tons at each lower joint. The reaction at each support is 7 tons, and the upper figure shows the shearing force diagram for the span. The shearing force is constant, and equal to 7 tons from A to B. It then falls to 5 tons across the panel BC, 3 tons across CD, and 1 ton across panel DE. Since the function of the bracing bars is to resist the shearing force, these shears of 7, 5, 3, and 1 tons will be the vertical components of the stress in the four diagonal ties from A to the centre of the span. These figures may be at once written down in a vertical position across the ties in question, the minus sign indicating tension, since the effect of the shearing force is to cause tensile stress in the diagonals which slope downwards from the ends towards the centre of the span. If the ties be assumed to slope at 45°, the horizontal component of the stress in each tie will be equal to the vertical component, and on the right-hand half of the girder the corresponding horizontal stress in each tie may be written down, taking care to write the figures horizontally. From these the stresses in the segments of the upper

[1] Completed 1917. At present the longest simple truss span in the world.

and lower booms are readily obtained by summation. Thus, the compression in KH = 7 tons, caused by the horizontal tension or pull in the diagonal K*h*. HG receives a thrust of 7 tons directly from KH, and an additional compression of 5 tons from the tie H*g*, or a total compression of 12 tons. Similarly, GF receives the thrust of 12 tons from HG, plus 3 tons from the tie G*f*, giving a total compression of 15 tons. Finally, the compression in FE = 15 + 1 = 16 tons. The loading in this case being symmetrical, the stresses on the left-hand side of the centre will be similar to those on the right.

In the lower boom the stress in *hk* is nothing, since the vertical K*k* has no horizontal component. *gh* resists the horizontal pull of 7 tons exerted by K*h*. *fg* resists the combined horizontal tensions in H*g* and *gh* = 5 + 7 = 12 tons, and *ef* resists the horizontal tensions in *f*G and *fg*. The compression in vertical D*d* is 1 ton caused by the downward pull of 1 ton in the tie D*e*. In C*c* the compression is 3 tons due to the tension in tie C*d*. Similarly, the compressions in B*b* and A*a* are 5 and 7 tons respectively. Finally, the direct or actual stresses acting along the sloping direction of the ties is equal to the horizontal tension × ratio of slope to horizontal length of the tie, which for an inclination of 45° = $\sqrt{2}$. Thus, the real or *direct* tensions in the ties are—

$$eF = \sqrt{2}, fG = 3\sqrt{2}, gH = 5\sqrt{2}, \text{ and } hK = 7\sqrt{2} \text{ tons.}$$

Generally, it is unnecessary to draw the shear force diagram, since the shearing force in each panel is readily estimated by successively subtracting the panel loads from the reaction at either end of the girder. In the subsequent examples the shear diagram is therefore omitted.

If the above girder be inverted, it becomes a Howe truss. The diagonals are then in compression, and the verticals in tension. Assuming the same loads, the stresses are as follow (Fig. 164). The

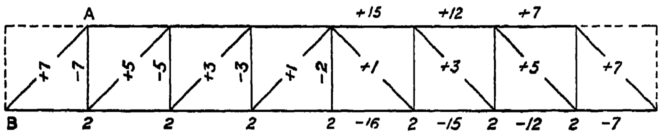


FIG. 164.

loading being the same, the shearing force in each panel is as before, but the sign of the stresses in the diagonals is changed to *plus* to denote compression. Inserting the corresponding horizontal stresses in the diagonals on the right-hand side, and summing up the boom stresses as before, it will be seen that the lower boom stresses are equal in amount to those of the upper boom in the previous example, whilst those in the upper boom equal those in the lower boom in the previous case. The stresses in corresponding vertical members are a little higher than before, but the end verticals, shown dotted, may be omitted, since the final shear of 7 tons is transmitted directly to the abutment through the inclined end strut AB, Fig. 164. The middle vertical here acts as a suspender for the central load of 2 tons, and is in tension to that

amount. The stresses in the diagonals are the same as before, but being compressive instead of tensile, and the diagonals being the longer members, their sectional area, and consequently their weight, will be appreciably greater than that of the vertical struts in the N-truss under similar conditions of span and load. It is principally this feature which places the Howe truss at a disadvantage as regards economy of material.

Stresses in an N-truss of Seven Panels carrying a Load of 6 tons at each Lower Joint. *Ties inclined 45°.*—In Fig. 165 the total load is

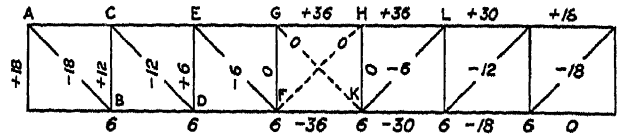


FIG. 165.

36 tons, which, being symmetrically disposed, gives a reaction or shearing force of 18 tons at each end of the span. The compression in the end vertical is therefore 18 tons. This member pressing upwards causes a vertical tension of 18 tons in the first diagonal. At B the diagonal AB exerts an uplift of 18 tons on the lower end of strut BC, but the load of 6 tons at B is pulling in the opposite direction, and so reduces the compression in BC to $18 - 6 = 12$ tons. This upward pressure of 12 tons in BC stretches the second diagonal CD with a vertical tension of 12 tons. CD then exerts an uplift of 12 tons on the lower end of strut DE, which is again reduced by the 6 tons load at D, giving a net compression of $12 - 6 = 6$ tons in DE. This compression in DE causes a vertical tension of 6 tons in tie EF, which just balances the load of 6 tons at F. There is therefore no stress in FG, as might be expected, since there is no diagonal member attached at G to carry the stress forward. If the shear-force diagram be drawn for this case of loading, it will exhibit no shear in the panel GH, thereby indicating no stress in any diagonals which may be inserted in that panel. In an actual girder as built, two diagonals, FH and GK, would be employed, but these will suffer no stress under a symmetrical system of loads such as under consideration. If, however, one half, say the left-hand half of the girder, be more heavily loaded than the other, the tendency of the load will be to distort the girder, as shown in Fig. 166, when the diagonal FH would be put into tension and GK into compression. Similarly, if the right-hand half be the more heavily

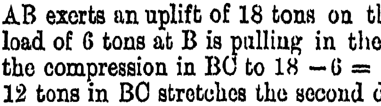


FIG. 166.

loaded, GK will be in tension and FH in compression. The duty of these central diagonals is therefore to make the girder capable of carrying an unsymmetrical system of loads, such as most usually occurs in practice. In Fig. 165 the horizontal stresses are written against the corresponding diagonals on the right-hand half of the girder, from which the boom stresses are readily obtained. The stress of 36 tons in HL is transmitted directly through GH, there being no increment

applied by the diagonal FH. As before, the *direct* stresses in the sloping bars are obtained by multiplying the horizontal stresses by $\sqrt{2}$ or 1·414.

Stresses in an N-truss loaded with 16 tons at each Lower Joint and 2 tons at each Upper Joint. Diagonals inclined 50° with Horizontal.—This example represents more closely the case of a practical girder having the flooring of a bridge attached to the lower joints, whilst a proportion of the dead weight of the girder with overhead bracing is applied directly at the upper joints.

It should be noticed (Fig. 167) the loads on the two end lower joints are neglected, since they are applied immediately over the

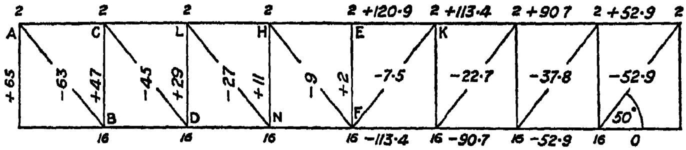


FIG. 167.

supports and do not give rise to any stress in the members of the girder. The total load is 130 tons, giving a vertical reaction or shear of 65 tons at each support. The compressive stress in the end strut is therefore 65 tons. Two tons of this is caused by the 2 tons load at A, leaving a net uplift of 63 tons as the vertical tension in tie AB. At B the tie AB exerts an upward vertical pull of 63 tons, which alone would cause a similar compression in strut BC. The downward acting load of 16 tons at B, however, reduces this to $63 - 16 = 47$ tons for the compressive stress in BC. Two tons of this is again due to the load at C, leaving 45 tons vertical tension in CD. By successive subtraction of the loads, the vertical stresses in the other members are obtained as indicated on the left-hand side of Fig. 167. The vertical EF here suffers a compression of 2 tons due to the load at E, which is transmitted from E to F, and thence as tension to the two ties HF and KF. At joint F, the downward acting forces are $2 + 16 = 18$ tons, whilst the tie HF exerts an uplift of 9 tons, giving $18 - 9 = 9$ tons as the vertical tension in tie FK necessary to maintain equilibrium. The diagonals in this case being inclined at 50° with the horizontal, the horizontal stress in any diagonal will be less than the vertical stress. In Fig. 168 the diagonal CD is shown separately. The horizontal stress in CD will bear the same ratio to the vertical stress as the length BD

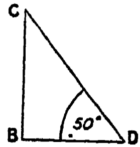


FIG. 168.

does to the length BC, or $\frac{\text{horizontal stress}}{\text{vertical stress}} = \frac{\text{BD}}{\text{BC}}$, which in a triangle of $50° = 0·84$. $\therefore$ horizontal stress = vertical stress $\times$ 0·84, and multiplying the vertical stresses already written down on the left-hand side of the girder by 0·84, the corresponding horizontal stresses are obtained as indicated on the right-hand half. The fraction $\frac{\text{BD}}{\text{BC}} = 0·84$ may be obtained directly from trigonometrical tables, it being = cotangent 50°, or by scaling off BD and BC from the elevation of the girder and

dividing the one by the other. From the horizontal stresses in the diagonals, the stresses in top and bottom booms are readily obtained by summation as before. The *direct* stresses in the ties are obtained from the vertical stresses as follows. In Fig. 168 the ratio of direct stress (*i.e.* acting along the *slope* CD) to vertical stress in diagonal CD $= \frac{CD}{CB}$, which for $50^\circ = 1 \cdot 3$.

$$\therefore \frac{\text{direct stress}}{\text{vertical stress}} = 1 \cdot 3, \text{ or direct stress} = \text{vertical stress} \times 1 \cdot 3.$$

Multiplying the vertical stresses in the diagonals by 1·3, the direct stresses are $63 \times 1 \cdot 3 = 81 \cdot 9$, $45 \times 1 \cdot 3 = 58 \cdot 5$, $27 \times 1 \cdot 3 = 35 \cdot 1$, and $9 \times 1 \cdot 3 = 11 \cdot 7$ tons tension in AB, CD, LN, and HF respectively.

Stresses in N-girder Unsymmetrically Loaded.—Fig. 169 represents an N-girder of 8 panels with ties at 45°. The upper and lower

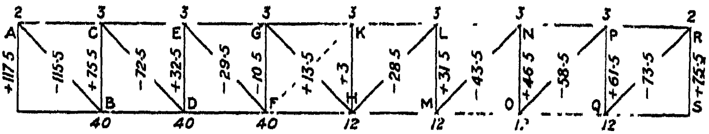


FIG. 169.

joints carry loads as indicated. Such a condition of loading would correspond with the case of a main bridge girder when the rolling load extends from the left-hand abutment up to and loading panel point F. The joints B, D, and F then carry both dead and live load, and the remaining lower joints dead load only, composed of part weight of main girders, cross-girders, flooring, and permanent way. The loads on the upper joints represent part weight of main girders and weight of over-head wind bracing.

The reaction at left-hand end of span

$$= \tfrac{25}{2}(\text{half upper loads}) + (\tfrac{7}{8} + \tfrac{6}{8} + \tfrac{5}{8}) \times 40 + (\tfrac{4}{8} + \tfrac{3}{8} + \tfrac{2}{8} + \tfrac{1}{8}) \times 12$$
$$= 117 \cdot 5 \text{ tons. Total load} = 25 \text{ (top)} + 168 \text{ (bottom)} = 193 \text{ tons.}$$
$$\therefore \text{Reaction at right-hand end} = 193 - 117 \cdot 5 = 75 \cdot 5 \text{ tons.}$$

Compression in left-hand vertical = 117·5 tons. Two tons of this is due to the load at A, ∴ vertical tension in AB = 115·5 tons. Subtracting 40 tons at B, compression in BC = 75·5 tons. Tension in CD = 75·5 − 3 = 72·5. Compression in DE = 72·5 − 40 = 32·5. Tension in EF = 32·5 − 3 = 29·5. At F a load of 40 tons is carried, of which 29·5 tons is supported by the tie EF, leaving 40 − 29·5 = 10·5 tons *tension* in vertical FG, which, for this position of the rolling load, acts as a *tie* instead of a strut. At G the vertical FG exerts a downward pull of 10·5 tons on the end of tie GH, which is augmented by the load of 3 tons at G, giving a vertical *compression* of 10·5 + 3 = 13·5 tons in GH, which now acts as a strut instead of a tie, *provided* it be built of a suitable section to resist compression. The 3 tons load at K is transmitted to H, causing a compression of 3 tons in KH. At H the downward forces are the 3 tons thrust in KH, 13·5 tons in GH,

and the 12 tons load at H, all tending to produce tension in HL and totalling 28·5 tons. At L the 3 tons load and vertical tension of 28·5 tons in HL both produce compression in LM = 31·5 tons. Similarly, tension in MN = 31·5 + 12 = 43·5, compression in NO = 43·5 + 3 = 46·5, tension in OP = 46·5 + 12 = 58·5, compression in PQ = 58·5 + 3 = 61·5, tension in QR = 61·5 + 12 = 73·5, and lastly, compression in RS = 73·5 + 2 = 75·5 tons, which agrees with the previously determined reaction at S.

With diagonals at 45°, the horizontal stresses in the diagonals are equal to the vertical stresses. In Fig. 170 these are written against

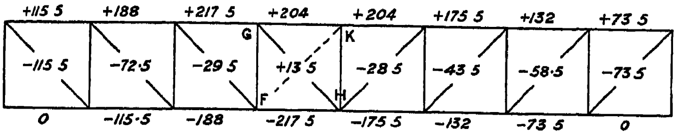


FIG. 170.

the corresponding bars, from which the top and bottom boom stresses, as indicated, are readily obtained. The direct stresses in the diagonals are equal to the horizontal stresses multiplied by $\sqrt{2}$, the ratio of inclined length of tie to horizontal breadth of panel. It will be noticed that the stress in diagonal GH is compressive. This is due to the rolling load being in the position which develops the maximum compression in GH, which compression is more than sufficient to neutralize the permanent tensile stress set up in GH by the dead load alone. The tie GH therefore undergoes a reversal of stress when the rolling load passes this position. A similar reversal would take place in the tie HL under the action of a rolling load advancing from the right. Usually, the diagonal ties consist of a pair of flat bars, Fig. 193, which are incapable of resisting compression, and in such cases the panel GFHK would be counter-braced by inserting a second tie, FK, sloping in the opposite direction to GH. The stresses in the panel GFHK are then as shown in Fig. 171. The main tie GH, being incapable of resisting any compression, falls into a state of no stress and becomes relatively slack. The upper 3 tons load at G is transmitted down GF as compression, whilst at F, the downward forces being 43 tons, and upward pull of tie EF 29·5 tons, the difference, 13·5 tons, determines the vertical tension in the counter-tie FK. This, increased by the 3 tons at K, gives a compression of 16·5 tons in KH, and adding the 12 tons at H, the vertical tension in HL = 16·5 + 12 = 28·5 tons, as previously obtained in Fig. 169. From L to the right-hand support the stresses are then the same as already determined in Fig. 170. For this position of the rolling load the stress in the other counter-tie KM is zero, it only coming into action when the rolling load extends from the *right-hand* abutment up to and loading joint M. The girder may be built without counter-ties, provided the members GH and HL are constructed of a suitable

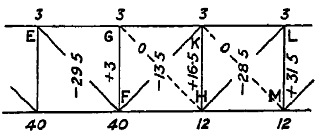


FIG. 171.

section for resisting both tension and compression, and this is occasionally done, although greater economy is realized by inserting counterties. It should be noticed the tension in the counter-tie is the same in *amount* as the compression which would be produced in the main tie of the same panel if not counter-braced.

Stresses in N-truss with Double System of Bracing.—Suppose the girder in Fig. 172 to be loaded with 2 tons and 8 tons respectively at

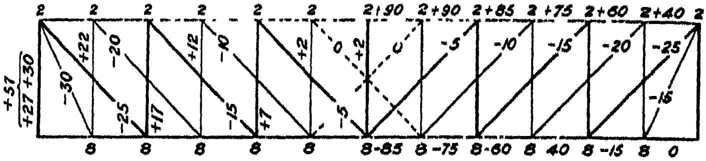


FIG. 172.

each upper and lower joint. The N-truss, with double system of bracing, may be regarded as consisting of two separate girders, superposed as indicated by the light and heavy lines in the figure. Considering the heavily-lined system first, the total load is 40 tons on the lower joints + 14 tons on the upper joints, counting the two end loads of 2 tons as carried by this system. The load being symmetrically disposed the reaction at each end for this system alone is 27 tons. The vertical stresses are written down on the left-hand side in the same manner as those in Fig. 167, giving, for the bars taken in order from left to right, + 27, − 25, + 17, − 15, + 7, and − 5 tons, and for the central vertical + 2 tons. At the central lower joint the 8 tons load + 2 tons compression from the central vertical make up 10 tons, which is resisted by the vertical tension of 5 tons in each of the two central ties.

Considering the lightly-lined system, the total load is 48 tons on lower joints + 12 tons on upper joints = 60 tons, giving a reaction at each end of 30 tons. An additional compression is set up in the end vertical, since this member is common to both systems, the total compression in the end vertical being, therefore, 27 + 30 = 57 tons, which of course equals half the total load on the complete girder. The vertical stresses are written down as before, taking care not to deduct the end upper load of 2 tons, since this has already been allotted to the other system. These stresses are, from left to right, + 30, − 30, + 22, − 20, + 12, − 10, and + 2 tons. Assuming the diagonals to slope at 45°, the horizontal components of stress are equal to the vertical in all the ties excepting the two end ones. These have an inclination of one horizontal to two vertical, and therefore the horizontal stress for these two members equals *half* the vertical stress. The summation of the boom stresses is effected as in the previous examples, the end section of the upper boom receiving a compression of 25 + 15 = 40 tons, applied by the end ties of both systems, since both are attached at the upper end of the boom. The direct stresses in the 45° diagonals equal the horizontal stresses $\times \sqrt{2}$. For the direct stress in the two end diagonals, proceed as follows—

$$\frac{\text{inclined length of diagonal}}{\text{horizontal length of diagonal}} = \frac{\sqrt{5}}{1}.$$

Hence the inclined or direct stress = horizontal stress $\times \sqrt{5}$
$= 15 \times \sqrt{5} = 33{\cdot}54$ tons.

Double system N-girders occasionally have the end members disposed as in Fig. 173. Assuming this to be a modified arrangement of the previous girder, the number of panels and loads remaining the same, the stresses in the end panel will be altered as follows: AC now acts simply as a suspender for the 8 tons load at C, and AB being reversed, acts as a strut and transmits the vertical pull of 20 tons in diagonal A8, + 8 tons in AC, together with the 2 tons load at A, making 30 tons in all, to the abutment at B. The remaining stresses in the web are unaltered. The segment EA of the upper boom resists the horizontal pull of 25 tons in EF. At A the segment AG receives a pressure of 25 tons from EA, a horizontal pressure of 15 tons from BA, and resists the horizontal pull of 20 tons in the tie A8, making 60 tons compression in all. From B to F the lower boom resists the outward horizontal pressure of 15 tons in strut AB. F8 resists the combined pull of 15 tons and 25 tons in FC and FE respectively. The remaining boom stresses are unaltered. The advantage of this arrangement is a simpler junction at E, where three members only, instead of four, have to be joined.

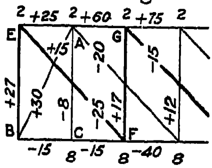


FIG. 173.

Note.—In the N-girder as usually constructed, it will be noticed in Figs. 163, 165, 167, 170, and 172, that the end segment of the lower boom is not stressed under the action of vertical loads. This segment, however, is always stiffened in such a manner as to be capable of resisting compression as well as tension. It comes into action in cases where application of the brakes on a train is made during its passage over the bridge. The longitudinal racking tendency thus created compresses the end segment of the lower boom in advance of the train, and puts into tension the end segment in rear of the train, the opposite effect resulting when a train passes in the reverse direction. Heavy end pressure, due to wind or the momentum only of the rolling load, also stresses these members. The amount of stress produced may be roughly calculated from the weight of rolling load and coefficient of friction, but it is not capable of exact determination. The end segments of the lower boom are, in practice, usually carried through of the same cross-section as obtains in the second panel, and in girders having lower booms of vertical plates only, these are braced together to form a lattice box strut. As this appears to be a satisfactory provision, the longitudinal stress is evidently not excessive.

Stresses in Warren Girder with Symmetrical Load—Fig. 174 represents a Warren girder with single system of bracing, carrying 10 tons at each lower, and 2 tons at each upper joint. The total load is 62 tons, giving a reaction of 31 tons at each end. The vertical stresses are written against the web members on the left-hand half of the girder, commencing with 31 tons in the end strut, and subtracting each load in order. The corresponding horizontal stresses on the

right-hand side are obtained as follows. In any inclined bar AB, Fig. 174A,

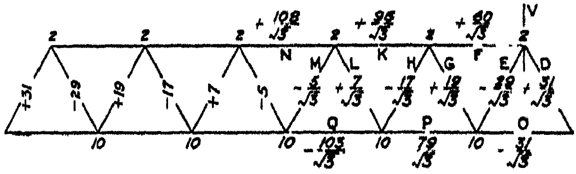


Fig. 174.

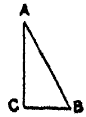


Fig. 174A.

$$\frac{\text{horizontal stress}}{\text{vertical stress}} = \frac{BC}{AC}$$

If the bars be assumed inclined at 60° with the horizontal,

$$\frac{BC}{AC} = \frac{1}{\sqrt{3}}$$

$$\therefore \text{horizontal stress} = \frac{\text{vertical stress}}{\sqrt{3}}$$

Dividing the vertical stresses by $\sqrt{3}$, the horizontal stresses are obtained as indicated. The end segment F of the upper boom receives a horizontal thrust of $\frac{31}{\sqrt{3}}$ tons from the strut D and an additional thrust of $\frac{29}{\sqrt{3}}$ tons caused by the tie E *pulling* from the opposite side of the vertical V, making $\frac{31}{\sqrt{3}} + \frac{29}{\sqrt{3}}$ or $\frac{60}{\sqrt{3}}$ tons in all. Segment K receives the direct thrust of $\frac{60}{\sqrt{3}}$ tons from F $+ \frac{19}{\sqrt{3}}$ from G $+ \frac{17}{\sqrt{3}}$ from H, or a total of $\frac{96}{\sqrt{3}}$ tons. Similarly the compression in N $= \frac{96}{\sqrt{3}}$ from K $+ \frac{7}{\sqrt{3}}$ from L $+ \frac{5}{\sqrt{3}}$ from M $= \frac{108}{\sqrt{3}}$ tons.

In the lower boom, the outward horizontal thrust of $\frac{31}{\sqrt{3}}$ tons in D produces a tension of $\frac{31}{\sqrt{3}}$ in O. Segment P resists the direct pull of $\frac{31}{\sqrt{3}}$ tons in O + horizontal pull of $\frac{29}{\sqrt{3}}$ in E + outward thrust of $\frac{19}{\sqrt{3}}$ in G, or a total of $\frac{79}{\sqrt{3}}$ tons. Similarly the tension in segment Q $= \frac{79}{\sqrt{3}} + \frac{17}{\sqrt{3}} + \frac{7}{\sqrt{3}} = \frac{103}{\sqrt{3}}$ tons.

Finally, the direct stresses in the inclined bars are obtained by doubling the horizontal stresses, since for an inclination of 60°, the inclined length AB = twice the horizontal length BC. The resulting direct or inclined stress in D therefore $= \frac{31}{\sqrt{3}} \times 2 = 35{\cdot}8$ tons; in E,

33·5 tons; in G, 21·9 tons; in H, 19·6 tons; in L, 8·1 tons, and in M, 2·9 tons.

Stresses in Warren Girder with Unsymmetrical Load.—Fig. 175, A, represents a singly braced Warren girder with a greater intensity of

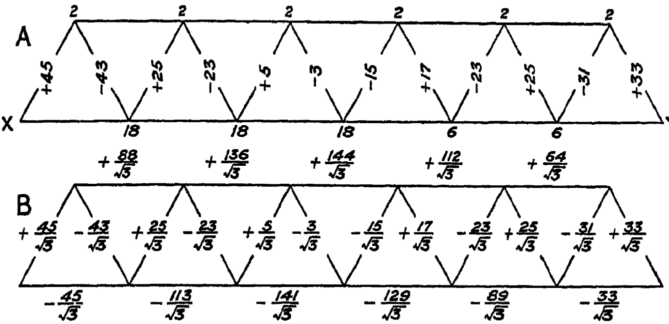


FIG. 175

load covering the left-hand portion of the span, up to and including the central lower joint. The reaction at $X = \frac{1}{2} \times 12$ tons (on upper joints) $+ 18(\frac{5}{6} + \frac{4}{6} + \frac{3}{6}) + 6(\frac{2}{6} + \frac{1}{6})$, being the proportions of the loads on the lower joints borne by X, and $= 45$ tons. Total load $= 78$ tons, $\therefore$ reaction at $Y = 78 - 45 = 33$ tons. Commencing at X, the vertical stresses are written down against *all* the inclined bars from X to Y, since in this case the stresses will not be symmetrical. The vertical stress in the last inclined strut at Y must, of course, equal the reaction of 33 tons at Y, which affords a check on the calculation. The corresponding horizontal stresses are inserted in Fig. 175, B, from which the boom stresses are readily obtained by summation. As before, the direct stresses in the inclined bars are obtained by doubling the horizontal stresses for an inclination of 60°.

Stresses in Warren Girder with Double System of Bracing and Unsymmetrical Load.—Fig. 176 represents a double Warren girder

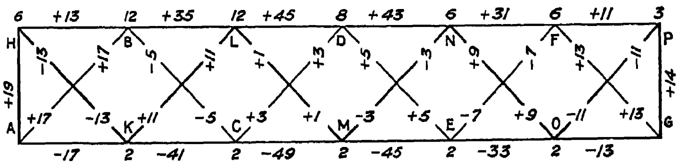


FIG. 176

with unequal loading on the upper joints, and uniform load on the lower joints, the lattice bars sloping at 45°. The girder possesses two distinct systems of bracing, AHKLM, etc., and ABCDE, etc. Considering first the system AHKLMNOP, the reaction at A due to the loads carried by this system $= 6$ (at H) $+ \frac{1}{2}(3 \times 2)$ at K, M, and O $+ \frac{2}{3} \times 12$ (at L) $+ \frac{1}{3} \times 6$ (at N) $= 19$ tons. Hence reaction at G due to these loads $= 33 - 19 = 14$ tons. The vertical stresses in the diagonal bars of this system are therefore as indicated. Considering

system ABCDEFG, reaction at A = $\frac{1}{2}(2 + 2)$ at C and E + $\frac{7}{8} \times 12$ (at B) + $\frac{1}{2} \times 8$ (at D) + $\frac{1}{8} \times 6$ (at F) = 17 tons, and reaction at G due to these loads = 30 − 17 = 13 tons. The vertical stresses for this system are + 17, − 5, + 3 tons, etc. The inclination being 45°, the horizontal stresses equal the vertical stresses. Writing these down horizontally on the respective bars, the stresses in the horizontal booms readily follow. The direct stresses in the lattice bars are obtained by multiplying the horizontal stresses by $\sqrt{2}$. It will be noticed that the member LM, which under a symmetrical load would be in tension, is put into compression to the extent of $1 \times \sqrt{2} = 1 \cdot 41$ tons, under the system of loads considered, and would therefore require to be designed as a strut.

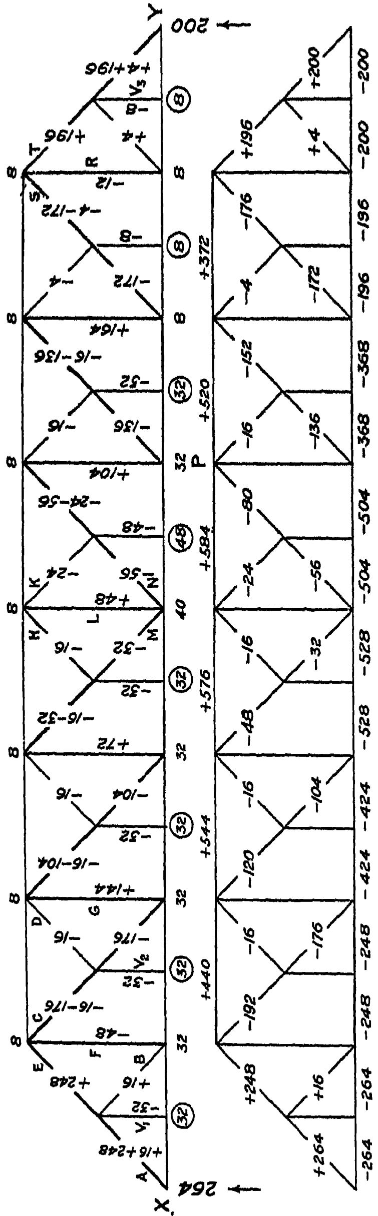


Fig. 177.

Stresses in Baltimore Truss with Unsymmetrical Loads.—Fig. 177 indicates a Baltimore truss of eight main panels, subdivided into sixteen panels by the sub-verticals supporting the intermediate loads marked by the circles. The loads assumed represent fairly closely the case of a large span girder carrying a rolling load extending from X up to and loading the main panel point P, the remainder of the girder carrying dead load only.

Reaction at X = $(\frac{1}{2} \times 7 \times 8)$ tons, due to loads on upper joints $+ 32(\frac{15}{16} + \frac{14}{16} + \frac{13}{16} + \frac{12}{16} + \frac{11}{16} + \frac{10}{16} + \frac{9}{16} + \frac{6}{16} + \frac{5}{16}) + \frac{1}{2} \times 40 + \frac{7}{16} \times 48 + 8(\frac{4}{16} + \frac{3}{16} + \frac{2}{16} + \frac{1}{16})$ tons, due to loads on lower joints = 264 tons. Total load = 464 tons, ∴ reaction at Y = 464 − 264 = 200 tons. The sub-verticals V act as suspenders for transferring the intermediate loads (ringed) to the main joints of the girder. Each is

consequently in tension to the extent of the load supported at its lower end. The tension of 32 tons in V_1 is shared equally by the struts A and B, giving a vertical compression of 16 tons in each. The tension of 32 tons in V_2 is divided between the upper half C of the *main* tie of the second panel and the sub-tie D of the same panel, giving 16 tons vertical tension in each. Similarly the other tensions are divided between the pairs of diagonal members supporting the sub-verticals. The stresses in the *main* members of the N-system may now be taken out. These members are shown by heavy lines. Commencing at X, the upward reaction is 264 tons, which produces a vertical compression of 264 tons in strut A. As 16 tons of this has already been written against this member, the remainder, 248 tons, is added on. This will also be the vertical compression in E. The vertical F acts as a suspender for the 32 tons load at its lower end, and also resists the downward vertical thrust of 16 tons in B, and therefore suffers a tension of $32 + 16 = 48$ tons. At the first upper joint the strut E exerts an uplift of 248 tons, whilst ties F and C exert a downward pull of $48 + 16 = 64$ tons, which together with the 8 tons of load, make a total downward force of 72 tons. The excess upward thrust of $248 - 72 = 176$ tons must be resisted by the vertical tension in tie C, over and above the 16 tons tension already caused in it by the intermediate load on V_2. This vertical tension of 176 tons is written against both upper and lower halves of the main tie C. At the second lower joint the vertical tension of 176 tons in C is balanced by the downward thrust in strut G + the 32 tons load, giving $176 - 32 = 144$ tons compression in G. The other panels are similarly treated. At the central upper joint the vertical tensions of 16 and 24 tons in H and K, together with the 8 tons load cause a compression of 48 tons in L. At the central lower joint the downward force is 88 tons, consisting of the 48 tons thrust in L + the 40 tons load. The upward pull in M = 32 tons, leaving $88 - 32 = 56$ tons to be resisted by the vertical tension in N. The last main vertical R resists the vertical thrust of 4 tons in the last sub-strut, and also acts as a suspender for the load of 8 tons at its lower end, its tension being therefore $4 + 8 = 12$ tons. At the upper end of the inclined end strut T, the downward forces are 176 tons in tie S + 12 tons in R + 8 tons of load on the joint, giving a vertical compression in T of 196 tons, which added to the 4 tons due to the sub-vertical V_3 gives 200 tons compression in the lower half of T, which checks with the reaction of 200 tons at Y.

The horizontal stresses in all the inclined bars, assuming their inclination as 45° are written on the lower diagram of Fig. 177, from which the indicated boom stresses are easily summed up. As before, the direct stresses in all the inclined bars equal the horizontal stresses $\times \sqrt{2}$.

Stresses in K-Truss.—The K-truss provides an alternative method of subdividing the original panels of a single triangulation girder. It is economical of material, largely on account of the relative shortness of its compression members, the detailed connections are simple and regular, and in riveted spans the secondary stresses are lower than in the Pennsylvania and Baltimore types. The stresses are more closely determinable than in the Whipple or double N-truss. K bracing is

employed in the cantilevers of the Quebec Bridge, and other large span bridges of this type have recently been erected.

Fig. 178 represents a K-truss of 240 feet span, 20 feet panel width and 40 feet depth, under a symmetrical panel load of 48 tons.

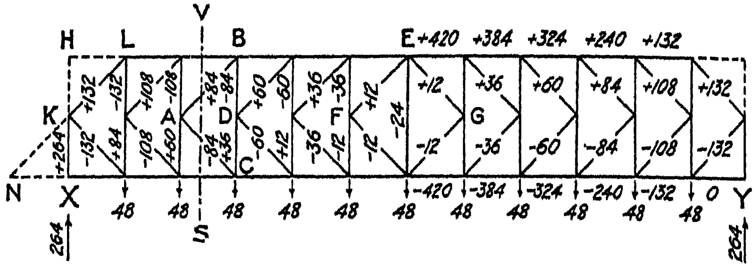


FIG. 178.

Reactions at X and Y = $\frac{1}{2} \times 11 \times 48 = 264$ tons. At any vertical section VS in a parallel girder of this type the vertical shear force is resisted by the two inclined web members AB and AC. These being equally inclined, each resists one-half of the vertical shear of $264 - (2 \times 48) = 168$ tons. Hence vertical component of stress in $AB = + (\frac{1}{2} \times 168) = + 84$ tons and in $AC = - 84$ tons. Proceeding in a similar manner for each panel, the vertical stresses in the diagonal members are obtained. Member CD resists a compression of 84 tons due to the upward vertical pull in AC less 48 tons due to the downward pull of the load at $C = 84 - 48 = 36$ tons compression. Member BD resists in tension the upward thrust of 84 tons in AB (less any load which may be imposed at B). The central vertical resists in tension the combined upward thrust of the two diagonal members EF and EG attached to its upper end.

The corresponding horizontal components of stress in the diagonals (in this case equal to the vertical components since inclined at 45°) are written on the right-hand side of Fig. 178, from which the boom or chord stresses are summed up in the usual manner. The dotted members HK and HL are unnecessary unless required in connection with a vertical system of portal bracing. These girders are frequently built with inclined end members as at LN, members HK and HL being then omitted.

If the girder be built as in Fig. 179, less economy of material is

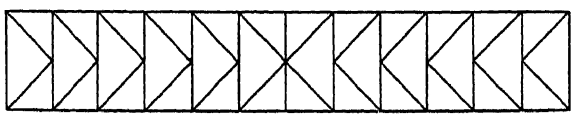


FIG. 179.

realised for the same span and loading, the increase of dead weight being about twelve per cent.

K-Girder with Unsymmetrical Load.—Taking a girder of the same

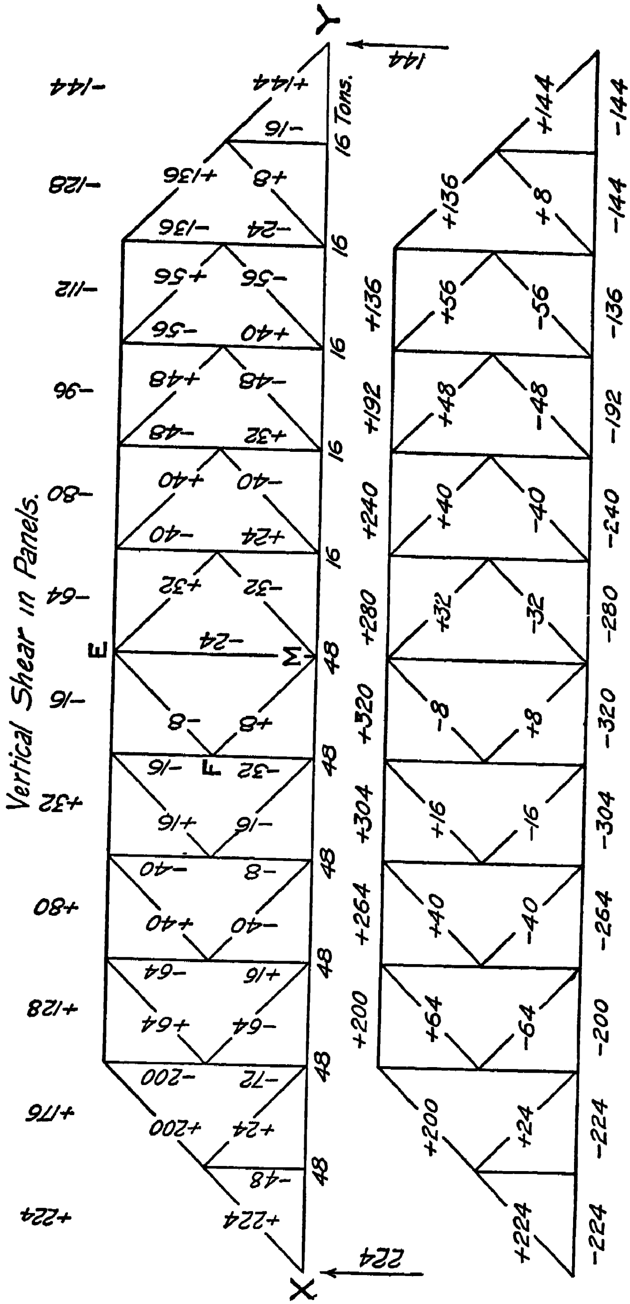


Fig. 180.

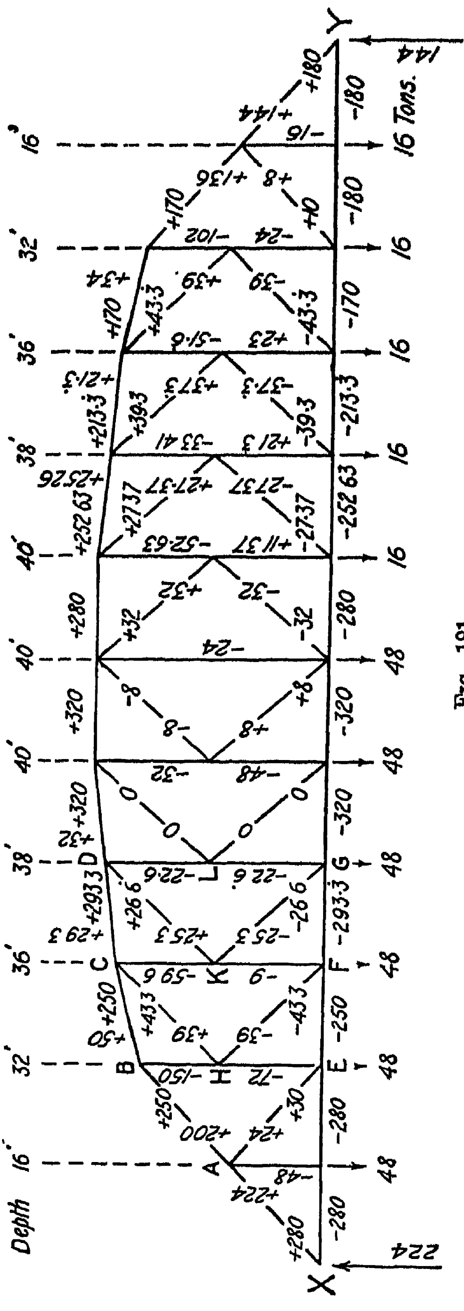

Depth
16'
32'
36'
38'
40'
40'
40'
38'
36'
32'
16'
16 Tons.
144
224
X
Y
A
B
C
D
E
F
G
H
K
L
48
48
48
48
48
48
16
16
16
16
16
+280
-280
-280
+224
-48
+200
+250
+24
+30
-150
-72
+50
+250
+43.3
+39
-39
-43.3
-250
-59.6
-9
+29.3
+293.3
+26.6
+25.3
-25.3
-26.6
-293.3
-22.6
+32
+320
-320
-32
-48
+320
-320
-8
+8
-24
+280
-280
+32
-32
+252.63
+25.26
+27.37
-27.37
-52.63
+11.37
-252.63
+213.3
+21.3
+39.3
-39.3
+37.3
-37.3
-33.41
+21.3
-213.3
+170
+34
+43.3
-43.3
+39
-39
-51.6
+23
-170
+170
+136
-102
+8
-24
+40
-180
+144
-16
+180
-180

FIG. 181.

dimensions and inclined end members, but with unsymmetrical loading as in Fig. 180, the reaction at

$$X = 48\left(\frac{11+10+9+8+7+6}{12}\right) + 16\left(\frac{5+4+3+2+1}{12}\right) = 224 \text{ tons.}$$

Total load = 368 tons. Reaction at Y = 368 − 224 = 144 tons. The vertical shear force in each panel is written down from inspection starting with R_X = 224 tons and deducting the loads in turn, working from left to right. The shear force changes sign in the sixth panel from X, hence EF is in tension and FM in compression.

The vertical stresses in the first two panels are obtained as in the Baltimore truss in Fig. 177. In K-girders members which suffer reversal of stress are usually built as compression members instead of introducing counterbraces. The chord stresses are summed up as before from the horizontal stresses in the diagonal web members and are given in the lower figure.

Stresses in K-Girder of Variable Depth.—Fig. 181 represents a K-girder of the same span and loading as in Fig. 180 but with the depth reduced from 40 feet at the centre to 32 feet near the ends as shown. The vertical stresses in the first two panels are the same as before. In the third, fourth, and fifth panels the inclined upper chord resists part of the vertical shear and the remainder only is shared between the diagonal web members. Converting the vertical stress of 200 tons in AB into its corresponding horizontal stress,—horizontal stress in AB = $200 \times \frac{20}{16} = 250$ tons.

This is also the horizontal stress in BC. The vertical stress in BC = $250 \times \frac{4}{20} = 50$ tons. Vertical shear in panel EF = 224 − (2 × 48) = 128 tons. Of this 50 tons is resisted by BC, leaving 128 − 50 = 78 tons to be equally divided between HC and HF if of equal inclination as is usual. Hence vertical stress in HC = + 39 tons and in HF = − 39 tons.

Horizontal stress in HC = $39 \times \frac{20}{18} = 43\cdot\dot{3}$ tons.
Hence, „ „ CD = $250 + 43\cdot\dot{3} = 293\cdot\dot{3}$ tons.
and Vertical „ CD = $293\cdot\dot{3} \times \frac{2}{20} = 29\cdot\dot{3}$ tons.

Vertical shear in panel FG = 224 − (3 × 48) = 80 tons, of which $29\cdot\dot{3}$ tons is resisted by CD, leaving $80 - 29\cdot\dot{3} = 50\cdot\dot{6}$ tons to be equally divided between KD and KG, giving $25\cdot\dot{3}$ tons for vertical stress in KD and KG. The remaining top chord and diagonal stresses are obtained similarly. Those for the other members are then readily filled in by summation. Thus, stress in HE = 24 + 48 = 72 tons tension. Stress in HB = 39 + 39 + 72 = 150 tons tension. Stress in FK = 48 − 39 = 9 tons tension. Stress in KC = $25\cdot\dot{3} + 25\cdot\dot{3} + 9 = 59\cdot\dot{6}$ tons tension. The stresses in the right-hand half of the girder have been obtained similarly by working from Y towards the central panel where the stresses check.

Stresses in Parallel Lattice Cantilever with Symmetrical Load.—The stresses in parallel cantilevers are readily obtained by the method of the previous examples. Assuming the loads indicated in Fig. 182, the vertical stresses are written down on the left-hand half of the girder, commencing with the 60 tons load at the outer end, and terminating with the supporting leg L, in which the compression is 100 tons, or half the total load, which is here symmetrically disposed

about the centre. The horizontal stresses in the diagonals (for bars at 45°) are shown on the right-hand half, from which the boom stresses readily follow. The upper boom is, of course, in tension, and the lower in compression. There will be no stress in the dotted diagonals of the central panel, these coming into action under an unsymmetrical load.

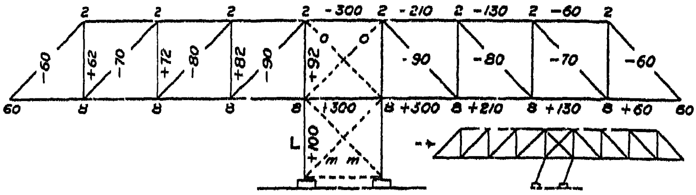


FIG. 182.

The lower dotted members, if the girder be supported on a braced pier as shown, will be required to resist longitudinal displacing forces, such as end wind pressure and longitudinal racking force, caused by the application of brakes to a rolling load traversing the girder. These forces would, in the absence of members *m*, *m*, tend to rack the structure as shown in the smaller figure.

Stresses in Parallel Cantilever with Unsymmetrical Load.—In Fig. 183 the same cantilever is taken with greater loads on the lower joints of the left-hand half. Proceeding as before, the vertical stresses may be written down for both halves until arriving at the upper points A and B. Inserting then the corresponding horizontal stresses in all the

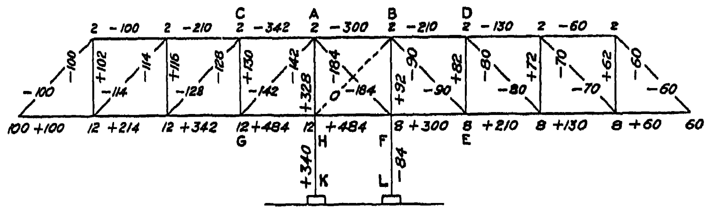


FIG. 183.

diagonals outside the points A and B, the boom stresses are summed up working from the outer ends towards the pier. The stress in AC is thus found to be 312 tons of tension, and that in BD, 210 tons. Next, the combined horizontal pulls of 210 and 90 tons in BD and BE respectively will create a tension in AB of 300 tons. As this is insufficient to balance the combined horizontal pull of $312 + 142 = 484$ tons exerted by AC and AG, the difference $= 484 - 300$, or 184 tons, will give the horizontal tension in AF, which thus comes into action as a tie, whilst HB remains lax. The 184 tons of horizontal stress in AF will be accompanied by a vertical stress of 184 tons (if inclined at 45°), inserting which, the remaining vertical stresses may be computed. At A the downward pull on the upper end of strut $AH = 142$ tons in $AG + 184$ tons in AF, which, with the 2 tons load at A, make up 328 tons of compression in AH. Adding the 12 tons

load at H, the compression in the pier leg HK = 328 + 12 = 340 tons. At B the downward pull of 90 tons in BE, together with the 2 tons load at B, causes a compression of 92 tons in BF. Lastly, at F the vertical uplift in tie AF is 184 tons, whilst the downward forces are the thrust of 92 tons in BF + the 8 tons load at F, or 100 tons. The resultant uplift is, therefore, 184 − 100, or 84 tons, which must be resisted by 84 tons of tension in the pier leg FL. This leg, for the loading in question, would therefore require to be anchored down by foundation bolts capable of resisting 84 tons of tension.

The loads on a large cantilever would greatly exceed those assumed above, and would result in an excessive uplift or tension in the pier legs, according as one or the other half of the girder were the more heavily loaded. In order to avoid such excessive uplift on the foundation, the legs forming the supports may be placed further apart, as in Fig. 184. The modification in the stresses will then be as follows.

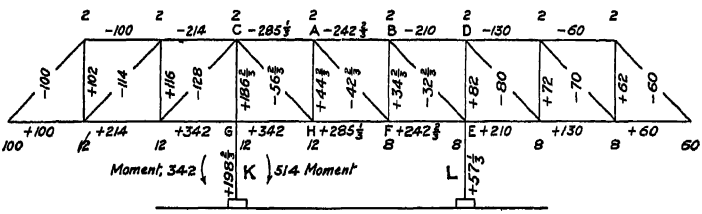


FIG. 184.

Assuming the same loads, but placing the supports at G and E instead of at H and F, the reactions in the two legs GK and EL are first found by taking moments about either.

Taking moments about G, and denoting the panel width by unity, the moment of the loads to the left of G = 100 × 3 + 14(2 + 1) = 342.

Moment of loads to right of G = 60 × 6 + 10(5 + 4 + 3 + 2) + 14 × 1 = 514.

The excess of the right-hand moments = 514 − 342 = 172, which, tending to pull *down* the portion of the girder to the right of G, will have to be resisted by an *upward thrust* in leg EL, applied at a leverage GE = 3 panel lengths.

Hence, compression in EL = $\frac{172}{3}$ = $57\frac{1}{3}$ tons.

The total load on the girder = 256 tons, ∴ compression in leg GK = 256 − $57\frac{1}{3}$ = $198\frac{2}{3}$ tons.

Commencing again with the vertical stresses, on reaching point C, the upward thrust in GC = $198\frac{2}{3}$ tons in GK − the 12 tons load at G = $186\frac{2}{3}$ tons. This is partly resisted by the downward pull of 128 tons to the left of C + the 2 tons load at C = 130 tons. The difference of $186\frac{2}{3}$ − 130 = $56\frac{2}{3}$ tons, will be resisted by the vertical tension in tie CH. It will be noticed the inclined member in this panel has been reversed as compared with the previous example. If the member AG be retained it would take a vertical *compression* of $56\frac{2}{3}$ tons, whilst CG would take 130 tons compression instead of $186\frac{2}{3}$ tons. The tie from C to H is the preferable arrangement. The remaining vertical stresses call for no special comment. The horizontal stresses in the booms,

assuming the diagonals inclined at 45°, are summed up as usual. The figures for the *horizontal* stresses in the diagonals, having the same value as the vertical stresses, have been omitted in Fig. 184 for the sake of clearness.

Stresses in Lattice Girders of Variable Depth.—In girders of the types shown in Fig. 162, Nos. 4, 5, 8, 12, 13, and 14, the curved or inclined boom resists a portion of the vertical shearing force, so that the inclined ties and struts are not called upon to resist the whole shear, and the variation in depth of any particular girder will determine what proportion of the total shear is borne by the booms or flanges, whilst the remainder only will be resisted by the web members. It is impossible, therefore, to write down the vertical stresses in the lattice bars merely from an inspection of the loads, as in the case of parallel girders, and a slightly modified method of analysis becomes necessary.

Stresses in Hog-backed Lattice Girder with Symmetrical Load.—Fig. 185 represents the elevation of one half of a girder of 80 feet span having 8 panels of 10 feet breadth, a central depth of 12 feet, and end depth of 6 feet, carrying the loads indicated on upper and lower joints. Under symmetrical loading only one half of the girder need be considered.

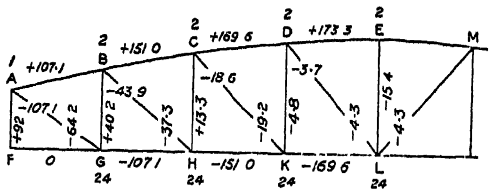


Fig. 185.

The reaction at each end $= 4 \times 2 + 3 \times 24 + \frac{1}{2} \times 24 = 92$ tons.

Bending moment at G $= (92 - 1) \times 10 = 910$ foot-tons.

" " $H = 91 \times 20 - 26 \times 10 = 1560$ foot-tons.

" " $K = 91 \times 30 - 26(20 + 10) = 1950$ foot-tons.

" " $L = 91 \times 40 - 26(30 + 20 + 10) = 2080$ foot tons.

The curve of the upper boom being drawn in from A to E, the depths intercepted at G, H, and K are $8\frac{1}{2}$, $10\frac{1}{3}$, and $11\frac{1}{2}$ feet respectively. Dividing the B.M. at each section by the vertical depth of the girder, the horizontal boom stress is obtained. Thus—

$$\text{Horizontal stress in AB} = \frac{\text{B.M. at G}}{\text{depth BG}} = \frac{910}{8\frac{1}{2}} = 107{\cdot}1 \text{ tons.}$$

$$\text{,, \quad ,, \quad BC} = \frac{\text{B.M. at H}}{\text{depth CH}} = \frac{1560}{10\frac{1}{3}} = 151{\cdot}0 \quad \text{,,}$$

$$\text{,, \quad ,, \quad CD} = \frac{\text{B.M at K}}{\text{depth DK}} = \frac{1950}{11\frac{1}{2}} = 169{\cdot}6 \quad \text{,,}$$

$$\text{,, \quad ,, \quad DE} = \frac{\text{B.M. at L}}{\text{depth EL}} = \frac{2080}{12} = 173{\cdot}3 \quad \text{,,}$$

These horizontal stresses are now written against the respective members, with a *plus* sign to denote compression, whence the horizontal

stresses in the diagonal ties at once follow. Thus, the horizontal compression of 107·1 tons in AB can only be caused by the horizontal pull applied by the tie AG, since the member AF is vertical and has no horizontal component of stress. The horizontal tension in AG is therefore 107·1 tons. Similarly the increase of horizontal compression from 107·1 tons in AB to 151 tons in BC is caused by the horizontal pull of tie BH = 151 − 107·1 = 43·9 tons. Also horizontal tension in CK = 169·6 − 151 = 18·6 tons, and in DL = 173·3 − 169·6 = 3·7 tons. The lower boom stresses now follow, being summed up as in a parallel girder. The *vertical* stresses in the ties are next required, and are obtained by multiplying the horizontal stress by the ratio $\frac{\text{vertical length}}{\text{horizontal breadth}}$ for each tie. Thus—

$$\text{Vertical stress in AG} = 107{\cdot}1 \times \frac{AF}{FG} = 107{\cdot}1 \times \frac{6}{10} = 64{\cdot}2 \text{ tons.}$$

$$\text{,, ,, BH} = 43{\cdot}9 \times \frac{BG}{GH} = 43{\cdot}9 \times \frac{8\frac{1}{2}}{10} = 37{\cdot}3 \text{ ,,}$$

$$\text{,, ,, CK} = 18{\cdot}6 \times \frac{CH}{HK} = 18{\cdot}6 \times \frac{10\frac{1}{3}}{10} = 19{\cdot}2 \text{ ,,}$$

$$\text{,, ,, DL} = 3{\cdot}7 \times \frac{DK}{KL} = 3{\cdot}7 \times \frac{11\frac{1}{2}}{10} = 4{\cdot}3 \text{ ,,}$$

These stresses are written vertically against the respective members. The stresses in the verticals easily follow. In AF the compression is of course equal to the reaction of 92 tons. At G the vertical uplift of 64·2 tons in AG is partly balanced by the downward pull of the 24 tons load at G, leaving a compression of 64·2 − 24 = 40·2 tons in BG. Similarly compression in CH = 37·3 − 24 = 13·3 tons. At K the vertical uplift in tie CK is only 19·2 tons, whilst the load to be supported at K is 24 tons. The difference = 24 − 19·2 = 4·8 tons is therefore taken by DK in *tension*. At the centre L the uplift in both ties DL and ML = 4 3 + 4·3 = 8·6 tons, and the load supported at L is again 24 tons. The difference 24 − 8·6 = 15·4 tons is therefore taken by EL in *tension*.

It should be noted that whether the vertical DK is in compression or tension depends on the outline given to the upper boom. A very slight increase of depth at CH, say to $10\frac{2}{3}$ instead of $10\frac{1}{3}$ feet, would result in a horizontal stress in BC of 146·2 tons and in CK of 23·8 tons, giving a vertical stress in CK of 25·4 tons. This being greater than the 24 tons load at K would result in DK being subject to 1·4 tons of compression instead of 4 8 tons tension.

It remains to convert the horizontal stresses in the ties and inclined segments of the upper boom into the corresponding inclined or direct stresses. This is done in each case by multiplying the horizontal stress in the member by the ratio $\frac{\text{inclined length}}{\text{horizontal breadth}}$. The inclined lengths may be calculated or scaled off, provided the elevation of the girder is carefully drawn to scale. The inclined lengths are as follows :—AB, 10·3 ft. ; BC, 10·2 ft. ; CD, 10·1 ft. ; DE, 10·02 ft. ; AG, 11·66 ft. ; BH, 13·12 ft. ; CK, 14·38 ft. ; DL, 15·24 ft. Hence direct stresses are,

$$AB = 107{\cdot}1 \times \frac{10{\cdot}3}{10} = 110{\cdot}3 \text{ tons.} \quad AG = 107{\cdot}1 \times \frac{11{\cdot}66}{10} = 124{\cdot}8 \text{ tons.}$$

$$BC = 151 \times \frac{10{\cdot}2}{10} = 154{\cdot}0 \;\;,, \quad BH = 43{\cdot}9 \times \frac{13{\cdot}12}{10} = 57{\cdot}6 \;\;,,$$

$$CD = 169{\cdot}6 \times \frac{10{\cdot}1}{10} = 171{\cdot}3 \;\;,, \quad CK = 18{\cdot}6 \times \frac{14{\cdot}38}{10} = 26{\cdot}7 \;\;,,$$

$$DE = 173{\cdot}3 \times \frac{10{\cdot}02}{10} = 173{\cdot}7 \;\;,, \quad DL = 3{\cdot}7 \times \frac{15{\cdot}24}{10} = 5{\cdot}6 \;\;,,$$

Any condition of unsymmetrical loading may be treated in the same manner by calculating the bending moment at each panel point throughout the girder, as exemplified in the following case.

Stresses in Cantilever of varying Depth with Unsymmetrical Load.—Fig. 186 represents the outline of a cantilever girder supported on piers 100 feet apart and overhanging 250 feet on either side. Each arm contains five panels of 50 feet horizontal breadth. The loads indicated are assumed as acting at the various panel points and represent closely the state of loading for a steel girder of these dimensions, forming one of a pair for carrying a double line of railway, when the left-hand half from P, up to and including joint N is loaded with the rolling load plus dead load, whilst the right-hand half is assumed to carry dead load only.

B.M. at A = 47 × 50 = 2350 foot-tons.
,, B = 47 × 100 + 103 × 50 = 9850 foot-tons.
,, C = 47 × 150 + 103 × 100 + 109 × 50 = 22,800 foot-tons.
,, D = 47 × 200 + 103 × 150 + 109 × 100 + 113 × 50
= 41,400 foot-tons.
,, E = 47 × 250 + 103 × 200 + 109 × 150 + 113 × 100 + 117 × 50
= 65,850 foot-tons.
,, F = 22 × 250 + 53 × 200 + 59 × 150 + 63 × 100 + 67 × 50
= 34,600 foot-tons.
,, G = 22 × 200 + 53 × 150 + 59 × 100 + 63 × 50 = 21,400 foot-tons.
,, H = 22 × 150 + 53 × 100 + 59 × 50 = 11,550 foot-tons.
,, K = 22 × 100 + 53 × 50 = 4850 foot-tons.
,, L = 22 × 50 = 1100 foot-tons.

In the following table are entered the bending moments, depth of girder at each panel point, and horizontal stresses in booms, obtained by dividing the B.M. by depth of girder. The horizontal stresses in diagonals are obtained by successive differences of the horizontal stresses in the boom segments, and the vertical stresses in diagonals by multiplying the horizontal stress by the ratio,

$$\frac{\text{vertical height of diagonal}}{\text{horizontal breadth of diagonal.}}$$

The resulting stresses are indicated on Fig. 186, the compression in any vertical being obtained by subtracting the load at its lower end from the vertical tension in the diagonal tie attached at that point.

At	B M. in foot-tons	Depth. Feet.	Horizontal stress in boom segments in tons.	Horizontal stress in diagonals.	Vertical stress in diagonals.
				PA, 58·8	
A	2,350	40	AB, 58·8		
				OB, 120·3	132·3
B	9,850	55	BC, 179·1		
				RC, 146·6	205·2
C	22,800	70	CD, 325·7		
				SD, 134·3	241·7
D	41,400	90	DE, 460·0		
				TE, 88·8	213 1
E	65,850	120	EF + EM, 548·8		
				EM, 260 5	416·8
F	34,600	120	EF, 288·3		
				FV, 50 5	121·2
G	21,400	90	FG, 237·8		
				GW, 72·8	131·1
H	11,550	70	GH, 165·0		
				HX, 76·8	107·5
K	4,850	55	HK, 88 2		
				KY, 60·7	66·8
L	1,100	40	KL, 27·5		
				LZ, 27·5	

The compression of 288·3 tons in MV is obtained by summing up from Z inwards or is taken directly from the calculation, being $= \frac{\text{B.M. at F}}{120 \text{ ft.}}$. The compressive stress in MN $= \frac{\text{B.M. at E}}{120 \text{ ft.}} = 548{\cdot}8$ tons. The difference $548{\cdot}8 - 288{\cdot}3 = 260{\cdot}5$ tons gives the horizontal tension in EM which comes into action when the left-hand cantilever arm is the more heavily loaded, NF being then inoperative. The pressures on the supports are, for the

$$\begin{aligned}
\text{left-hand pier} &= \text{vertical compression in EN} + \text{load at N} \\
&= 836{\cdot}7 + 108{\cdot}0 = 944{\cdot}7 \text{ tons, and for the} \\
\text{right-hand pier} &= \text{compression in FM} - \text{vertical uplift in EM} \\
&\qquad + \text{load at M} \\
&= 298{\cdot}9 - 312\ 6 + 58{\cdot}0 = 44\ 3 \text{ tons.}
\end{aligned}$$

These may be verified if desired, by taking moments about either pier. The following table gives the calculated inclined lengths of the sloping ties and segments of the upper boom, from which the tabulated direct stresses have been calculated as follows. Direct stress in any tie, say SD,

$$= \text{horizontal stress} \times \frac{SD}{ST} = 134{\cdot}3 \times \frac{103}{50} = 276{\cdot}6 \text{ tons.}$$

Direct stress in any segment of the upper boom, say DE,

$$= \text{horizontal stress} \times \frac{DE}{TN} = 460{\cdot}0 \times \frac{58{\cdot}3}{50} = 536{\cdot}3 \text{ tons.}$$

The horizontal and vertical stresses in members which are themselves horizontal or vertical, constitute of course the direct stresses for those members. The direct stress in *any* sloping member of any girder may, if preferred, be obtained as follows.

Let H = horizontal stress. V = vertical stress, then direct stress $= \sqrt{H^2 + V^2}$. In large girders, the lengths of the longer struts as CS,

DT, NM, and EN being unavoidably great, these members would be stiffened by auxiliary struts *ss*, shown by dotted lines, in order to

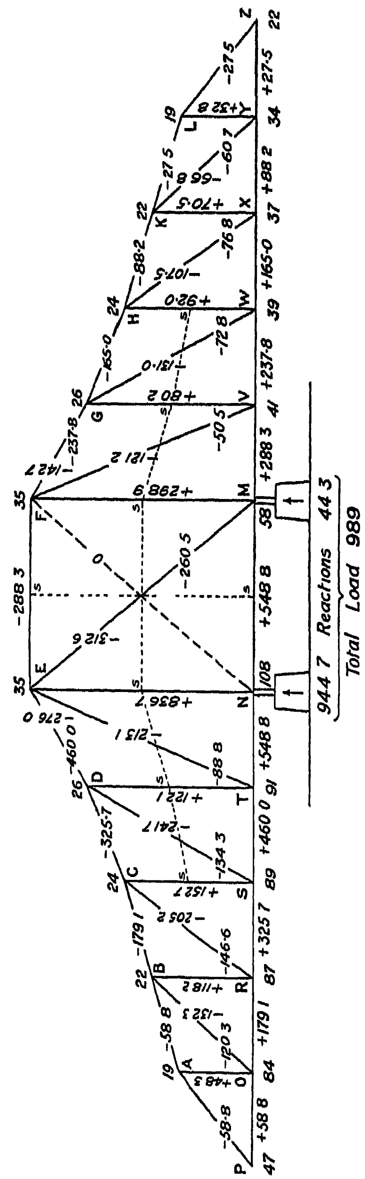


Fig 186.

reduce their tendency to buckle. Such members, however, do not enter into the calculations for the primary stresses.

DIRECT STRESSES IN INCLINED MEMBERS OF CANTILEVER GIRDER IN FIG. 186.

Member	Horizontal stress	Inclined length.	Horizontal length.	Direct stress.
	tons	ft.	ft	tons
PA	58·8	64·0	50	75·3
AB	58 8	52 2	50	61·7
BC	179 1	52·2	50	188 0
CD	325·7	53·9	50	351 1
DE	460 0	58 3	50	536·3
FG	237·8	58 3	50	277 2
GH	165·0	53 9	50	177 8
HK	88 2	52 2	50	92 1
KL	27 5	52 2	50	28 7
LZ	27 5	64 0	50	35 2
OB	120 3	74 3	50	178 7
RC	146·6	86 0	50	252·1
SD	134·3	103·0	50	276 6
TE	88 8	130 0	50	230 8
EM	260 5	156 2	100	406·9
FV	50 5	130·0	50	131 3
GW	72 8	103 0	50	149·9
HX	76 8	86 0	50	132 1
KY	60·7	74 3	50	90 2

General Method for Rolling Load Stresses in Girders of Constant Depth.—The calculation of the maximum stresses caused by a rolling train load passing over a bridge span is considerably simplified if the actual axle loads be replaced by the "equivalent distributed load," that is, by a uniformly distributed load which would produce at least as large bending moments or shearing forces as the actual axle loads of the type of train in question.

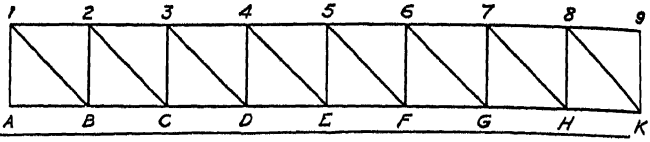
The following method is conveniently applicable to the most generally employed types of single triangulation bridge girders. Let Fig. 187 represent the outline of an N-girder of eight panels. The "equivalent distributed rolling load" will be specified as so many tons per foot run (see Table 22). The "panel rolling load" will therefore = equivalent distributed rolling load × panel length. As this value will be variable for different types of locomotives and varying panel lengths, it is convenient to assume a "panel rolling load" of one ton and afterwards to multiply the stresses resulting from unit panel load by the actual panel load for any particular case.

Considering a rolling distributed load equivalent to one ton per panel advancing from A towards K, let the load advance until panel point B is loaded with one ton. In this position seven-eighths of a ton of vertical shear will be transmitted through diagonal 1—B acting in tension, to abutment A, and one-eighth of a ton of vertical shear through all the remaining diagonals acting in compression to abutment K.

These values are entered along the first horizontal line of the table beneath Fig. 187, the negative sign indicating tension and the positive sign compression. If panel point C be loaded with one ton, six-eighths of a ton of vertical shear will be transmitted through diagonals 2—C and 1—B as tension, to A, and two-eighths of a ton through the

remaining diagonals as compression, to K. The completion of the table as the unit load occupies successively the different panel points is

FIG. 187.



Position of Unit Load.	Vertical Components of Stress in Diagonals.							
	1—B	2—C	3—D	4—E	5—F	6—G	7—H	8—K
At B . .	$-\frac{7}{8}$	$+\frac{1}{8}$	$+\frac{1}{8}$	$+\frac{1}{8}$	$+\frac{1}{8}$	$+\frac{1}{8}$	$+\frac{1}{8}$	$+\frac{1}{8}$
„ C . .	$-\frac{6}{8}$	$-\frac{6}{8}$	$+\frac{2}{8}$	$+\frac{2}{8}$	$+\frac{2}{8}$	$+\frac{2}{8}$	$+\frac{2}{8}$	$+\frac{2}{8}$
„ D . .	$-\frac{5}{8}$	$-\frac{5}{8}$	$-\frac{5}{8}$	$+\frac{3}{8}$	$+\frac{3}{8}$	$+\frac{3}{8}$	$+\frac{3}{8}$	$+\frac{3}{8}$
„ E . .	$-\frac{4}{8}$	$-\frac{4}{8}$	$-\frac{4}{8}$	$-\frac{4}{8}$	$+\frac{4}{8}$	$+\frac{4}{8}$	$+\frac{4}{8}$	$+\frac{4}{8}$
„ F . .	$-\frac{3}{8}$	$-\frac{3}{8}$	$-\frac{3}{8}$	$-\frac{3}{8}$	$-\frac{3}{8}$	$+\frac{5}{8}$	$+\frac{5}{8}$	$+\frac{5}{8}$
„ G . .	$-\frac{2}{8}$	$-\frac{2}{8}$	$-\frac{2}{8}$	$-\frac{2}{8}$	$-\frac{2}{8}$	$-\frac{2}{8}$	$+\frac{6}{8}$	$+\frac{6}{8}$
„ H . .	$-\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$+\frac{7}{8}$
− Total .	$-3\frac{1}{2}$	$-2\frac{5}{8}$	$-1\frac{7}{8}$	$-1\frac{1}{4}$	$-\frac{3}{4}$	$-\frac{3}{8}$	$-\frac{1}{8}$	0
+ Total .	0	$+\frac{1}{8}$	$+\frac{3}{8}$	$+\frac{3}{4}$	$+1\frac{1}{4}$	$+1\frac{7}{8}$	$+2\frac{5}{8}$	$+3\frac{1}{2}$
Alg. Total	$-3\frac{1}{2}$	$-2\frac{1}{2}$	$-1\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+1\frac{1}{2}$	$+2\frac{1}{2}$	$+3\frac{1}{2}$

simple. The algebraic sum of the quantities in any vertical column from the top down to any particular panel point will evidently give the vertical component of stress in the diagonal at the head of that column at the instant when the load occupies the panel points in question. Thus, when the distributed load extends from A over panel points B, C, and D, the vertical stress in diagonal 2—C $= +\frac{1}{8} - \frac{6}{8} - \frac{5}{8} = -\frac{10}{8}$ ton, and in diagonal 6—G $= +\frac{1}{8} + \frac{2}{8} + \frac{3}{8} = +\frac{6}{8}$ ton. It will be noted that the signs of all the quantities above the stepped line are positive and those below the stepped line negative. It is readily seen, therefore, that in order to produce the maximum rolling load stress in 1—B, *all* the panel points must be loaded, when the vertical stress in 1—B would be $\frac{-7-6-5-4-3-2-1}{8} = -3\frac{1}{2}$ tons. Also to produce the maximum vertical stress in 3—D, the points D, E, F, G, and H must be loaded, this stress then being $= \frac{-5-4-3-2-1}{8} = -1\frac{7}{8}$ tons. If the points B and C be also loaded the negative stress of $1\frac{7}{8}$ tons in 3—D would be diminished by $\frac{+1+2}{8} = +\frac{3}{8}$ ton. It is evident there are specific positions of the load which produce the maximum tensile and compressive stresses in the diagonals and these positions are defined by the stepped line separating the positive and negative quantities. The totals of the negative and positive quantities in each column are figured at the foot of the table, as also the algebraic totals.

The negative totals give the vertical components of the maximum tension which the rolling load may produce in any diagonal. The positive totals give the vertical components of the maximum com-

pression which the rolling load may produce in any diagonal. The algebraic totals give the vertical components of the stresses in the diagonals when every panel point is loaded, that is, when the rolling load covers the span.

In order to compute the actual maxima stresses in the diagonals it is only necessary to multiply these positive and negative totals by the actual "panel rolling load," and to convert the resulting vertical stresses into their corresponding inclined or direct stresses. It will be noticed that all the diagonals, excepting those in the two end panels, undergo reversal of stress due to rolling load only, according as the load approaches from one or the other end of the span.

A similar table may quickly be constructed for a girder of other than eight panels, and for practical use only one-half of the table is required, since the totals are the same to right and left of the centre but with the signs changed. When the diagonals 5F, 6G, 7H, and 8K are reversed as in an actual girder, the signs of the vertical stresses pertaining to those members are thereby changed and the totals to right and left of the centre become identical both in sign and magnitude.

The maxima stresses in the upper and lower chords occur when the rolling load covers the whole span, increasing from zero to the maximum as the head of the rolling load moves from A to K or from K to A. The *rate* of this increase upon which the impact allowance partly depends is governed by the speed of the advancing load. There is, of course, no reversal of stress in the chords.

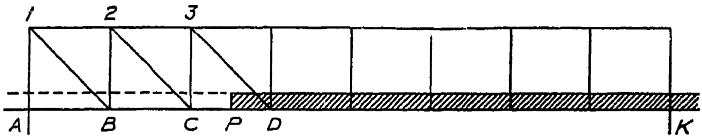


FIG. 188.

Rolling Load Stresses in Verticals.—The stresses in the vertical members, excepting the two end ones 1A and 9K, also undergo reversal during the passage of the rolling load. The maximum values are readily deduced from the totals of the above table.

In Fig. 188 the maximum tension is assumed to occur in diagonal 3—D when the rolling load extends from K up to a point P such that panel point D is fully loaded whilst C is unloaded. Actually this is not possible with a uniform load, since any portion of the load as PD which enters the panel CD is necessarily partly carried by the panel point C in advance of D. For the present purpose P may be regarded as the centre of panel CD.[1] The vertical stress in 3—D at this instant is $1\frac{7}{8}$ tons tension. The vertical 3—C receives this downward pull of $1\frac{7}{8}$ tons applied at its upper end and therefore suffers a *compression* of $1\frac{7}{8}$ tons, which is transmitted as tension through 2—C, compression through 2—B, tension through 1—B and compression through 1—A to the abutment A. Hence the vertical next in advance of the head of the load receives its maximum compression simultaneously with the occurrence of the maximum tension in the diagonal attached to its upper end.

[1] See chapter on Influence Lines, p. 276.

Again, when the load extends from A to P, diagonal 3—D receives its maximum compression, the vertical component of which (from the table) is $\frac{3}{8}$ ton. This vertical uplift exerted on the upper end of 3—C applies a *tension* of $\frac{3}{8}$ ton to 3—C. These occurrences may be summarised thus :—

Load.	Diagonal 3—D.	Vertical 3—C.
From K to P	Max. vert. tension = $1\frac{7}{8}$ tons.	Max. comp. = $1\frac{7}{8}$ tons.
„ A to P	„ „ comp. = $\frac{3}{8}$ „	„ tension = $\frac{3}{8}$ „

The application of this method to the calculation of the combined dead and rolling load stresses in a bridge girder will now be shown.

EXAMPLE 31A.—*An N-girder of* 128 *feet span,* 16 *feet depth, and having eight panels of* 16 *feet width, carries a dead load of* $\frac{3}{4}$ *ton per foot run due to floor and track supported on lower panel points and* $\frac{5}{8}$ *ton per foot run due to weight of girder, divided equally between upper and lower panel points. The equivalent rolling load is* 2 *tons per foot run carried on lower panel points. Required the maximum and minimum stresses in the members.*

Dead Load per panel on upper panel points $= \frac{5}{16} \times 16 = 5$ tons.

„ „ „ lower „ „ $= (\frac{3}{4} + \frac{5}{16}) \times 16 = 17$ tons.

$$R_A = R_K = \tfrac{1}{2}(7 \times 17 + 7 \times 5 + 2 \times 3) = 80 \text{ tons.}$$

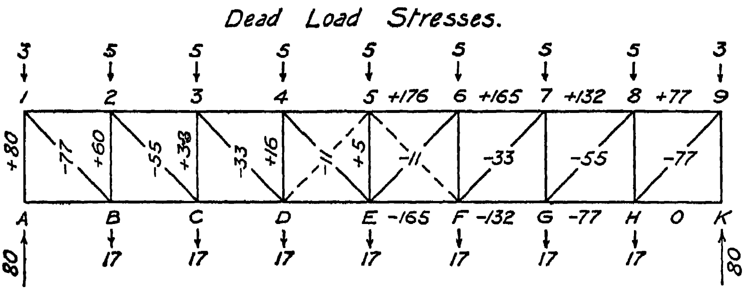


FIG. 189.

Dead Load Stresses.—The dead load stresses, taken out in the usual manner, are shown on Fig. 189, and are transferred to the "dead load" column of Stress Sheet after multiplying the figured diagonal stresses by $\sqrt{2}$.

Maximum and Minimum Rolling Load Stresses in Chords.—The maximum stresses in the chord members due to rolling load occur when the rolling load covers the whole span. The minimum stresses are zero since no reversal occurs.

Panel rolling load $= 16 \times 2 = 32$ tons.

Fig. 190 shows the stresses caused by a load of 32 tons imposed on each lower panel point. Only the chord stresses from this figure are transferred to the Stress Sheet, since the diagonal and vertical stresses are not the maximum stresses for those members.

Maximum and Minimum Rolling Load Stresses in Diagonals.—The negative and positive totals in the foregoing table apply to a panel load of one ton. The actual panel rolling load is 32 tons. Hence,—

Max. stress in 1B $= -3\frac{1}{2} \times 32 \times \sqrt{2} = -158{\cdot}4$ tons.
Min. „ 1B $= 0$
Max. „ 2C $= -2\frac{5}{8} \times 32 \times \sqrt{2} = -118{\cdot}8$ „
Min. „ 2C $= +\frac{1}{8} \times 32 \times \sqrt{2} = +5{\cdot}7$ „
Max. „ 3D $= -1\frac{7}{8} \times 32 \times \sqrt{2} = -84{\cdot}9$ „
Min. „ 3D $= +\frac{3}{8} \times 32 \times \sqrt{2} = +17{\cdot}0$ „
Max. „ 4E $= -1\frac{1}{4} \times 32 \times \sqrt{2} = -56{\cdot}6$ „
Min. „ 4E $= +\frac{3}{4} \times 32 \times \sqrt{2} = +34{\cdot}0$ „

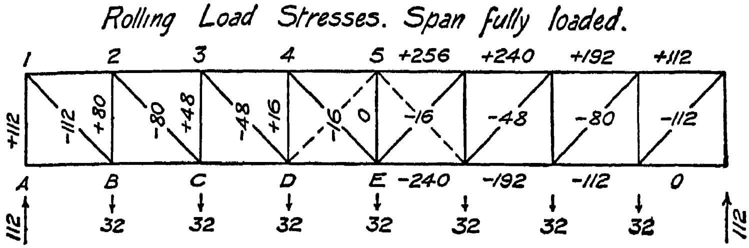


FIG. 190.

Maximum and Minimum Rolling Load Stresses in Verticals.

Max. stress in 1A $= +3\frac{1}{2} \times 32 = +112$ tons.
Min. „ 1A $= 0$
Max. „ 2B $= +2\frac{5}{8} \times 32 = +84$ „
Min. „ 2B $= -\frac{1}{8} \times 32 = -4$ „
Max. „ 3C $= +1\frac{7}{8} \times 32 = +60$ „
Min. „ 3C $= -\frac{3}{8} \times 32 = -12$ „
Max. „ 4D $= +1\frac{1}{4} \times 32 = +40$ „
Min. „ 4D $= -\frac{3}{4} \times 32 = -24$ „

Stresses in Counterbrace 5D and Central Vertical 5E.—The maximum and minimum stresses due to the combined dead and rolling loads are obtained by taking the algebraic sum of *first* the dead load stress and maximum rolling load stress, and *secondly* the dead load stress and minimum rolling load stress for each member. The resulting stresses are entered with their proper sign in the two end columns of the Stress Sheet. It is now evident that the stresses in the chord members fluctuate between higher and lower values of the *same* kind of stress. In diagonals 1B, 2C, and 3D the stresses fluctuate between higher and lower values of *tension*. In diagonal 4E the maximum stress is 72·2 tons of tension, whilst under the worst position of the rolling load for creating compression in this member, the incidental compression of 34 tons is more than sufficient to neutralise the 15·6 tons of permanent tension caused by the dead load. Hence during the passage of the rolling load from A up to and fully loading panel point D, the stress in 4E first rapidly diminishes from 15·6 tons of tension to zero and then changes to compression which rapidly increases to 18·4 tons. An instant later panel point E also becomes loaded when the rolling load stress in 4E is seen to be (from Table,

STRESS SHEET.

Member	Dead Load.	Rolling Load.		Combined Load.	
		Max.	Min	Max.	Min.
1—2	+ 77	+ 112	0	+ 189	+ 77
2—3	+ 132	+ 192	0	+ 324	+ 132
3—4	+ 165	+ 240	0	+ 405	+ 165
4—5	+ 176	+ 256	0	+ 432	+ 176
A—B	0	0	0	0	0
B—C	− 77	− 112	0	− 189	− 77
C—D	− 132	− 192	0	− 324	− 132
D—E	− 165	− 240	0	− 405	− 165
1—B	− 108·9	− 158·4	0	− 267·3	− 108·9
2—C	− 77·8	− 118·8	+ 5·7	− 196·6	− 72·1
3—D	− 46·7	− 84·9	+ 17·0	− 131·6	− 29·7
4—E	− 15·6	− 56·6	+ 34·0	− 72·2	+ 18·4 0 if c.b.
5—D	0	− 18·4	0	− 18·4	0
1—A	+ 80	+ 112	0	+ 192	+ 80
2—B	+ 60	+ 84	− 4	+ 144	+ 56
3—C	+ 38	+ 60	− 12	+ 98	+ 26
4—D	+ 16	+ 40	− 24	+ 56	− 8 0 if c.b.
5—E	+ 5	+ 13	0	+ 18	+ 5

Fig. 187) $= 32 \times (+\frac{1}{8} + \frac{2}{8} + \frac{3}{8} - \frac{4}{8}) \times \sqrt{2} = +11{\cdot}3$ tons, that is, still compression. But this compression of 11·3 tons is more than neutralised by the permanent tension of 15·6 tons due to the dead load, so that during the short interval of time occupied by the head of the load in reaching some point just beyond E the member 4E is restored to a state of tension. It will be realised that these fluctuations of stress in 4E take place very rapidly. Whilst the head of the load travels from A to just beyond E, say a distance of 70 feet, the stress in 4E passes through the following changes.

Head of advancing load.	Stress in 4E
At A	15·6 tons tension.
Between B and C	10·0 ,, ,,
,, C ,, D	1·3 ,, compression.
,, D ,, E	18·4 ,, ,,
,, E ,, F	4·3 ,, tension.

After which the tension in 4E continues to increase until the load covers the whole span. So long as the load covers the span, the stress in 4E remains sensibly constant, but again increases as the tail end of the load passes from X to panel D—E, at which instant the maximum tension of 72·2 tons occurs. Finally, as the tail end rolls off from panel D—E to K the stress falls from 72·2 tons tension to 15·6 tons tension.

For a load travelling at 60 miles an hour, the time period during which the stress in 4E changes from 15·6 tons tension to 18·4 tons compression and back again to 4·3 tons tension would be only about seven-ninths of a second, whilst the time occupied by the complete cycle of fluctuations of stress in the case of a train 500 feet long would be only $\frac{500 + 128}{88} = 7{\cdot}14$ seconds.

The actual stresses caused by the load travelling at high speed would moreover be greater than those indicated, which are merely the result of considering several individual *static* positions of the load.[1]

The diagonal 4E may either be built as a compression member capable of resisting both the tensile and compressive stresses indicated, or if consisting of two flat bars incapable of resisting compression, a counterbrace 5D will be inserted capable of resisting 18·4 tons of tension. The stress in the main tie 4E then fluctuates between 72·2 tons tension and zero, and the stress in the counterbrace between zero and 18·4 tons tension. The central vertical 5E which under the dead load takes 5 tons compression will, at the instant either counterbrace is in full extension, be required to resist also the downward vertical pull of the counterbalance on its upper end. Hence maximum rolling load compression in

$$5E = \text{vertical component of maximum stress in } 5D$$
$$= 18{\cdot}4 \div \sqrt{2} = 13{\cdot}01 \text{ tons.}$$

The reversal of stress in 4D from + 56 tons to − 8 tons is not of great moment, since this member is necessarily a strut, and will therefore satisfactorily resist a small amount of tension, whilst if a counterbrace be inserted this reversal will not take place.

Bowstring Girders.—The bowstring girder consists of one curved and one straight boom with verticals and diagonals. In the upright type, Fig. 162, No. 12, the curved boom is in compression and the horizontal boom in tension. These stresses are reversed if the girder be inverted as in Fig. 162, No. 13. The upright form is employed for through spans and the inverted form for deck spans. If the outline of the girder be made parabolic, then the horizontal stress in the curved boom is uniform throughout under the action of a uniformly distributed load. In Fig. 191, the parabola ACB represents the B.M. diagram for a uniformly distributed load. If the depth *cd* of the girder be everywhere proportionate to the ordinates of the parabola ACB, then the B M. CD, at any point, divided by the corresponding girder depth *cd*, will give a constant quotient for the horizontal flange stress whatever section be considered. This being the case, there is evidently no stress in the diagonal bracing, since the horizontal boom stress does not increase from panel to panel and the diagonals do not therefore apply any increment of horizontal stress to the booms. The diagonals being in a state of no stress, it is obvious that the whole of the shearing force is borne by the curved boom. The verticals under a uniform load simply act as suspenders for the panel loads applied at their lower ends and are in *tension*. The horizontal boom *adb* resists the outward thrust of the ends of the curved boom and is subject to a tensile stress equal in amount to the constant horizontal compression in the curved boom. These conditions of stress will obtain very closely in

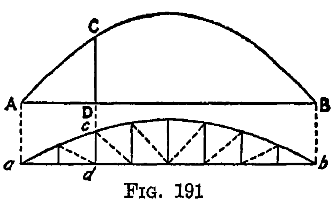


FIG. 191

[1] See Impact Formulæ, p 49

girders of circular outline provided the circular curve does not deviate far from the parabola. In girders the outlines of which are not parabolic there will, however, be small stresses in the bracing, the magnitude of which will vary with the extent of deviation of the outline from the parabola. Under an unsymmetrical system of loads, for which most practical girders must be designed, the diagonals will suffer considerable tensile or compressive stresses according to their direction of slope, as will be seen from the following examples.

Stresses in Parabolic Bowstring Girder under Uniform Load.—Fig. 192, represents a bowstring girder of 80 feet span having 8 panels of 10 feet width, a central depth of 10 feet, and carrying 2 tons and 8 tons respectively at each upper and lower joint. The depths of the girder at B, C, and D are respectively 4·375, 7·5, and 9·375 feet, being the correct values of the parabolic ordinates. The reaction at A is 35 tons. Then

$$\text{B.M. at B} = 35 \times 10 = 350 \text{ ft.-tons.}$$
$$\text{,,} \quad \text{C} = 35 \times 20 - 10 \times 10 = 600 \text{ ft.-tons.}$$
$$\text{,,} \quad \text{D} = 35 \times 30 - 10(20 + 10) = 750 \text{ ft.-tons.}$$
$$\text{,,} \quad \text{E} = 35 \times 40 - 10(30 + 20 + 10) = 800 \text{ ft.-tons.}$$

$$\therefore \text{Horizontal boom stress in AB} = \frac{350}{4\cdot375} = 80 \text{ tons.}$$
$$\text{,,} \quad \text{,,} \quad \text{BC} = \frac{600}{7\cdot5} = \text{,,}$$
$$\text{,,} \quad \text{,,} \quad \text{CD} = \frac{750}{9\cdot375} = \text{,,}$$
$$\text{,,} \quad \text{,,} \quad \text{DE} = \frac{800}{10} = \text{,,}$$

Since the horizontal compression in each segment of the upper boom is 80 tons, no increment of horizontal stress is applied by the diagonals, whence their stress is zero. The outward thrust of 80 tons in AB creates a tension of 80 tons in AF and this stress is passed on unaltered throughout the lower boom, since the diagonals do not affect it. The stress in each vertical is 8 tons of tension. Finally, the inclined lengths of AB, BC, CD and DE are respectively 10·9, 10·4, 10·2 and 10·02 feet, whence the direct stresses are

$$\text{in AB} = 80 \times \frac{10\cdot9}{10} = 87\cdot2 \text{ tons.}$$
$$\text{,, BC} = 80 \times \frac{10\cdot4}{10} = 83\cdot2 \quad \text{,,}$$
$$\text{,, CD} = 80 \times \frac{10\cdot2}{10} = 81\cdot6 \quad \text{,,}$$
and
$$\text{,, DE} = 80 \times \frac{10\cdot02}{10} = 80\cdot16 \text{ ,,}$$

It will be noticed the greatest direct stress occurs in AB near the support, whereas in parallel girders the greatest boom stress is in the central panels, and further, that the direct stress in the curved boom

being only slightly greater near the ends than at the centre, the cross-section adopted for AB may be used throughout the boom with little

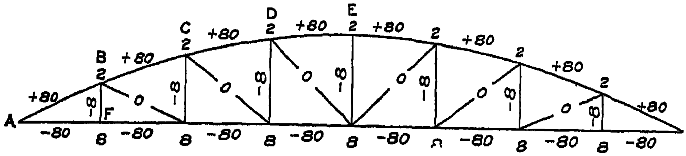


FIG. 192.

sacrifice of economy, whilst the practical advantage is considerable. For the same reason, the parabolic girder is a more economical type than the parallel girder, span for span. In a large span, the dead weight of the girder creates a considerable proportion of the total B.M. In the parallel type the heaviest portions of the booms are near the centre, that is, in the most disadvantageous position for creating B.M., and therefore stress, in the girder. In the bowstring girder, the weight of the booms being practically uniformly distributed, or actually a little greater towards the supports, the B.M. and stress due to dead weight is relatively less. The bracing of bowstring girders is also usually lighter than that of parallel types.

Stresses in Parabolic Bowstring Girder with Unsymmetrical Load.—In Fig. 193, the girder of the previous example is supposed loaded

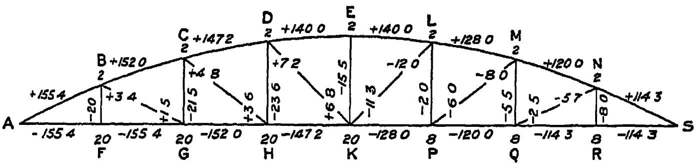


FIG. 193.

with an additional 12 tons at panel points F, G, H and K.

The reaction at $A = \frac{1}{2} \times 14$ (upper loads) $+ 20(\frac{7}{8} + \frac{6}{8} + \frac{5}{8} + \frac{4}{8}) + 8(\frac{3}{8} + \frac{2}{8} + \frac{1}{8}) = 68$ tons, and at $S = 118 - 68 = 50$ tons.

B.M. at $F = 68 \times 10 = 680$ ft.-tons.
,, $G = 68 \times 20 - 22 \times 10 = 1140$ ft.-tons.
,, $H = 68 \times 30 - 22(20 + 10) = 1380$ ft.-tons.
,, $K = 68 \times 40 - 22(30 + 20 + 10) = 1400$ ft.-tons.
,, $P = 50 \times 30 - 10(20 + 10) = 1200$ ft.-tons.
,, $Q = 50 \times 20 - 10 \times 10 = 900$ ft.-tons.
,, $R = 50 \times 10 = 500$ ft.-tons.

Dividing these bending moments by the depths at the corresponding sections, the horizontal boom stresses marked in the figure are obtained. At B the horizontal stress of 155·4 tons in AB is not balanced by that of 152 tons in BC, therefore the diagonal BG must exert a horizontal *thrust* of $155{\cdot}4 - 152 = 3{\cdot}4$ tons. Similarly CH and DK are found to be in compression to the extent of 4·8 and 7·2 tons respectively. The

horizontal thrust of 128 tons in LM increases to 140 tons in EL, so that the increment of $140 - 128 = 12$ tons must have been applied by a horizontal tension of 12 tons in diagonal LK. Similarly 8·0 and 5·7 tons of tension respectively are found to exist in MP and NQ. These horizontal stresses are converted into the corresponding vertical stresses as usual, by multiplying them by the ratio, $\dfrac{\text{vertical height}}{\text{horizontal breadth}}$ of each diagonal.

The stresses in the vertical members then follow. BF obviously acts as a suspender for the 20 tons load at F. At G, the load of 20 tons + the vertical downward thrust of 1·5 tons in BG, together produce a tension of 21·5 tons in CG. At H, $20 + 3{\cdot}6 = 23{\cdot}6$ tons tension in DH. At K the downward forces are 6·8 tons vertical thrust in DK + 20 tons load, making 26·8 tons, which is partly resisted by the vertical tension of 11·3 tons in LK, leaving $26{\cdot}8 - 11{\cdot}3 = 15{\cdot}5$ tons tension, in EK. Similarly LP takes 2 tons and MQ, 5·5 tons of tension, whilst NR simply suspends the 8 tons load at R. The direct stresses in the diagonals and upper boom segments are obtained as in previous examples and are as follow :—

Upper boom		Diagonals	
AB = 169·4	tons, compn.	BG = 3·7	tons, compn.
BC = 158·8	„	CH = 6·0	„
CD = 150·1	„	DK = 9·9	„
DE = 140·3	„	LK = 16·5	tons, tension.
EL = 140·3	„	MP = 10·0	„
LM = 130·5	„	NQ = 6·2	„
MN = 124·8	„		
NS = 124·6	„		

With a single system of diagonals inclined as in Fig. 193, each diagonal will be subject to both tension and compression under varying positions of the unsymmetrical portion of the load. If the diagonals be reversed in direction, then for the same arrangement of loads as in Fig 193, the horizontal and vertical stresses will be as indicated in Fig. 194.

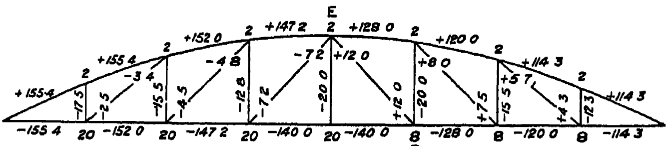


FIG. 194.

Here, again, the diagonals may be subject to either tension or compression according to the position of the unsymmetrical load.

The B.M. at each section is as before and the horizontal stresses in the boom segments and diagonals readily follow.

Stresses in Bowstring Girder with Crossed Diagonals in every Panel.—A third arrangement, and one frequently adopted, is to cross-brace every panel with flat tie-bars capable of resisting tension only. Assuming the same loads and dimensions as in the two previous

cases, the horizontal and vertical stresses would then be as indicated in Fig. 195.

In this case the diagonals being incapable of resisting compression, become lax under any tendency of the load to create compressive stress in them, whilst those diagonals inclined in the opposite direction come into action for resisting the tension. Thus, in Fig. 195, the dotted diagonals which *would* suffer compression if of suitable section to resist it, are in a state of no stress, whilst the full line diagonals are put in tension under the disposition of loads considered. The dotted diagonals,

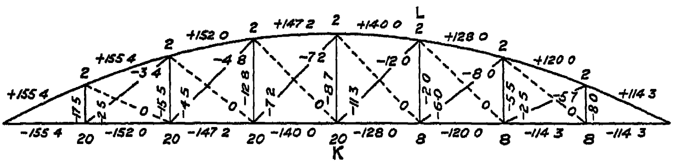


Fig. 195.

of course, come into action when the right-hand portion of the girder is the more heavily loaded. It is a mistake to cross-brace every panel with two diagonals capable of resisting compression since the bracing is then *redundant*, and an indeterminate amount of compression exists in one diagonal and an indeterminate amount of tension in the other, making it impossible to accurately compute the stresses.

Effect of Uniform Rolling Load on Parabolic Bowstring Girder.—In each of the three preceding cases, the first four panel points from the left-hand support were supposed loaded with 12 tons over and above the symmetrical dead load in Fig. 192. This additional load may be regarded as a uniform rolling load which has advanced from the left abutment up to the central panel. The maximum horizontal stress in the diagonals occurs in KL, Figs. 193 and 195, or in EP, Fig 194, that is, in the diagonal of the panel immediately in advance of the rolling load, and is equal to 12 tons of tension or compression according to the inclination of the diagonal. If the load be supposed to advance another panel so that P now becomes loaded with an additional 12 tons, and a similar analysis of the stresses be made, the diagonal MP will be found to be the most heavily stressed, and further, the amount of the horizontal stress in MP will again be 12 tons. This result is peculiar to the bowstring girder of parabolic outline and may be stated as follows. Under the action of a uniform rolling load advancing from one abutment, the maximum horizontal stress in any diagonal occurs when the head of the load reaches the panel in which the diagonal is situated, and the amount of the maximum horizontal stress is the same for every diagonal in the girder. When the rolling load covers the whole span, the loading is again symmetrical and the B.M. at the centre of the span due to 12 tons of *rolling load* on each lower joint, is then

$$= 42 \times 40 - 12(30 + 20 + 10) = 960 \text{ ft.-tons.}$$

Dividing this B.M. by the central depth of 10 ft., the horizontal boom stress at the centre, due to *rolling* load *covering* the span $= \frac{960}{10} = 96$ tons. It was shown above that the maximum horizontal

stress in each diagonal, as the rolling load reached it, was 12 tons. The number of panels is 8 and $\frac{96}{8} = 12$; or stated generally, the maximum horizontal stress in each diagonal

= Maximum horizontal boom stress when rolling load covers the whole span $\div$ number of panels.

This relation furnishes a ready method of calculating the maximum stresses in the diagonals due to the passage of a uniform rolling load. A strictly mathematical investigation gives the maximum horizontal stress in each diagonal

$$= \frac{\text{Maximum horizontal boom stress}}{\text{Number of panels} + 1}$$

The discrepancy is due to the fact that it is impossible to fully load any one panel point with a uniform load without, at the same instant, *partially* loading the next panel point in advance. In the above analysis the assumption is made that the panel point at the head of the advancing load is fully loaded, whilst the next panel point in advance is unloaded. This could only be effected by a *uniform* load extending from, say, A to B in Fig. 196, together with a *concentrated* load *at* B equal to half the panel length of uniform load, in which case B is fully loaded, whilst C is unloaded. The diagonal BD then suffers its maximum horizontal stress. As this represents more closely the practical condition of loading, especially in the case of train loads on bridges, where the uniform train load is headed by the heavy concentrated axle loads of the engine, the results are more nearly correct than if a perfectly uniform load be assumed throughout.

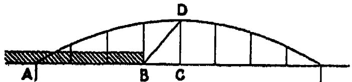


FIG. 196.

It is common practice to construct the upper boom with an actually curved outline. In designing the cross-section of the various segments, it should be noted there will be both bending and direct stress to provide for. Thus in Fig. 197 the calculated direct stress P acts along the straight dotted line AB. If the boom be curved, the B.M. at its centre = P $\times$ *d*, the case being similar to that of a deflected column in compression. The intensity of compressive stress is thereby augmented on the under side of the boom, and slightly relieved on the upper side. The weight of the boom segment itself sets up a small amount of B.M acting contrarily to that of P*d*, and so tends to equalize the stresses at upper and lower faces. Although the employment of curved booms is theoretically disadvantageous, the practical advantage resulting from greater facility of construction, especially in riveted girders, more than compensates.

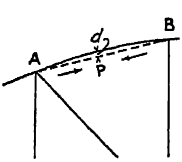


FIG 197

Stresses in Inclined Girders.—Girders in inclined positions are frequently employed for footbridges, supports for inclined conveyers and hoists, and for communication bridges between a quayside and floating landing stage. In the latter case the inclination of the girders may vary between wide limits due to a considerable tidal range.

Unless the inclination is appreciable the difference between the stresses in the inclined position and those obtaining when the girder is horizontal will not be serious, and most bridge girders on inclines usually prevailing on railways may safely be designed as horizontal spans. On rack railways the incline may be as steep as 1 in 4. The stresses in bascule and rolling lift bridges are subject to wide variations between the horizontal and raised positions.

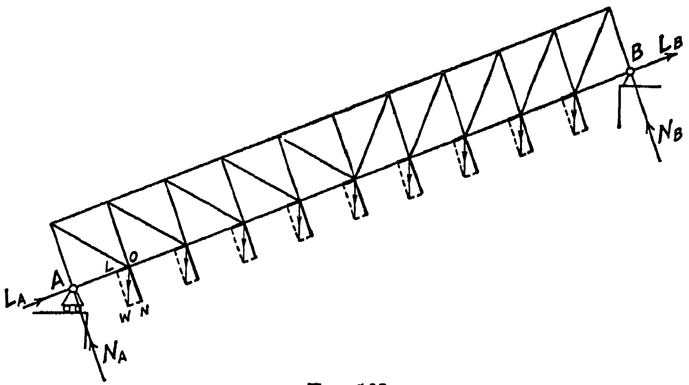


FIG. 198.

In Fig. 198 suppose the inclined girder AB loaded at each lower joint with a vertical load OW. Each vertical load may be resolved into a normal component ON and a longitudinal component OL. The normal components will create stresses in the members due to traverse bending action on the inclined span AB, and will set up normal reactions N_A and N_B, the values of which will be determined by the magnitude and distribution of the normal components of load. The longitudinal components will modify the stresses in the lower chord (in this case) and will set up longitudinal reactions L_A and L_B, the values of which will depend upon the manner in which the girder ends are supported at A and B.

If both A and B are fixed, say by pin bearings on bolted down shoes, reactions L_A and L_B will be indeterminable since a minute difference of fit at A or B may bring the whole of the longitudinal loading as a thrust against A or as a pull against B. Moreover, the question of temperature stresses would be introduced. One end of the girder should be allowed to move freely in some pre-determined direction, so that the longitudinal reactions may be defined and made automatically adjustable.

Fig. 199 shows the possible methods of support practically available. In Fig. 199 *a*, the lower end A has a fixed pin bearing and the upper end B is free to slide or roll in a direction parallel to the girder. The reaction R_B is normal to the girder and R_A is drawn through O, the intersection of W and R_B. The magnitudes of R_A and R_B are obtainable by the triangle of forces and L_A the longitudinal component of $R_A = W_L$. The whole longitudinal loading W_L comes on the lower bearing A as a thrust. There is no longitudinal reaction at B, and

there will be a uniform augmentation of *pressure* through the lower chord from B to A, which will *reduce* the tension due to the normal loading to a greater extent in the lower than in the upper panels. It is here assumed that the load is applied at the lower panel points or on the lower flange in the case of a plate girder.

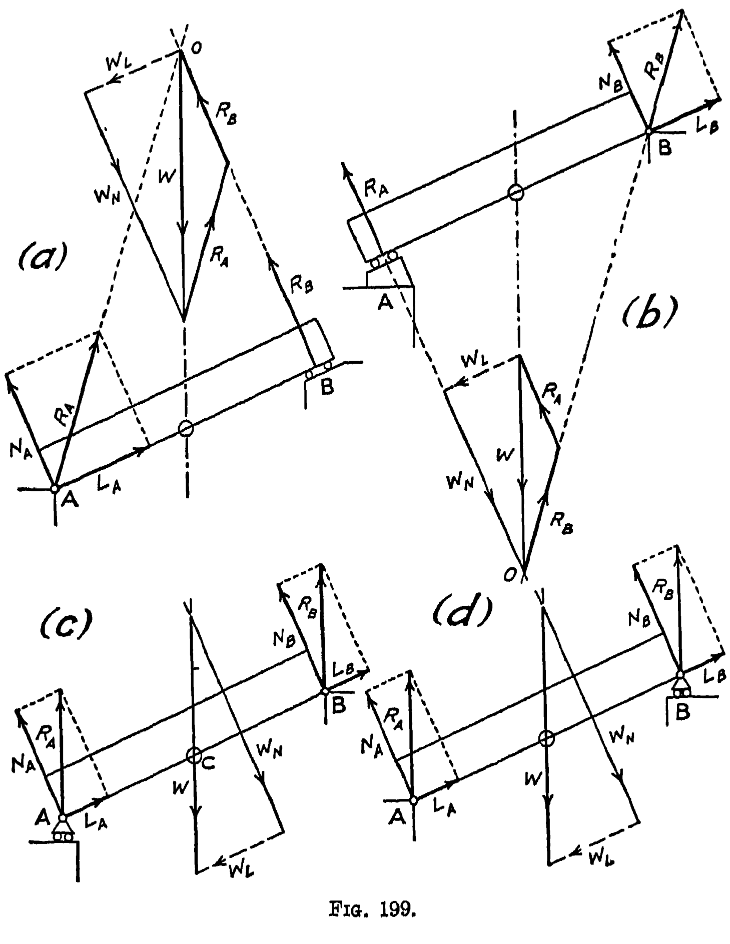


FIG. 199.

In Fig. 199 *b*, A is free to move longitudinally and B is a fixed pin bearing. R_A is normal and the longitudinal reaction L_B balances the whole longitudinal loading W_L which comes on B as a pull. There is thus an augmentation of *tension* throughout the lower chord from A to B which will *increase* the tension due to the normal loading to a greater extent in the upper than in the lower panels. These effects will be appreciated in studying Examples 31B, C, and D.

In Fig. 199, *c*, B is a fixed pin bearing and A is free to slide or roll horizontally. R_A is therefore vertical. N_A and L_A are respectively the

normal and longitudinal components of R_A. Under symmetrical or uniform loading, W passes through the centre of span C and $N_A = N_B = \frac{1}{2}W_N$, whence $L_A = L_B = \frac{1}{2}W_L$ and R_B is also vertical and equal to R_A and from the geometry of the figure $R_A = R_B = \frac{1}{2}W$.

In Fig. 199, *d*, A is fixed and B is free to move horizontally. The same relations hold for the reactions. The fixed bearings are preferably placed on the immovable support should the other end be on a floating stage.

Fig. 200 shows the case for unsymmetrical loading. The resultant load W intersects the span line at P, whence $N_A : N_B :: PB : PA$. R_A and R_B are vertical and therefore $L_A : L_B :: PB : PA$.

EXAMPLE 31B—*In Fig.* 201 *AM represents one main girder of an inclined bridge supported as in Fig.* 199, *c. The girder is hinged at M to fixed bearings and is supported at A on saddles free to move horizontally*

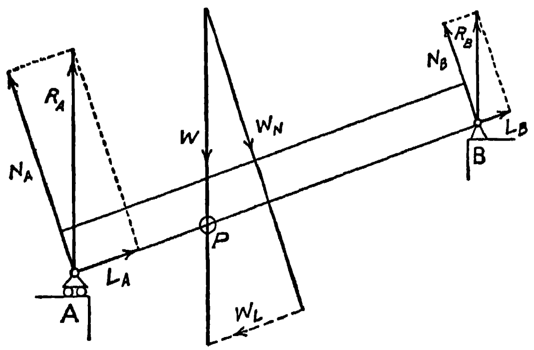


FIG. 200.

on a floating pontoon. In its lowest position it is inclined 1 *vertically to* 4 *horizontally. Inclined span* 100 *feet. Depth* 12 *ft* 6 *ins. Flooring and live load supported on lower chord. Panel load* 10 *tons. Required the stresses in the members.*

It will be convenient to suppose each lower joint loaded with 4 tons acting normally to the girder and 1 ton acting longitudinally. This will correspond to a *vertical* load at each joint of $\sqrt{17}$ tons. The resulting stresses will then require multiplying by $10 \div \sqrt{17} = 2{\cdot}425$ to give those due to a vertical panel load of 10 tons. The normal reactions at A and M each equal 18 tons, and the stresses due to these and the normal loads of 4 tons are written down as for a horizontal girder, leaving out for the moment the lower chord stresses.

The longitudinal reactions for this method of support will be 4·5 tons at M acting away from the girder and 4·5 tons at A acting towards the girder. The lower chord stresses may now be summed up. The reaction 4·5 tons at M creates a tension of 4·5 tons in LM. At L there is a longitudinal pull to the right of 14·4 tons in LY and 4·5 tons in LM = 18·9 tons. The 1 ton of longitudinal load at L acting to the left balances 1 ton of this, leaving 17·9 tons for the

tension in KL. A similar reasoning at K, H, etc., deduces the stresses as figured. The direct stresses in the diagonals are obtained by multiplying the normal components by $\frac{OB}{OA} = \frac{\sqrt{41}}{5} = 1{\cdot}28$. Thus, direct stress in OB $= -18 \times 1{\cdot}28 = -23{\cdot}04$ tons. The stresses shown in Fig. 201 are those due to a vertical load of $\sqrt{17}$ tons per panel. Multiplying them all by $\frac{10}{\sqrt{17}} = 2{\cdot}425$, the stresses due to a panel load of 10 tons are obtained. These are figured in the table on p. 260 and side by side with them are given the stresses for the same girder under the same loading when lifted into a horizontal position. The important differences occur, of course, in the lower chord. If the load were carried on the upper joints the upper chord segments would be most affected by the inclination. Also in that case the reactions would be slightly unequal, since the resultant load line would be nearer A than M.

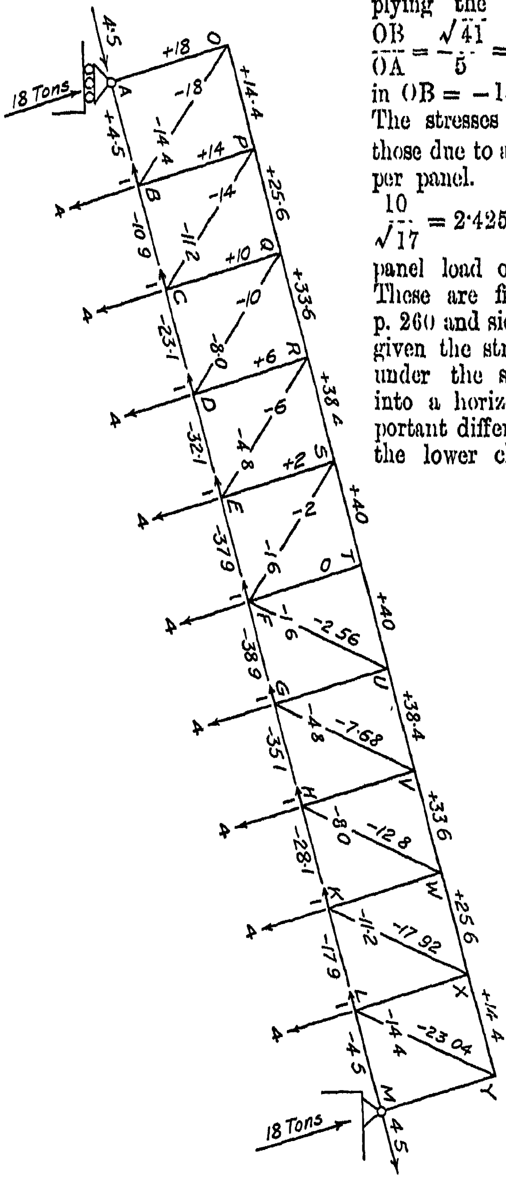


FIG. 201.

EXAMPLE 31C *Case* 1. —*Fig.* 202 *shows the outline of a bascule bridge girder raised to an inclination of* 3 *vertical to* 1 *horizontal. Suppose each lower chord joint loaded with* 1 *ton acting normally and* 3 *tons acting longitudinally to the girder. This corresponds with a vertical panel load of* $\sqrt{10}$ *tons. The stresses due to the normal loads are written down in the usual manner working from D to B. For stress in BC take moments about A.*

Longitudinal component of stress in CB $\times$ CA $= 1 \times 20\ (6 + 5 + 4 + 3 + 2 + 1)$. CA $= 40$ ft. $\therefore$ Longitudinal component of stress in

CB = 10·5 tons tension, whence longitudinal component in BA = 4·5 tons to balance the 15 tons combined pull in BE and BF. The longitudinal stresses in upper chord readily follow. The compression in the lower chord is cumulative from D to A. Thus—

Compn. in DG = 1 (from DK) + 3 (longitudinal load) = 4 tons.
„ GH = 4 (from DG) + 2 (from GL) + 3 (load) = 9 tons.
„ HM = 9 (from GH) + 3 (from HN) + 3 (load) = 15 tons, etc.

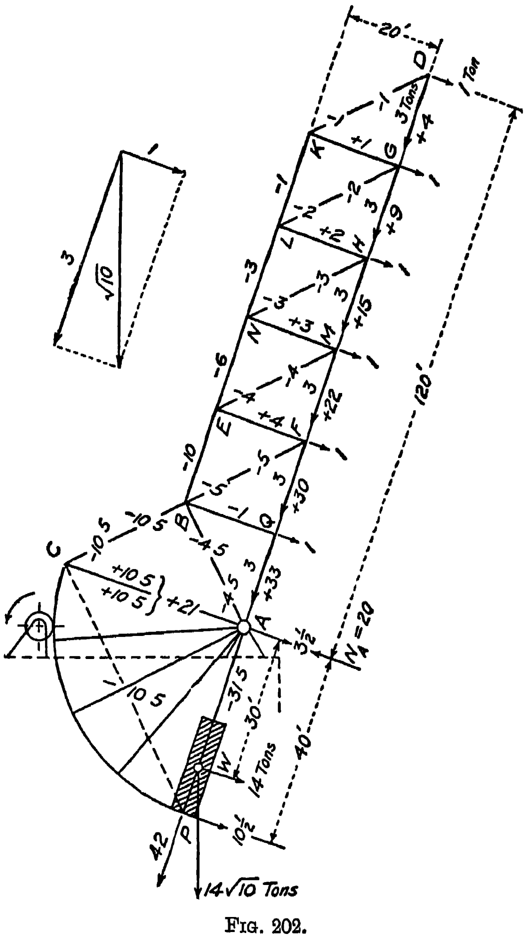


Fig. 202.

Stress in CA.—Normal component of balance weight applied 30 ft. from A × 30 = 1 × 20 (6 + 5 + 4 + 3 + 2 + 1);

whence, Normal component of balance weight = 14 tons,
and Longitudinal „ „ „ = 14 × 3 = 42 tons.

Actual balance weight = 14 × $\sqrt{10}$ tons.

The longitudinal component acts through the trunnions at A and does not influence the stress in CA. Consider the normal component. The 14 tons at W is equivalent to $\frac{30}{40} \times 14 = 10\cdot5$ tons at P and $\frac{10}{40} \times 14 = 3\cdot5$ tons at A.

The 10·5 tons at P would create a normal component of 10·5 tons

Member.	Girder inclined 1 to 4.	Girder horizontal.
AB	+ 10·9	0
BC	− 26·4	− 36·0
CD	− 56·0	− 64·0
DE	− 77·9	− 84·0
EF	− 91·9	− 96·0
FG	− 94·4	− 96·0
GH	− 85·2	− 84·0
HK	− 68·1	− 64 0
KL	− 43 4	− 36·0
LM	− 10·9	0
OP and XY	+ 34·9	+ 36·0
PQ ,, WX	+ 62 1	+ 64·0
QR ,, VW	+ 81·5	+ 84·0
RS ,, UV	+ 93 2	+ 96·0
ST ,, TU	+ 97 0	+ 100 0

Member.	Girder inclined 1 to 4.	Girder horizontal.
OA and YM	+ 48·7	+ 45·0
PB ,, XL	+ 34·0	+ 35·0
QC ,, WK	+ 24 3	+ 25·0
RD ,, VH	+ 14 6	+ 15·0
SE ,, UG	+ 4·9	+ 5·0
TF	0	+ 0
OB and YL	− 55·8	− 57·6
PC ,, XK	− 43·4	− 44·8
QD ,, WH	− 31·0	− 32·0
RE ,, VG	− 18·6	− 19·2
SF ,, UF	− 6·2	− 6·4

This Table refers to Example 31B.

tension in a direct tie PC if inserted. However the quadrant CAP may be constructed, this stress will be transmitted through it as shear and will create a normal pull on CA of 10·5 tons, which together with the normal component of 10·5 tons in CB (CB being inclined 45°) causes a compression of 21 tons in CA.

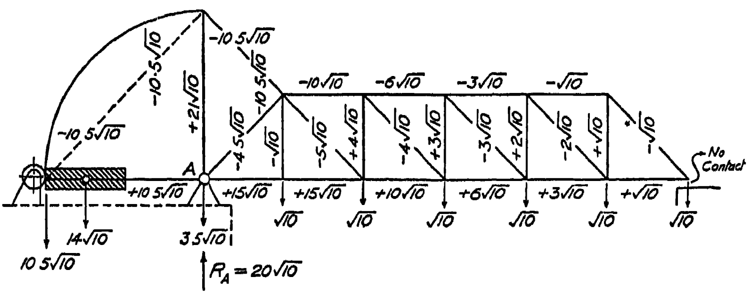


FIG. 203.

Stress in PA.—The direct stress in PA is given by the difference between the longitudinal component of the balance weight, 42 tons, which in this position tends to stretch PA and the longitudinal component of 10·5 tons in the virtual tie PC which tends to compress PA.

Hence, longitudinal stress in PA $= 42 - 10\cdot5 = 31\cdot5$ tons tension. PA is also subject to bending stress due to the balance weight being distributed over part of its length.

Pressure on Trunnion Bearings at A.—Normal pressure = 21 tons pressure from CA + $3\frac{1}{2}$ tons (share of balance weight) − 4·5 tons (normal upward pull in AB) = 20 tons. This checks with $(6 \times 1) + 14 = 20$ tons total normal load on girder. Longitudinal pressure = 33 tons from QA − 4·5 tons (longitudinal upward pull in AB) + 31·5 tons (downward pull in AP) = 60 tons. This checks with $(6 \times 3) + 42 = 60$ tons total longitudinal load on girder. Vertical *resultant* pressure on bearings $= 20 \times \sqrt{10} = 63{\cdot}25$ tons. If there should be a panel load *at* A it should be added, but such load would not affect the stresses in the girder.

Case 2.—Fig. 203 shows the stresses in the girder when just attaining the horizontal position but before the end D is actually in contact with the abutment.

The girder is now acting as a horizontal cantilever. The evaluation of the stresses will be readily made if the reasoning of the last case has been carefully followed.

Case 3.—Fig. 204 shows the stresses in the girder when the end D is resting on the abutment and the stress in CB relieved by blocking up

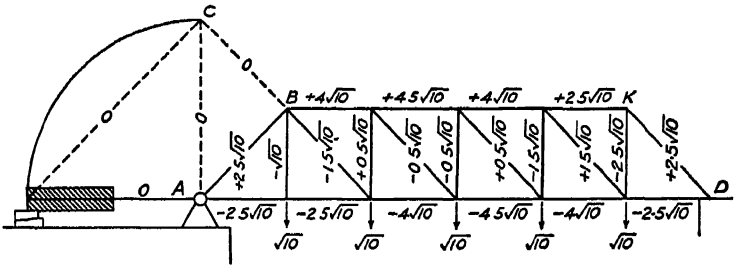


Fig. 204.

the quadrant and balance weight. ABKD is then acting as a simply supported span carrying dead load.

The stresses in the above three cases are the dead load stresses for a panel dead load of $\sqrt{10}$ tons. For any other panel load w tons, these stresses will be multiplied by $(w \div \sqrt{10})$. If additional live load b imposed at each panel point, the live load stresses will be those set up in the simply supported girder ABKD and the resultant stresses will be the algebraic sum of the live load stresses and the dead load stresses of Case 2, if the balance weight is freely acting on the tail end, or the sum of the live load stresses and the dead load stresses of Case 3 if the balance arm is blocked up until the stress in CB is zero.

The resultant dead load in the above cases passes through the trunnion axis, so that when lowered with balance weight free the girder is still delicately balanced and the minimum of power is required to rotate it. The line of action of the resultant dead load may be arranged to fall a short distance to the right of A, so that a small pressure is exerted on D when the bridge is lowered. This increases the power required for operating but prevents chattering as the live load rolls on and off at D.

EXAMPLE 81c.—*Fig. 205 represents the outline of an inclined elevator girder for raising material from the boot C and delivering to a hopper or conveyer at D. Inclination 2 vertical to 3 horizontal. The girder is supported on fixed pin bearings at B and horizontally rolling or sliding bearings at A.*

Suppose each lower joint, Fig. 206, loaded with a normal load of 3 tons and a longitudinal load of 2 tons, corresponding with a vertical panel load of $\sqrt{13}$ tons. Resultant load $W = 14\sqrt{13}$ tons. The line of action of W divides the distance between supports in the ratio of 9 to 7. Hence $R_A = \frac{7}{16} W = 22{\cdot}05$ tons and $R_B = \frac{9}{16} W = 28{\cdot}35$ tons. Total normal load $= 14 \times 3 = 42$ tons. Normal reaction at $A = \frac{7}{16} \times 42 = 18\frac{3}{8}$ tons. Normal reaction at $B = \frac{9}{16} \times 42 = 23\frac{5}{8}$ tons. Total

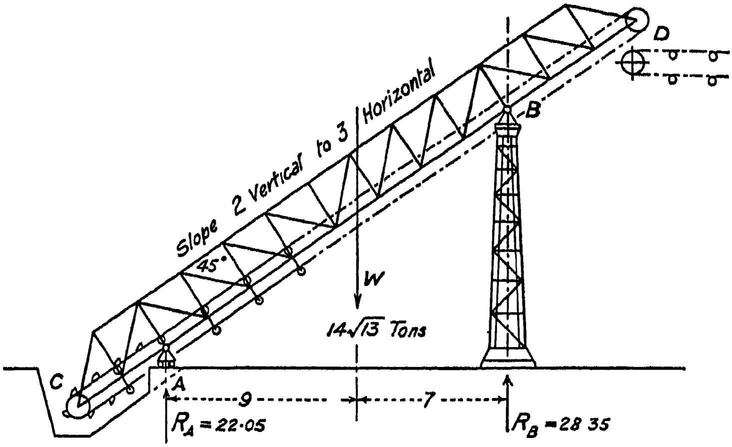


FIG. 205.

longitudinal load $= 14 \times 2 = 28$ tons. Longitudinal reaction at $A = \frac{7}{16} \times 28 = 12\frac{1}{4}$ tons and longitudinal reaction at $B = \frac{9}{16} \times 28 = 15\frac{3}{4}$ tons.

In Fig. 206 the normal stress components due to the normal loading are first written down and converted into the corresponding longitudinal components, which for diagonals inclined at 45° are equal. From these the chord stresses are summed up in the usual manner. Fig. 206 shows the resulting stresses. For any other panel load w tons these stresses will be multiplied by $(w \div \sqrt{13})$. In designing a girder of this type a reasonable allowance should be made for the longitudinal tension in the elevator chains or belt which will apply additional compression throughout the loaded chord.

Design for Lattice Crane Girder.—The preceding methods will now be applied for obtaining the maxima stresses in the members of the lattice crane girder shown in outline in Fig. 207, and in detail in Fig. 208. Lattice girders for travelling cranes are usually of the Warren type with intermediate verticals for reducing the panel spans

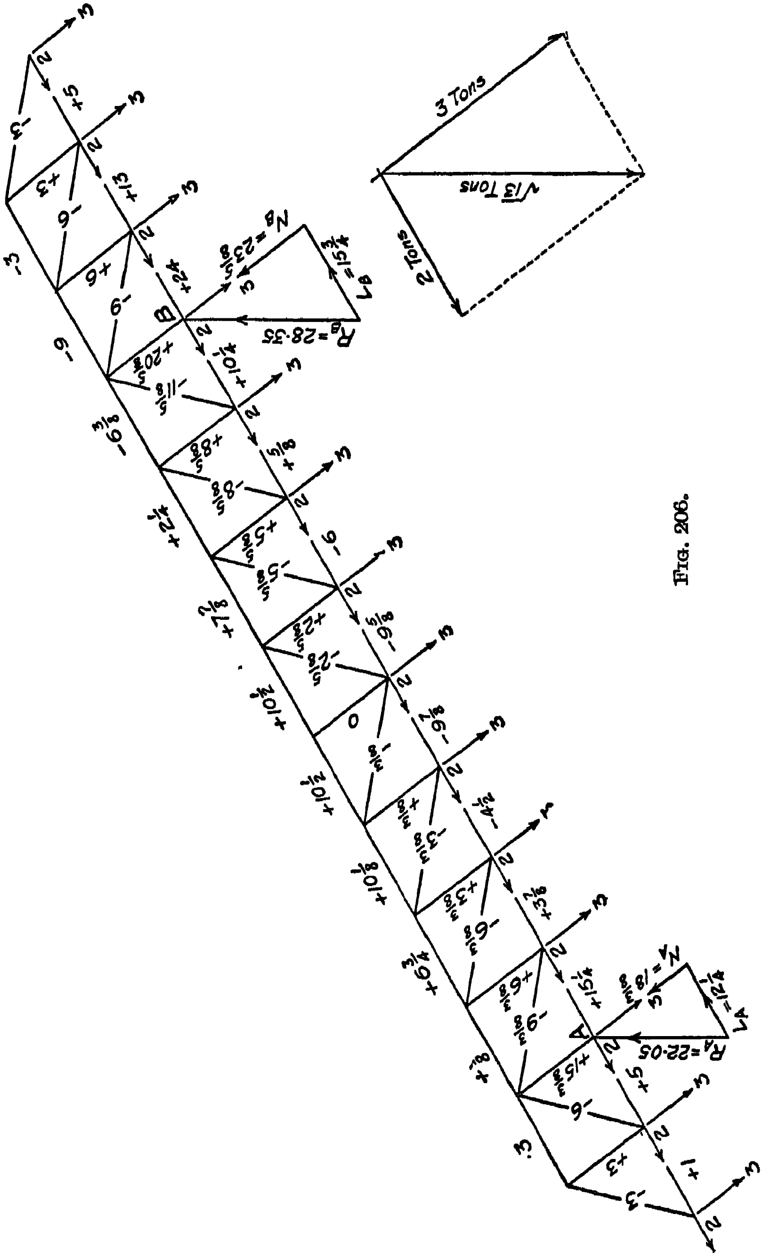


FIG. 206.

of the upper boom over which the crab travels, and which are therefore subject to bending moment in addition to direct compression. In this example, the span is 46 ft. 6 in., depth of girder 5 ft., and breadth of panels 5 ft. The useful crane load is 70 tons, weight of crab assumed as 18 tons, and weight of each main girder 10 tons, of which 1 ton is apportioned to each upper joint, the remaining 1 ton being divided between the end supports, and consequently not appearing in the

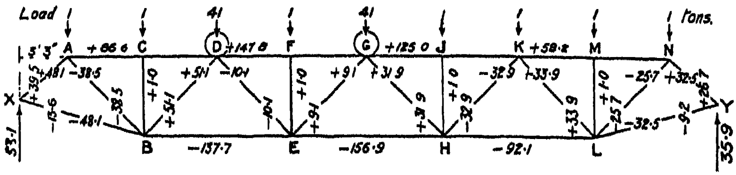


FIG. 207.

calculations for the stresses in the lattice members, since it does not contribute to the bending moment. The distance between centres of crab axles is 10 ft. The load lifted will be doubled and treated as equivalent dead load. Then equivalent crab load = 70 × 2 + 18 = 158 tons, *i.e.* $\frac{158}{4}$ = 39·5, say 40 tons per wheel. It should be noted that by doubling the crane load an outside allowance is made for shock due to possible slipping of the tackle, and in designing the sectional areas of members a relatively high working stress may be taken.

The following table shows the stresses caused by placing the crab in five different positions; first, with the left-hand axle over point A, and right-hand axle over D, afterwards moving it 5 ft. at a time, so that the left-hand axle comes successively over C, D, F, and G. The transit over the right-hand half of the span will give rise to similar stresses in corresponding members, and need not be treated. As the method of calculation for each position is a repetition of the one before, it is here stated for one position only of the load, namely, when the left-hand axle is at D, and right-hand axle at G.

STRESSES IN MEMBERS OF LATTICE CRANE GIRDER.

Member	Direct stress when left-hand axle of crab is at,				
	A	C	D	F	G
XA	+ 82·0	+ 72 3	+ 62·8	+ 52·2	+ 26 7
AD	+ 74·4	+100·7	+ 86·6	+ 72 4	+ 58·2
DG	+ 90·0	+139·1	+147 8	+156·4	+125·0
XB	− 67·3	− 58·5	− 50·0	− 41·9	− 33 9
BE	−102 7	−120·4	−137·7	−114 9	− 92·1
EH	− 76·3	−116·8	−156·9	−156·9	−156·9
AB	− 15·5	− 63·2	− 54·3	− 45·3	− 36·2
BD	+ 39 9	+ 27 8	+ 72·1	+ 59 9	+ 47 8
DE	+ 17 9	− 26·4	− 14·3	− 58·5	− 46·4
EG	− 19 3	− 31 5	+ 12·8	+ 0·7	+ 45·0
BC	+ 1·0	+ 41·0	+ 1·0	+ 1 0	+ 1·0
EF	+ 1·0	+ 41 0	+ 1·0	+ 41·0	+ 1·0

$$\text{Reaction at X} = \frac{33\frac{1}{4}}{46\frac{1}{2}} \text{ of } 40 + \frac{23\frac{1}{4}}{46\frac{1}{2}} \text{ of } 40 + 4\tfrac{1}{2} = 53{\cdot}1 \text{ tons.}$$

$$\text{B.M. at A} = 53{\cdot}1 \times 3\tfrac{1}{4} = 172{\cdot}6 \text{ ft.-tons.}$$

$$\text{Vertical depth of girder at A} = 3{\cdot}59 \text{ ft.}$$

$$\therefore \text{Horizontal stress in XA or XB} = \frac{172{\cdot}6}{3{\cdot}59} = 48{\cdot}1 \text{ tons.}$$

Vertical stress in XB = $48{\cdot}1 \times \frac{28''}{99''}$ = 13·6 tons tension, 28 in. and 99 in. being respectively the vertical height and horizontal length of XB. Since the downward vertical pull in XB and thrust in XA together make up the reaction of 53·1 tons, vertical compression in XA = 53·1 − 13·6 = 39·5 tons.

Writing these vertical stresses against XB and XA, Fig. 207, the remaining vertical stresses for this position of the load are easily written down. Thus, at A, 1 ton of the vertical compression of 39·5 tons in XA is due to the dead load at A, leaving a vertical tension of 38·5 tons to be applied by AB. The 1 ton of dead load at C is transmitted down CB as compression, so that at B there is an uplift of 38·5 + 13·6, exerted by ties AB and XB, = 52·1 tons − 1 ton due to the thrust in BC, giving 51·1 tons as the vertical down-thrust or compression in BD. At D the 41 tons of load creates 41 of the 51·1 tons compression in BD, whence the remaining 10 1 tons is due to the vertical tension in DE. The other vertical stresses will be readily followed, care being taken not to omit the loads transmitted down the intermediate verticals from top to bottom joints. The stresses beyond G are not required for insertion in the table, but are followed through in order to check with the reaction at Y. On arriving at L, the vertical thrust of 33 9 tons in KL + 1 ton in LM = 34·9 tons. This represents the *combined* vertical tensions in LN and LY. The *separate* tensions in these members are not required, as they will not be the maxima stresses. They may be found, if desired, from the B.M. at N in a similar manner as adopted for the stresses in XB and XA. Adding the last 1 ton load at N, the pressure on Y = 34 9 + 1·0 = 35 9 tons. The total load on girder = 89 tons, and reaction at Y = 89 − reaction at X = 89 − 53·1 = 35·9 tons, which of course agrees with the 35·9 tons pressure applied by members NY and LY. The inclination of the lattice bars, excepting XA and XB, is 45°, and the horizontal stresses equal the vertical stresses for bars AB, BD, DE, and EG. The horizontal stress in XB and XA has already been calculated, and equals 48·1 tons. From the horizontal stresses the flange stresses are readily computed, and finally, the direct stresses for the inclined members are obtained in the usual way.

$$\text{Direct stress in XB} = 48{\cdot}1 \times \tfrac{103''}{99''} = 50 \text{ tons,}$$

$$\text{,,} \qquad \text{XA} = 48{\cdot}1 \times \frac{50{\cdot}5''}{39''} = 62{\cdot}3 \text{ tons,}$$

103 in. and 50·5 in. being the inclined lengths of XB and XA respectively. The direct stresses in AB, BD, DE, and EG = horizontal stresses × $\sqrt{2}$. The direct stresses are entered in column D of the

table, and the stresses due to the other positions of the crab being similarly calculated and inserted, the maxima stresses are indicated by the figures in heavy type.

Fig. 208 shows the sections adopted, and general arrangement of the girder. The upper boom from C to G consists of one horizontal plate $16'' \times \frac{5}{8}''$ two vertical channels $9'' \times 3\frac{1}{2}'' \times 25{\cdot}89$ lbs., and two vertical plates $7\frac{1}{2}'' \times \frac{5}{8}''$. From C to the end of the girder the two vertical plates are suppressed. The C.G. of the section is 3·7 in. from the upper surface, moment of inertia = 498·4, and modulus of section for compressive stress at upper surface therefore $= \dfrac{498{\cdot}4}{3{\cdot}7} = 135$. The segments of the upper boom act as beams when the rolling load travels over them, and are subject to bending stress in addition to the direct compression indicated in the table. With 40 tons midway between two panel points, the B.M. $= \dfrac{40 \times 5}{4} = 50$ foot-tons $= 600$ inch-tons. This, it will be noted, is an outside estimate, since the continuity of the upper boom will tend to diminish the B.M., which is here calculated as for a simply supported span of 5 ft.

Therefore compressive stress due to bending only $= \frac{600}{135} = 4{\cdot}45$ tons per square inch.

Total sectional area = 34·3 sq. in. Maximum direct compression in segment DG (from table) = 156·4 tons. Hence direct compression per sq. in. $= \dfrac{156{\cdot}4}{34{\cdot}3} = 4{\cdot}56$ tons, and total compressive stress per square inch at upper edge of section due to bending *and* direct compression $= 4{\cdot}45 + 4{\cdot}56 = 9{\cdot}01$ tons. Remembering that the rolling crane load was originally doubled, this stress represents the outside maximum which may occur under a sudden shock, due to a possible slip of tackle whilst slinging the load. Under normal working conditions the maximum compression in the top boom will not exceed 5 to $5\frac{1}{2}$ tons per square inch. The compression members here are all relatively "short columns," and no reduction in working stress is necessary on the score of slenderness.

The lower boom consists of two vertical plates $8'' \times \frac{5}{8}''$, two angles $7'' \times 3\frac{1}{2}'' \times 17{\cdot}81$ lbs. with 7 in. leg vertical, and two horizontal plates $3\frac{1}{2}'' \times \frac{1}{2}''$. Sectional area = 24 sq. in. Maximum stress = 156·9 tons in EH, and maximum stress per sq. in. $= \dfrac{156{\cdot}9}{18} = 8{\cdot}7$ tons, after deducting 6 sq. in. for six $\frac{7}{8}$ in. rivets—four in the vertical and two in the horizontal portion of the boom. Each vertical member suffers 41 tons of compression as an axle passes over it. These consist of two $7'' \times 3'' \times 17{\cdot}56$ lbs. channels, back to back. Sectional area = 10·32. Maximum working stress $= \dfrac{41{\cdot}0}{10{\cdot}32} =$ say 4 tons per square inch. Diagonal EG has the same section as the verticals. Its maximum stresses are 45 tons compression and 31·5 tons tension, and maximum working stress $= \dfrac{45}{10{\cdot}32} = 4{\cdot}4$ tons per square inch compression. Diagonals AB, BD, and DE all have the same section, consisting of two $7'' \times 3''$ channels, with one $7'' \times \frac{1}{2}''$ plate between. Maximum stress in

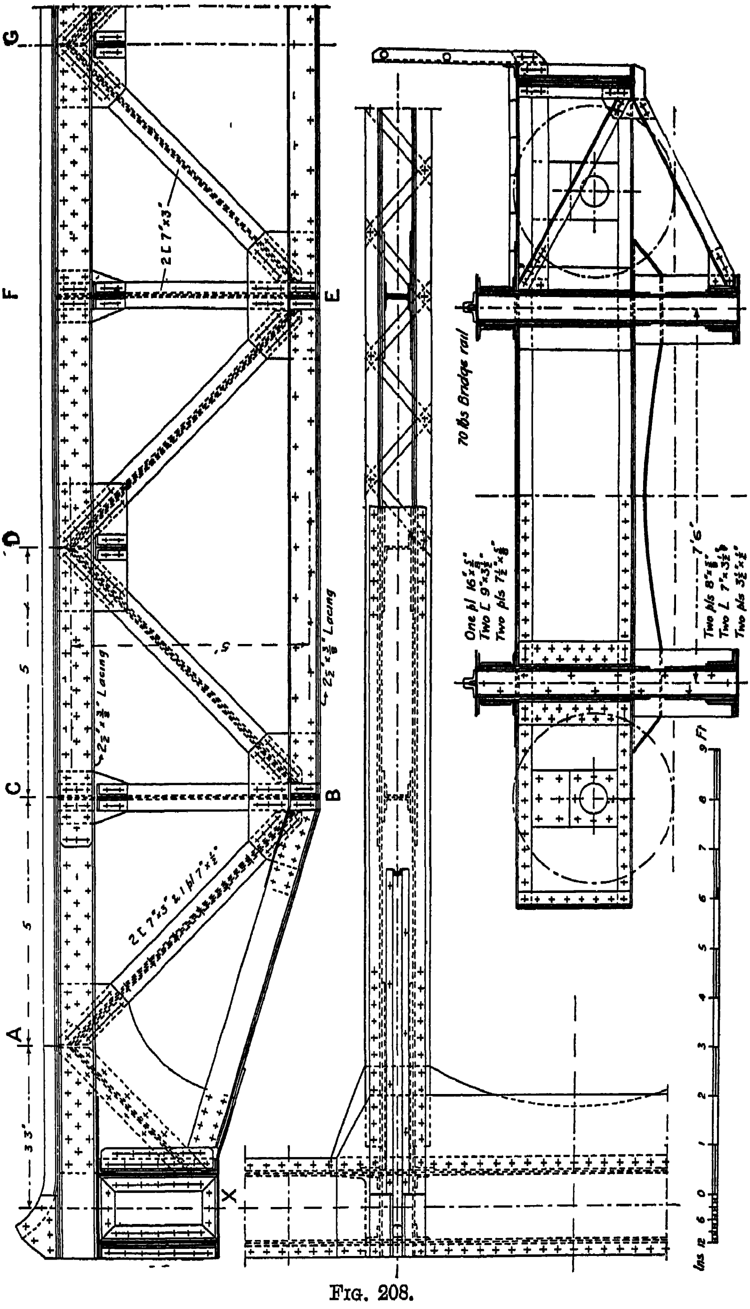


FIG. 208.

AB = 63·2 tons tension. Gross sectional area = 13·82 sq. in., and deducting, say, 5 rivet holes, four through gusset plates and one through webs of channels and sandwich plate, nett sectional area = 11 sq. in., giving a maximum working stress = $\frac{63 \cdot 2}{11}$ = 5·8 tons per square inch. In BD maximum compression = 72·1 tons, and maximum working stress = $\frac{72 \cdot 1}{13 \cdot 82}$ = 5·3 tons per square inch. DE, with a maximum tension of 58·5 tons, is obviously of ample section. In XB the vertical angles and horizontal plates only are carried through, the two 8″ × $\frac{5}{8}$″ vertical plates being suppressed on the left of joint B. The maximum tension is 67·3 tons. The gross section is 14 sq. in., and deducting 6 rivet holes, nett section = 10·5 sq. in. Maximum working stress = $\frac{67 \cdot 3}{10 \cdot 5}$ = 6·4 tons per square inch. The short strut XA is enclosed between two deep $\frac{5}{8}$ in. gusset plates, to which it is riveted throughout its length. A considerable proportion of its stress will therefore be transmitted by the gusset plates. Its maximum stress is 82 tons. The strut, apart from the gussets, consists of two 7″ × 3″ channels, having a sectional area of 10·32 sq. in. As sufficient rivets are provided to pass quite half the stress to the gussets this section will be ample. The riveted connections have been designed on a basis of 3 tons single shearing resistance per $\frac{7}{8}$ in. rivet, and a maximum bearing stress of 10 tons per square inch. All gussets are $\frac{5}{8}$ in. thick. On the cross-section a light gantry is shown on the right-hand side, carried by a lattice girder 2 ft. 6 in. deep, bracketed out from the main girder.

Practical Arrangement of Details of Lattice Girders.—Fig. 209

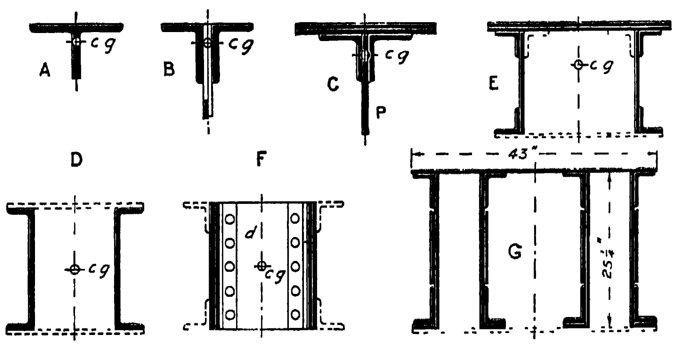


FIG 209.

illustrates sections most commonly employed for the booms of lattice girders.

Sections A and B are used for the flanges of light girders in roof construction, or for parapet girders having webs of closely spaced intersecting flat bars. These girders are now practically obsolete for large spans. C is the usual flange section for girders of medium length. The bracing consists of flats, channels, angles, or tees riveted on either side of the vertical web plate P. The compression booms of large span

girders are generally of section E, the number of vertical and horizontal plates being increased according to the sectional area required. Section D is occasionally employed for the compression boom in light girders, the channels being laced together top and bottom. For the tension boom any of the above forms inverted may be used, whilst section F, consisting of vertical plates with or without the dotted angles, is very generally employed, and possesses the advantage of not accumulating dirt as does the trough section E The tension booms of pin-jointed trusses consist of several flat eye-bars placed side by side as in Fig. 214. The bracing bars of lattice girders are of flat section in the case of ties, and angle, tee, or channel section for the struts of small girders. In larger girders the struts may be of rolled beam section, or any of the column sections indicated in types 1, 7, 10, and 11, Chapter V. Type 10 is most commonly employed in English practice, whilst in pin-jointed girders of American design the struts are almost invariably formed of two channels laced together as in Fig. 214.

Fig. 210 shows the usual arrangement of details adopted in English practice for riveted main girders of bridges. The figure shows the elevation and cross-section of one panel of a main girder for a double-line railway bridge of a type suitable for spans of from 120 ft. to 150 ft. The upper boom A is of trough section similar to Fig. 209, E, a joint being indicated at J. This joint is a full butt, with internal and external covers to both horizontal and vertical plates. An alternative arrangement is to make the horizontal and vertical plates break joint, which however makes the work more difficult to handle in the shop. The under side of the boom is laced as shown at L, and the boom is stiffened transversely by plate diaphragms *d*, *d*. The lower boom B consists of two parcels of vertical plates, the arrangement of a joint in this boom being indicated at P. One or two plate diaphragms similar to *d* in Fig. 209, F, are usually inserted in each panel, whilst angles as shown dotted in Fig. 209, F, are often provided. If the girder be finished with a *vertical* end post, these angles are necessary in the end panels to allow of the vertical plates being laced together in order to resist longitudinal compression due to application of brakes. The vertical struts V are of four angles, back to back, with flat lacing bars. The ties T are of two or four flats, according to the section required, and are here shown butt-jointed to the gussets *g*, *g*, with double covers This joint is often lapped, but the butt joint is preferable. The riveting should be arranged with one rivet only on the leading section of the tie-bar, to avoid weakening the tie by more than one rivet hole. The cross-girders G are attached to the lower ends of the verticals V, which pass through the lower boom and carry a pair of suspension plates S, between which the web of the cross-girder is riveted. The vertical angles forming the strut are also carried through and riveted to the cross-girder web. This is probably the most desirable method of attachment of cross-girders, and its adoption is a strong argument in favour of the open type of lower boom. Other advantages are that the centre of gravity of cross-section is at the centre of depth, and the boom does not accumulate dirt and water. In Fig. 209 the centres of gravity of the various boom sections are indicated at *cg*. It is important in setting out the skeleton lines intended as the

axes of the members of a lattice girder that these lines intersect at the centres of gravity of cross-section of the members, otherwise secondary bending stresses will be set up in the region of the joints. In Fig. 210 the chain dotted line *x-x*, drawn on the upper boom in elevation, is

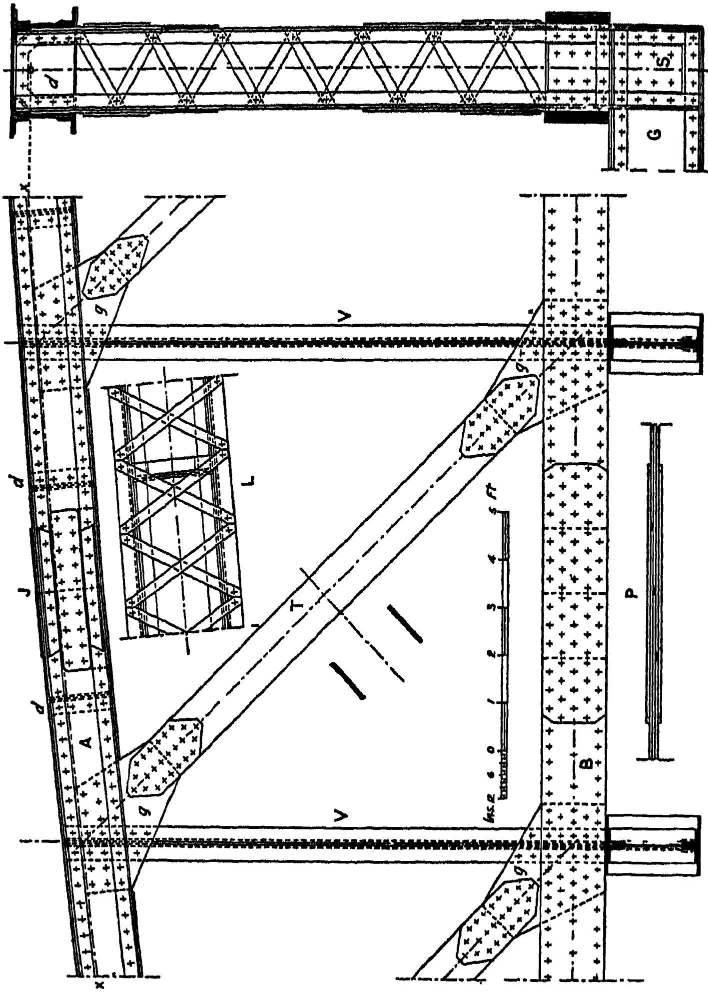


Fig. 210.

projected from the c.g. of the boom section, indicated by the small circle on the cross-section. This line is sometimes referred to as the *gravity line* or *gravity axis* of the boom, and it will be seen that the centre lines or axes of the ties and struts all intersect on this line. The details of jointing in larger girders with double systems of bracing are similar to those just mentioned.

Fig. 211 shows a vertical section through the end post or strut of

a girder similar in type to that of Fig. 210. The end post P consists of two side plates, one transverse plate, and eight angles arranged as indicated on the horizontal cross-section. The upper boom may be either lap- or butt-jointed to the end post. It is here shown lap-jointed, and the end tie T is connected to gusset plates G, packing being required as shown by the dotted shading. The two vertical plates of the lower boom B are riveted to the lower end of the post P, and laced together top and bottom by lacing bars L, L, for the length of the first panel of

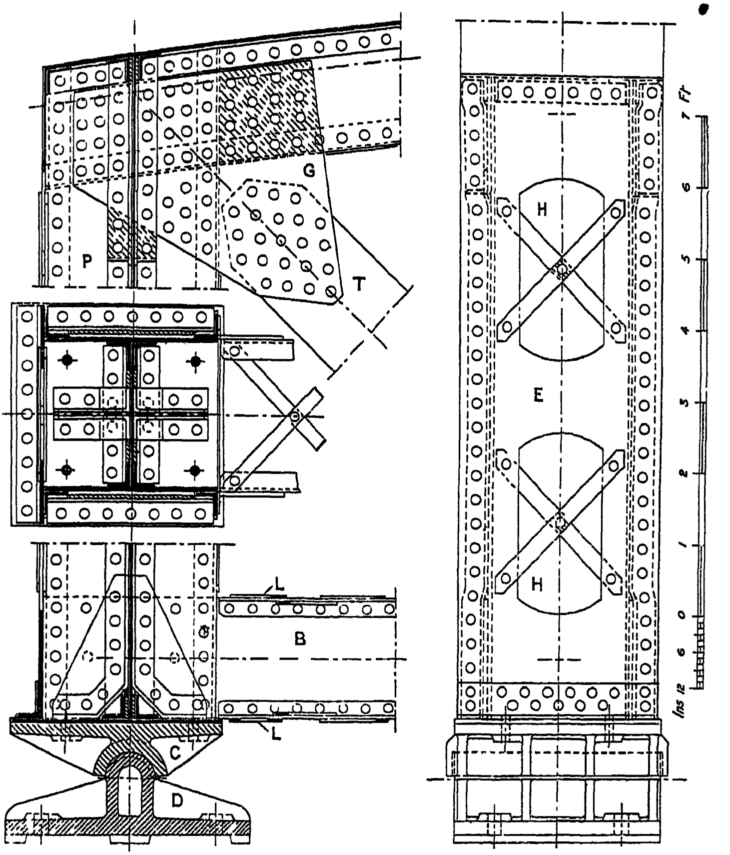


Fig. 211

the girder. The lower end of the end post carries a base or *bolster* plate, which is bolted to the upper casting C of the deflection bearing. The inner and outer faces of the end post are covered by end plates E, having hand-holes H for convenience of painting the interior.

End Bearings for Girders.—Girders exceeding 70 feet span should be provided with pin and roller bearings to allow of free deflection and expansion and contraction under changes of temperature. The usual arrangement is a combined roller and pin bearing B, Fig. 212, beneath

one end, and some form of pin bearing P, under the other end of the girder. Considerable variety exists in the detailed design of bearings.

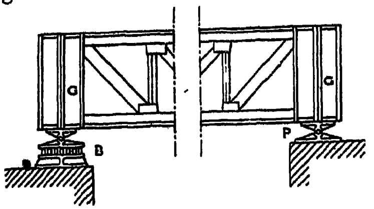


FIG. 212

Figs. 211 and 213 show two common types. In Fig. 211 the girder is carried on two steel castings C and D, having machined concave and convex surfaces respectively. In Fig. 213, which is the more usual type employed in English practice, the upper casting C bears on a steel pin carried by the intermediate casting D, which transmits the load to the nest of rollers R, these again resting on a lower casting L. The rollers should not be less than 4 in. diameter, and vary from 4 in. to 12 in. The safe load on rollers may

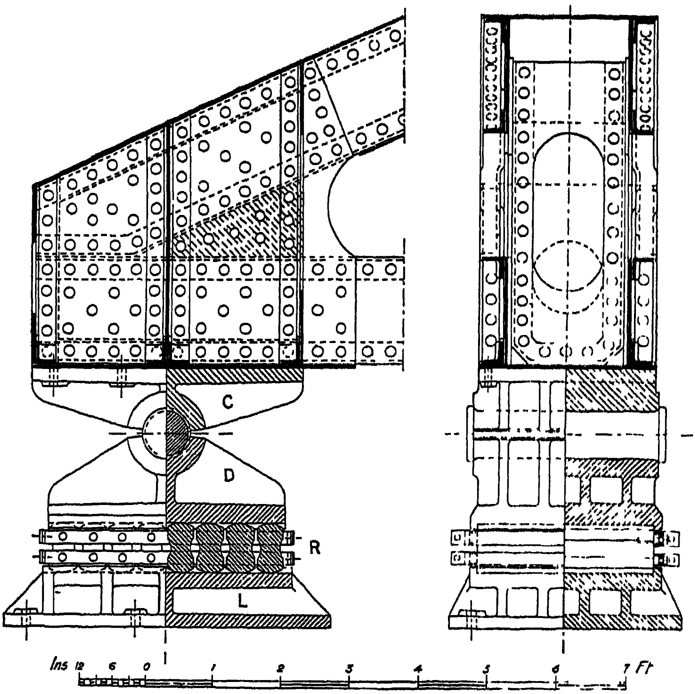


FIG. 213.

be $1000\sqrt{d}$ lbs. per lineal inch of bearing between rollers and castings. Hence, a bearing having, say, eight rollers of 9 in. diameter and 2 ft. long, may be safely loaded with—

$$\frac{24 \times 8 \times 1000\sqrt{9}}{2240} = 257 \text{ tons.}$$

The lower casting should be at least the same depth as the rollers,

and its base area increased if necessary to distribute the pressure over a suitable area of masonry. The pressure between lower bed-plate and masonry should not exceed 400 lbs. per square inch. Built-up pedestals are sometimes employed instead of castings, but the riveting is usually cramped and difficult. The rollers are carried in a pair of light bar frames, and either the rollers or castings are flanged to prevent lateral movement.

Girders of less than 70 ft. span should have a steel bolster plate not less than $\frac{3}{4}$ in. thick, or a casting 2 in. to 3 in. thick attached to the lower flange under the end of the girder, such bolster or casting sliding on a second cast bed-plate bolted to the masonry. One end only of the girder is free to slide, the other end being suitably prevented from moving longitudinally by a flange or stop on the bed-plate or by substantial bolts.

Fig. 213 also shows the detail of plating at the end of a bowstring girder of 200 ft. span. The construction is clearly indicated, and calls for no special remark.

Fig. 214 shows the detailed arrangement of one panel of a pin-connected truss of 160 ft. span. The upper boom A consists of two vertical plates and four angles laced together at upper and lower faces. The lower boom consists of eye-bars E, the number and sectional area being proportioned to the stress in each panel. The diagonal ties T are also eye-bars, two or four being employed as required. The members meeting at each panel point are assembled on a steel pin, the diameter varying from 4 in. or 5 in. in small spans, to 9 in. or 10 in. in large spans. Joints J in the upper boom are usually placed near the pin instead of at centre of panel. The vertical struts V′, V, in girders of moderate span consist usually of two channels laced together on inner faces. A horizontal section through strut V′ is shown at S, from which the assemblage of the members meeting on the lower pin P will be readily traced. The right-hand view is an end elevation and part vertical section through strut V, and shows the connection of a cross-girder G to the inner face of the strut, as well as the overhead transverse or *sway* bracing B. A diaphragm plate D is inserted in the plane of the web of the cross-girder between the two channels forming the strut V. The panel width, here 20 ft., being considerable, the cross-girders are much more heavily loaded than when spaced at 7 ft. or 8 ft., and require a correspondingly greater depth. They are cut away at X to clear the pin end and internal eye-bars of the lower boom. This necessitates the employment of a *hitch* plate H, which is riveted to the U-plate fitted between the lower ends of the channels of strut V. The hitch plate is required to resist the tendency of the horizontal diagonal wind braces to bend the lower end of the strut inwards. These wind braces are omitted in the figure, but are attached to the hitch plate, which is so shaped as to form a junction plate or gusset for the horizontal wind ties. A similar arrangement obtains at Y, where the sway bracing B is cut away to clear the upper boom. The upper system of horizontal wind braces are attached to the upper boom. The use of pin connections obviously modifies the section of boom which may be adopted. The upper boom plates bear edgewise on the pins, and in order to obtain sufficient bearing area, a considerable portion of the

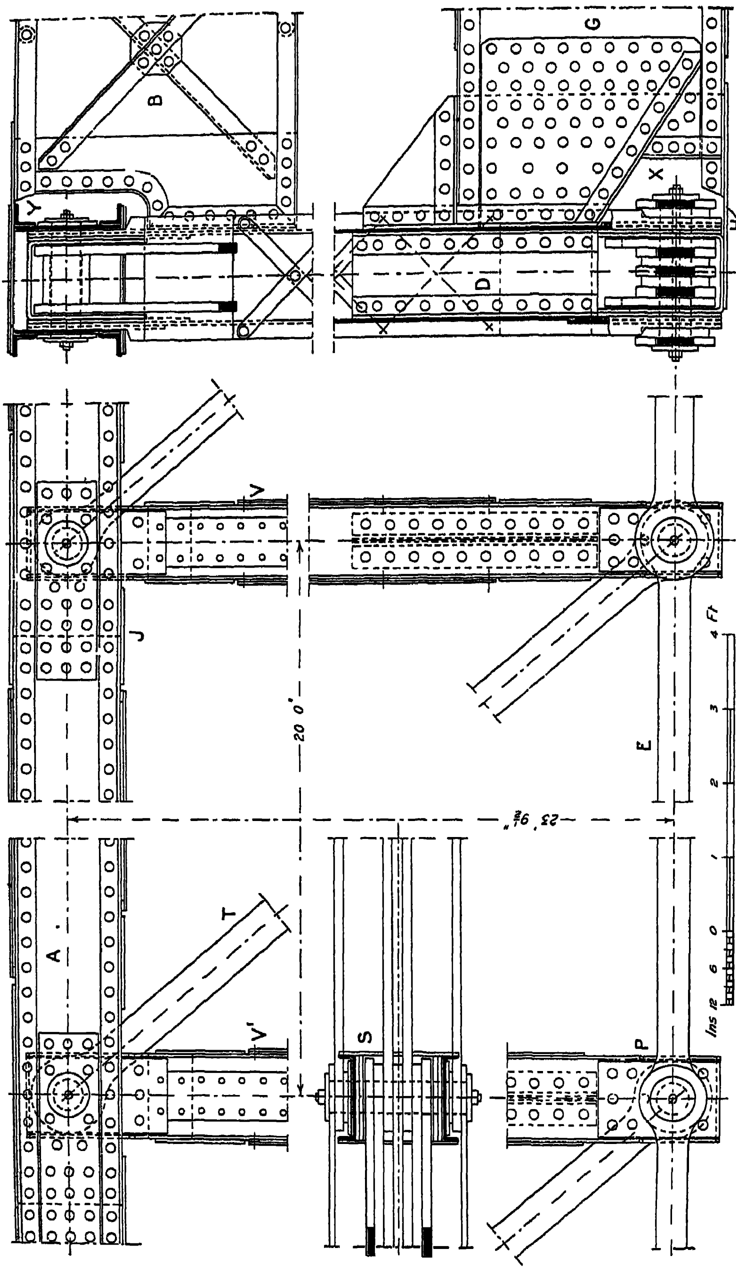


FIG 214.

upper boom section is made up of vertically disposed plates, especially in the case of very large girders. Fig. 209, G, shows the section of the upper boom of the 439 ft. 9¼ in. spans of the Bellefontaine bridge,[1] which in the central panel is made up of one horizontal plate 43″ × ½″, two vertical inside plates 25¼″ × ⅞″, two vertical outside plates 25¼″ × ¾″, four side plates 12″ × ¾″, eight angles 6″ × 4″ × ¾″, and four horizontal flats 4″ × 1″. The pins are 9 in. diameter, so that this disposition of plating gives 56¼ sq. in. of bearing area on the pin. Further consideration of pin-jointed trusses is beyond the scope of this work, but a comparison of Figs. 210 and 214 will illustrate the noticeable differences in detailed design of pin-jointed and riveted girders.

Fig. 215 shows the construction of a lattice girder such as commonly used for foot-bridges over railway tracks. The boom sections are

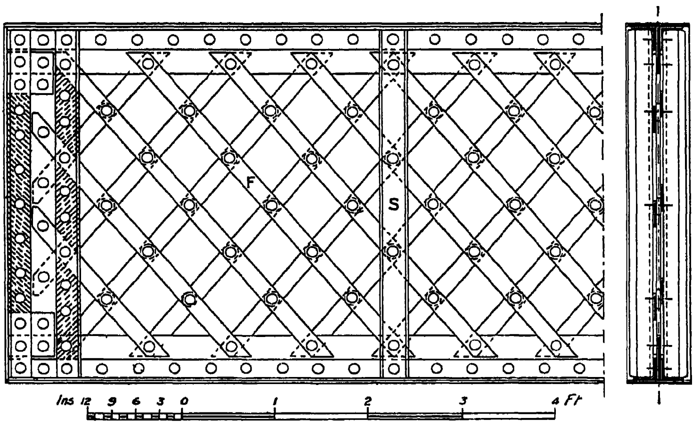


FIG. 215.

similar to Fig. 209, C, whilst the bracing consists of flat bars F, riveted at the intersections with washers for packing. Angle, tee, or channel stiffeners S are riveted to the web bracing and fitted well up to the flanges at suitable intervals. Parapet girders of deck bridges in open country are usually of this type. For bridges over town streets open-work parapet girders are generally prohibited.

[1] *Engineering*, Sept. 20, 1895.

CHAPTER VIII.

INFLUENCE LINES.

An influence line or diagram may be defined as a line or diagram plotted usually on a horizontal base line, the ordinates of which indicate the extent to which a reaction, shear, moment or stress in a member of a girder is affected or influenced during the passage of a concentrated rolling load across a span.

If it be desired to estimate exactly the stresses caused by a system of concentrated loads such as the axle loads of a locomotive, in the members of a girder, instead of replacing the axle loads by an equivalent distributed load, the method of influence lines may be adopted which considerably reduces the labour of calculation. Some of the simpler and more generally useful cases of influence lines will now be considered.

Influence Line of Reaction.—The simplest example of an influence line is that for finding the reaction at either end of a simply supported span.

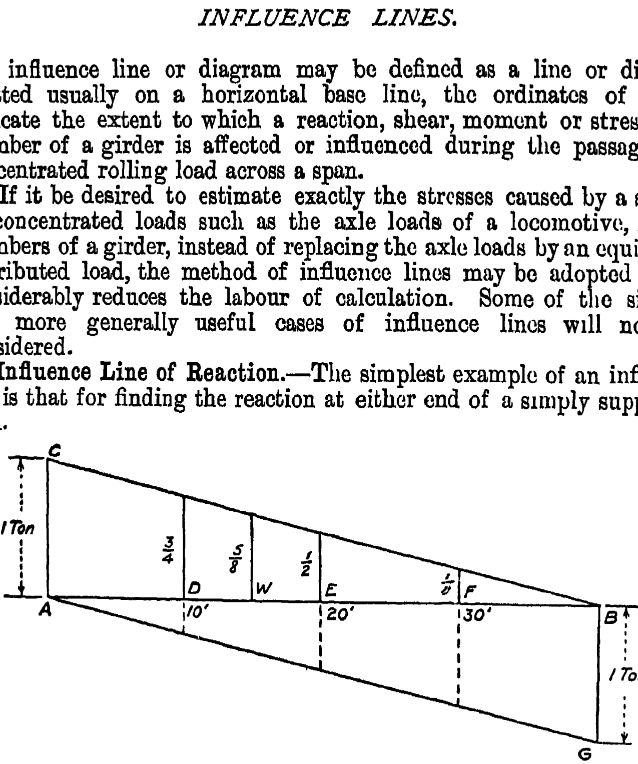


Fig. 216.

In Fig. 216 let AB represent a 40-ft. span. Suppose a concentrated load of one ton to roll from A to B. When just entering the span at A the whole weight is borne by the abutment A. Set up AC to represent one ton to any convenient scale. When the load is at D, $\frac{3}{4}$ ton is borne by A and $\frac{1}{4}$ ton by B. When at E, $\frac{1}{2}$ ton is the reaction at A and when at F, $R_A = \frac{1}{4}$ ton. On reaching B the reaction at A is zero. It is obvious the ordinates at D, E, and F represent $\frac{3}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$

ton to the scale of AC. The line CB is the reaction influence line for the support A. Similarly making BG = 1 ton, the dotted ordinates at D, E, and F indicate the reactions at B when the load occupies those successive positions. AG is the reaction influence line for support B.

If instead of 1 ton, a load of 5, 8, or 12 tons traverses the span, the reactions will be five, eight, or twelve times the value of the respective ordinates scaled from the unit diagram. Thus the reaction at A due to 12 tons at W $= \frac{5}{8} \times 12 = 7{\cdot}5$ tons. If several axle loads at fixed distances apart traverse the span the reaction at A or B is readily obtained for any desired position of the loads.

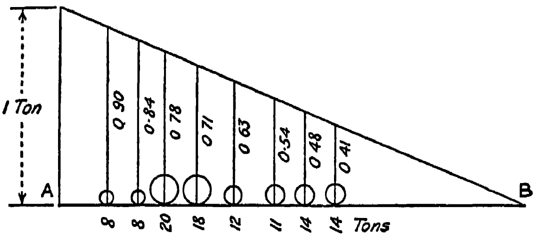


FIG. 217.

In Fig. 217 the axle loads of an Atlantic locomotive and tender are shown on a 100-ft. span with the leading bogie axle 10 feet from A. Scaling off the ordinates, reaction at A $= (8 \times 0{\cdot}90) + (8 \times 0{\cdot}84) + (20 \times 0{\cdot}78) + (18 \times 0{\cdot}71) + (12 \times 0{\cdot}63) + (11 \times 0{\cdot}54) + (14 \times 0{\cdot}48) \times (14 \times 0{\cdot}41) = 68{\cdot}26$ tons. Total load = 105 tons, whence $R_B = 105 - 68{\cdot}26 = 36{\cdot}74$ tons.

Maximum Rolling Load Reaction on a Cross-girder.—The maximum rolling load on a cross-girder due to the worst position of axle loads on the longitudinal girders in the adjacent panels, is quickly obtained from the reaction influence line.

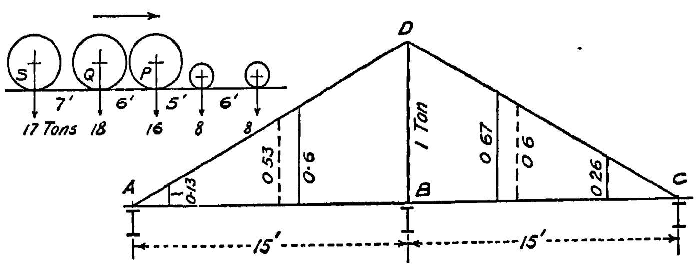


FIG. 218.

In Fig. 218 let A, B, and C be three consecutive cross-girders 15 feet apart, and suppose the given axle loads to roll over the span from A to C. Make BD = 1 ton. AD is the reaction influence line for the support B of span AB, and CD the reaction influence line for

support B of span BC. Placing the load P at B and the other loads in their resulting positions shown by full lines, reaction at B

$$
\begin{array}{lll}
= 16 \times 1 & = 16{\cdot}00 & \text{tons, due to load at B} \\
+ 18 \times 0{\cdot}6 & = 10{\cdot}80 & \\
+ 17 \times 0{\cdot}13 & = 2{\cdot}27 & \left.\right\} \text{,, ,, loads on AB} \\
+ 8 \times 0{\cdot}67 & = 5{\cdot}36 & \\
+ 8 \times 0{\cdot}26 & = 2{\cdot}13 & \left.\right\} \text{,, ,, ,, BC}
\end{array}
$$

$$R_B = 36{\cdot}56 \text{ tons}$$

It is possible that a greater reaction at B may result if the loads be moved to the right until axle Q arrives at B. The loads are then in the positions shown by the dotted lines, the leading 8 tons load having rolled off the span BC. R_B then

$$
\begin{array}{lll}
= 18 \times 1 & = 18{\cdot}00 & \text{tons due to load at B} \\
+ 17 \times 0{\cdot}53 & = 9{\cdot}01 & \text{,, ,, ,, on AB} \\
+ 16 \times 0{\cdot}60 & = 9{\cdot}60 & \\
+ 8 \times 0{\cdot}26 & = 2{\cdot}13 & \left.\right\} \text{,, . loads on BC}
\end{array}
$$

$$R_B = 38{\cdot}74 \text{ tons}$$

It is evident the second value must be the maximum, since if the load S be advanced to B a much greater reduction of the reaction due to loads P and Q will take place than may be balanced by the increase of reaction due to S. Hence the axle Q must be placed over cross-girder B to give the maximum reaction. It should be noted that *one* of the loads *must* be placed *at* B to give the maximum value. If the loads be arrested in some intermediate position, then on moving the group to right or left of that position, either an increase or decrease of reaction will take place, and such increase or decrease will continue until *a* load arrives under the highest point D of the influence line. It is further unnecessary to try different positions at random. The position which gives the maximum reaction may be readily located as follows. The slope of line AD determines the *rate* at which the ordinates *increase* over the span AB, and the slope of DC determines the *rate* at which the ordinates *decrease* over the span BC, for a movement of loads from left to right. In this case the slopes AD and DC are equal since the spans are equal. Hence if two *equal* connected loads on opposite sides of BD be moved either to right or left, the increase of reaction due to one will be exactly balanced by the decrease due to the other, since each moves through the same distance. But if a greater load be approaching B whilst a less load is receding from B there will be a net increase of reaction at B for this direction of motion.

Hence if P be supposed at B and a slight movement of the group of loads takes place towards the right, the loads become separated into two groups, Fig. 219, one of 32 tons gross moving under DC and tending to *decrease* R_B, and the other of 35 tons gross moving under AD and tending to *increase* R_B. Since the rate of increase per unit of load equals in this case the rate of decrease, the net result of a left to right movement will be an increase in the reaction. Continuing the movement until Q arrives at B a still further slight movement to the right may be imagined in order to ascertain if the reaction will continue

to increase after Q has passed the point B. The right-hand group of loads influencing the decrease now totals $18 + 16 + 8 = 42$ tons (the leading 8 tons having now rolled off the span), whilst the left-hand group influencing the increase comprises only the 17 tons load. The

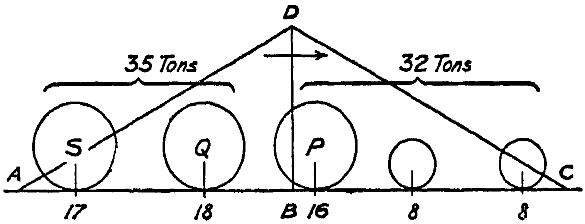


FIG. 219.

result of moving Q to the right of B is therefore to cause a decrease in the reaction, and evidently Q must be arrested at B for the position of maximum reaction.

Influence Line of Bending Moment.—The reader should carefully appreciate the following theorem before proceeding to influence lines for moments.

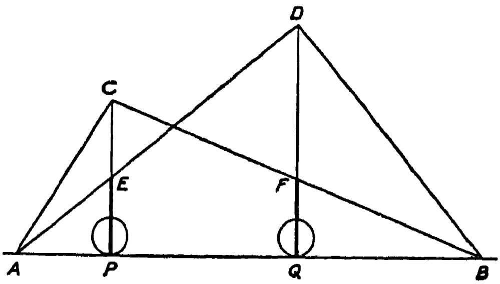


FIG. 220.

In Fig. 220 let AB be any span. Place a load of 1 ton at any point P.

Reaction at A due to 1 ton at P $= \frac{BP}{AB} \times 1 = \frac{BP}{AB}$ ton. Bending moment at P due to 1 ton at P $= \frac{BP}{AB} \times AP$ ft.-tons.

Make CP $= \frac{BP}{AB} \times AP$. Then ACB is the bending moment diagram for a load of 1 ton at P.

Now place another load of 1 ton at any other point Q. Reaction at A due to 1 ton at Q $= \frac{BQ}{AB} \times 1 = \frac{BQ}{AB}$ ton.

Bending moment at Q due to 1 ton at Q $= \frac{BQ}{AB} \times AQ$ ft.-tons.

Make $DQ = \frac{BQ}{AB} \times AQ$ to the same scale as used for CP.

Then ADB is the bending moment diagram for a load of 1 ton at Q. Consider the section at P. There is here a bending moment CP due to the 1 ton load at P + an additional bending moment EP due to the 1 ton load at Q. It may be proved that the intercept FQ of the ordinate through Q on the original diagram ACB is equal to EP. In other words, the total bending moment at any section P = the moment at P due to the load at P + the ordinate at Q to the bending moment diagram of P. For loads other than 1 ton, the ordinates are multiplied by their respective loads.

To prove FQ = EP.

From similar triangles $\frac{EP}{AP} = \frac{DQ}{AQ}$, $\therefore EP = \frac{AP}{AQ} \times DQ = \frac{AP}{AQ} \times \frac{BQ}{AB} \times AQ = \frac{AP \times BQ}{AB}$ (substituting value of DQ from above).

Again from similar triangles $\frac{FQ}{BQ} = \frac{CP}{BP}$, $\therefore FQ = \frac{BQ}{BP} \times CP = \frac{BQ}{BP} \times \frac{BP}{AB} \times AP = \frac{AP \times BQ}{AB}$ (substituting value of CP from above), whence FQ = EP.

ACB is called the Influence Line of Moments for section P.

Bending Moment at any Section of a Girder due to a Group of Axle Loads in any Position.—In Fig. 221 let AB be a 50-ft. span. Required the bending moment at a section P, 20 ft. from A, due to the given loads in the position indicated

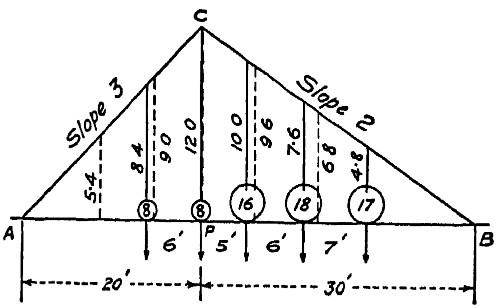


FIG. 221.

Place 1 ton at P. $R_A = \frac{30}{50} = 0·6$ ton.

Bending moment at P due to 1 ton at P = 0·6 × 20′ = 12 ft.-tons. Make PC = 12 ft.-tons to scale. ACB is the influence line of moments for section P. The figured heights of the ordinates give the bending moments in ft.-tons at P due to a 1 ton load at each of the five positions. Multiplying the ordinates by the loads, the total moment at P = (8 × 8·4) + (8 × 12) + (16 × 10) + (18 × 7·6) + (17 × 4·8) = 541·6 ft.-tons.

Position of Loads for Maximum Bending Moment at any Section.—In the last example, to determine the position of the loads for maximum bending moment at section P and the value of the maximum moment.

Slope of AC $= \frac{12}{20} = 0{\cdot}6$. Slope of BC $= \frac{12}{30} = 0{\cdot}4$. Hence relative slopes of AC and BC are as 3 to 2.

Starting from the position shown in Fig. 221 let the loads be moved slightly to the left. The ordinates of the two 8 tons loads will decrease at the relative rate 3, whilst the ordinates of the 16, 18, and 17 tons loads will increase at the relative rate 2. Hence,

Total relative decrease of moment $= (8 + 8) \times 3 = 48$
 „ „ increase „ $= (16 + 18 + 17) \times 2 = 102$

A movement of the group towards A will therefore cause a net increase of moment at P. Advance the 16 tons load to P and imagine a slight *further* movement to the left. The loads 8, 8, and 16 tons are now moving under the slope AC and are influencing a decrease of moment at P at the relative rate 3, whilst the 18 and 17 tons loads are influencing an increase of moment at P at the relative rate 2.

Total relative decrease of moment $= (8 + 8 + 16) \times 3 = 96$.
 „ „ increase „ $= (18 + 17) \times 2 = 70$.

Thus a further movement of the 16 tons load beyond P towards A will cause a net decrease of moment at P. The position for maximum moment at P is therefore when the 16 tons load is at P. Drawing the new ordinates (dotted) for the remaining loads when the 16 tons load is at P and scaling off their lengths, the maximum bending moment at section P $= (8 \times 5{\cdot}4) + (8 \times 9) + (16 \times 12) + (18 \times 9{\cdot}6) + (17 \times 6{\cdot}8) = 595{\cdot}6$ ft.-tons.

NOTE.—If these loads are able to traverse the span in the reverse order, as, for example, after having been rotated on a turn-table, it will

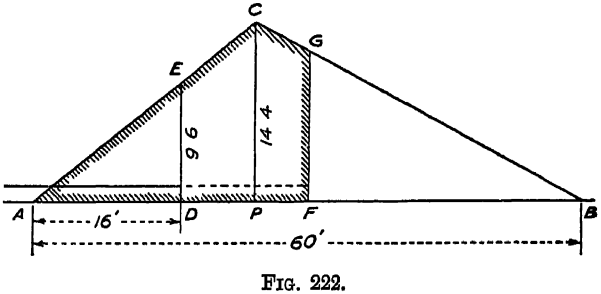


FIG. 222.

be found that the 18 tons load at P will give the maximum moment at P and its value will then be 604·6 ft.-tons.

Bending Moment at any Section due to advancing Distributed Load.—A uniformly distributed load may be regarded as an infinitely large number of concentrated loads so close together that their ordinates coalesce and constitute an area. The bending moment due to a distributed load is then measured by the *area* of the portion of

the influence diagram vertically over the load. In Fig. 222 let AB be a span of 60 feet, and suppose a distributed load of 2 tons per foot run to advance from A towards B. Consider the section P, 24 feet from A. Construct the influence diagram for section P as before, by placing 1 ton at P. $R_A = \frac{36}{60}$ ton and bending moment at $P = \frac{36}{60} \times 24 = 14{\cdot}4$ ft.-tons. Make PC = 14·4.

Suppose the distributed load to have advanced to D, say 16 feet from A. The bending moment at P due to this position of the load = area ADE × load intensity = $\frac{1}{2}$AD × DE × 2. The area must be computed in the proper units. DE is measured to the scale of PC and AD to the scale of the span AB. The resulting product of $\frac{1}{2}$AD × DE will be the bending moment at P due to a load of 1 ton per foot run or of *unit intensity*, since the influence diagram was constructed for *unit* load at P. For a load of 2 tons per foot run, the result will be multiplied by 2. Thus,

Area ADE = $\frac{1}{2}$AD × DE = $\frac{1}{2} \times 16 \times 9{\cdot}6 = 76{\cdot}8$ ft.-tons for a load of 1 ton per foot run, and

Area ADE × load intensity = $\frac{1}{2} \times 16 \times 9{\cdot}6 \times 2 = 153{\cdot}6$ ft.-tons for a load of 2 tons per foot run.

When the load has advanced to F the moment at P = area ACGF × 2 ft.-tons. Obviously the moment at P will continue to increase as the area over the load increases. If the load be long enough to cover the whole span, the maximum moment at P occurs when the head of the load reaches B and will then = area ACB × 2 = $\frac{1}{2} \times 14{\cdot}4 \times 60 \times 2 = 864$ ft.-tons.

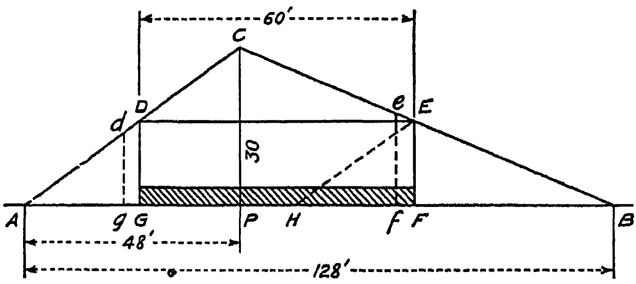


Fig. 223.

Maximum Moment at any Section due to a Distributed Load shorter than the Span.—Let AB, Fig. 223, be a span of 128 feet traversed by a distributed load of $1\frac{1}{2}$ tons per foot run, extending over a length of 60 feet. Let P be any section say 48 feet from A. Place 1 ton at P. $R_A = \frac{80}{128}$ ton and bending moment at $P = \frac{80}{128} \times 48 = 30$ ft.-tons = PC. ACB is the influence line of moments for section P. The position of the load which creates the maximum moment at P is obtained by locating the *horizontal* length DE = 60 feet which is intercepted by CA and CB, and projecting to G and F. The moment at P for this position of the load = area GDCEF × load intensity. The area GDCEF is evidently the maximum,

since a small movement $gG = fF$ of the load to the left (or right) results in a smaller area $gdCef$ over the load, the strip $eEFf$ cut off at F being greater than the strip $dDGg$ added at G. DE is located by making AH = 60 feet and drawing HE parallel to AC.

Maximum moment at P = area GDCEF $\times 1\frac{1}{2}$

= area (GDEF + CDE) $\times 1\frac{1}{2}$ ft.-tons.

GD scales 16 bare, its exact value being $15\frac{15}{16}$.

$\therefore$ Maximum moment at P = $(15\frac{15}{16} \times 60) \times 1\frac{1}{2}$

$+ (\frac{1}{2} \times 14\frac{1}{16} \times 60) \times 1\frac{1}{2} = 2067{\cdot}19$ ft.-tons.

Influence Lines of Stress.—Influence lines of stress may be constructed to indicate the fluctuation of stress in any member of a braced girder due to the passage of a specified system of loads. Such may be drawn to show the influence of the moving loads on the stress in an upper or lower chord member or in a tension or compression member of the web bracing.

Influence Line of Chord Stress.—In Fig. 224 let the girder AB be 128 feet span and 15 feet deep. Consider any member of the upper

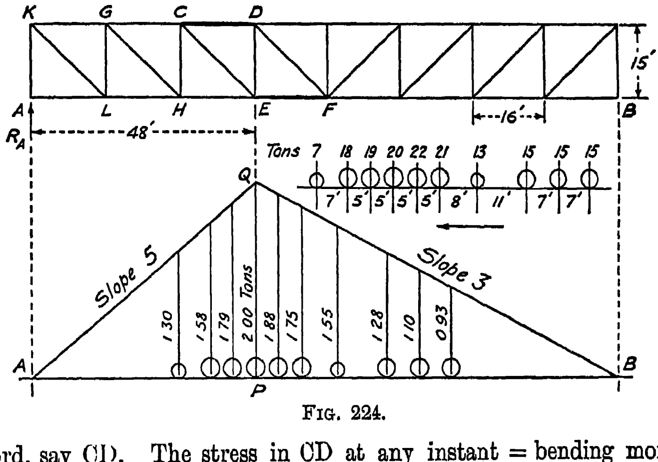


FIG. 224.

chord, say CD. The stress in CD at any instant = bending moment at section DE $\div$ depth of girder.

If a 1-ton load roll from A to B it will create the maximum bending moment at E and consequently the maximum stress in CD on reaching the point E. Placing 1 ton at E, $R_A = \frac{5}{8}$ ton. Bending moment at $E = \frac{5}{8} \times 48 = 30$ ft.-tons, and stress in CD $= \frac{30}{15} = 2$ tons compression. In the lower figure make PQ = 2 tons. AQB is the influence line of stress for CD. The ordinate of this diagram at *any* point measured to the scale of PQ gives the stress in CD when a load of 1 ton is *at that point*. The diagram is used in exactly the same manner as the preceding ones. Thus, suppose it be required to ascertain the maximum stress which may be created in CD by the locomotive and tender indicated by the given axle loads advancing from B towards A. The relative slopes of QA and QB are as 5 to 3.

From inspection of the loads it is probable that the 20 tons or 22 tons load at P may produce the maximum stress in CD. Place the

20 tons load at P and imagine a slight movement towards A. The ordinates of the 7, 18, 19, and 20 tons loads will be decreasing at the relative rate 5 and the ordinates of the remaining loads increasing at the relative rate 3. Hence, for movement to the left,

Total relative decrease of stress $= (7 + 18 + 19 + 20) \times 5 = 320$
" " increase " $= (22 + 21 + 13 + 15 + 15 + 15) \times 3 = 303$

A movement to the left from this position therefore causes a net *decrease* of stress in CD. Again placing the 20 tons load at P imagine a slight movement towards B. Loads 7, 18, and 19 tons are now influencing an increase of stress at the relative rate 5 and loads 20, 22, 21, 13, 15, 15, 15 are influencing a decrease at the relative rate 3. Hence for movement to right,

Total relative increase of stress $= (7 + 18 + 19) \times 5 = 220$
" " decrease " $= (20 + 22 + 21 + 13 + 15 + 15 + 15) \times 3 = 363$

That is, a net decrease of stress in CD occurs whether the loads be moved either to left or right of the position in which the 20-tons load is at P. This is therefore the position for maximum stress in CD.

NOTE.—In moving the loads to right or left of an assumed position, be careful to notice whether one or other of the end loads rolls off the span as a succeeding load is brought beneath the peak of the diagram for examination. This frequently occurs on small spans. Also note that the above numbers 220, 363, etc., do not represent any concrete units. They are simply comparative values indicating the occurrence of an increase or decrease of stress for a supposed movement.

Having ascertained the required position of the loads, the maximum stress in CD is obtained as follows. Mark the positions of the loads on the span with the 20 tons load at P. Draw the ordinates and scale off their values to the scale of PQ. Much labour will be saved if such diagrams are drawn fairly large on squared paper.

Maximum compression in CD $= (7 \times 1{\cdot}3) + (18 \times 1{\cdot}58) + (19 \times 1{\cdot}79) + (20 \times 2) + (22 \times 1{\cdot}88) + (21 \times 1{\cdot}75) + (13 \times 1{\cdot}55) + (15 \times 1{\cdot}28) + (15 \times 1{\cdot}10) + (15 \times 0{\cdot}93) = 259{\cdot}46$ tons.

Influence Line for EF.—Since, in an N-girder, the bending moment at E ÷ depth DE = either the compressive stress in CD or the tensile stress in EF, the influence diagram already drawn for CD also applies to EF, the values of the ordinates indicating tension instead of compression. Thus, the position of the loads which causes the maximum compression of 259·46 tons in CD will at the same instant cause a maximum tension of 259·46 tons in EF. Similarly the influence line for GC will also serve for HE and that for KG for LH.

Influence Line of Chord Stress in Girder of Variable Depth.—In girders of variable depth the same influence line does not apply to two chord members as CD and EF. The position of the unit load for maximum stress in either CD or EF is again at E (Fig. 225) but stress in EF = bending moment at E ÷ DE, whilst stress in CD = bending moment at E ÷ perpendicular distance EP.

In Fig. 225 AB = 90 ft. DE = 14 ft. and EP = 13·6 ft. Placing

1 ton at E, $R_A = \frac{2}{3}$ ton. Bending moment at $E = \frac{2}{3} \times 30 = 20$ ft.-tons.

Stress in CD $= 20 \div 13 \cdot 6 = 1 \cdot 47$ tons compression.

„ EF $= 20 \div 14 \cdot 0 = 1 \cdot 428$ tons tension.

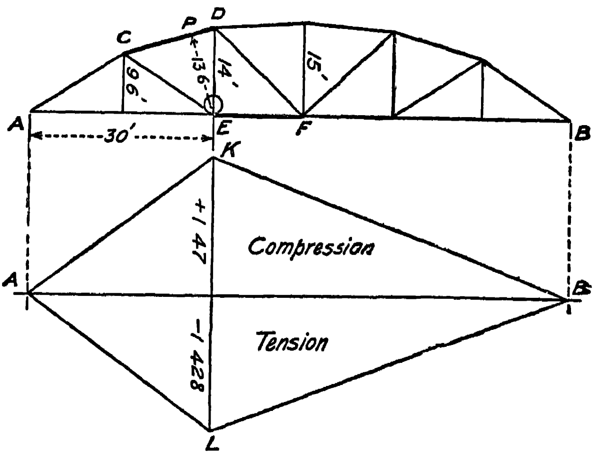


FIG. 225.

In the lower figure, AKB is the influence line of stress for CD and ALB that for EF.

Influence Line of Chord Stress in Warren Girder.—(*a*) *Through Bridge Girder*, with rolling load on lower joints, Fig. 226. For maximum stress in any lower chord member, say CD, the unit load must be placed in the vertical plane PQ, which position gives the maximum bending moment at P, whence maximum stress in CD = bending moment at section PQ ÷ depth *d*. But the 1-ton load at Q rests on the rail bearer bridging the interval between cross-girders C and D and is actually applied to the main girder as two concentrated loads of 0 5 ton each at the points C and D. The bending moment at PQ and therefore the maximum stress in CD is slightly less than if the whole load of 1 ton were applied *on* the main girder at P or Q. Let the span = 120 feet and depth = 18 feet. Suppose 1 ton actually applied at Q. $R_A = \frac{9}{12}$ ton. Bending moment at $Q = \frac{9}{12} \times 30' = 22 \cdot 5$ ft.-tons, and stress in CD $= 22 \cdot 5 \div 18 = 1 \cdot 25$ tons tension.

To construct the influence line for CD make EF $= 1 \cdot 25$, join FA and FB and project C and D to G and H. The influence line for CD is then AGHB.

In moving a group of axle loads to and fro under a diagram of this character having two peaks at G and H, the position of the loads which produces the maximum stress in CD will be such that *a* load *must be* under one or other of the peaks G or H. The relative slopes of AG, GH, and HB will be required. In this example, calling the panel length unity,

Slope AG $= \frac{5}{6}$ vertical to 1 horizontal $= \frac{5}{6} = \frac{15}{18}$
" GH $= (1\frac{1}{9} - \frac{5}{6})$ vertical to 1 horizontal $= \frac{5}{18}$
" HB $= 1\frac{1}{9}$ vertical to 4 horizontal $= \frac{5}{18}$

that is the relative slopes are as 3, 1 and 1.

Taking the locomotive and tender in Fig. 224, let it be required to find the position for maximum stress in CD. As a first trial place the 20 tons load at D under the peak H. Move the loads slightly to the left.

Relative decrease of stress $= (7 + 18 + 19 + 20) \times 1 = 64$
" increase " $= (22 + 21 + 13 + 15 + 15 + 15) \times 1$
$= 101$

Hence this movement causes an increase of stress in CD.

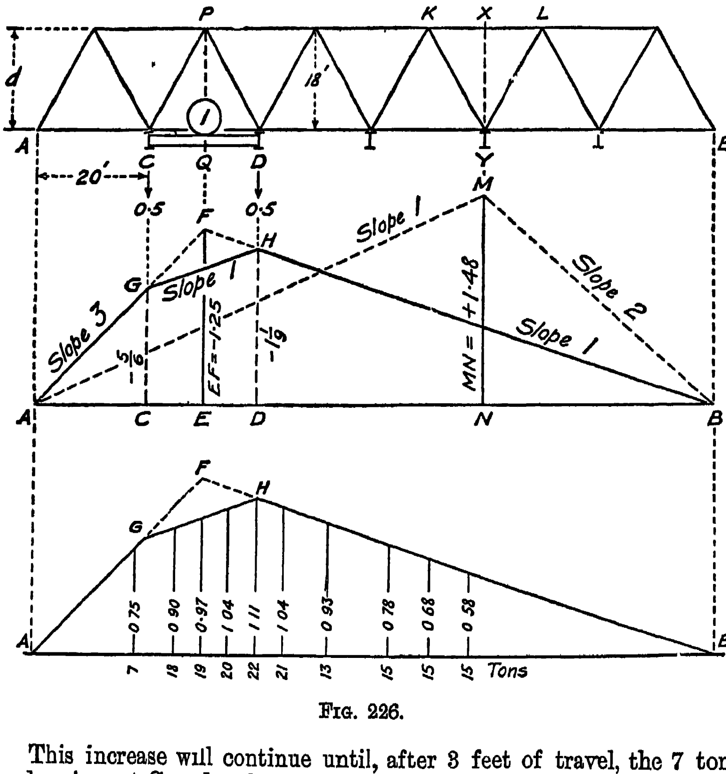


FIG. 226.

This increase will continue until, after 3 feet of travel, the 7 tons load arrives at C under the peak G. A further slight movement to the left now brings the 7 tons load beneath AG of slope 3, the 18, 19, and 20 tons loads are still moving beneath GH of slope 1, whilst the 22 tons load has not yet arrived at D.

Relative decrease $= (7 \times 3) + (18 + 19 + 20) \times 1 = 78$
" increase $= (22 + 21 + 13 + 15 + 15 + 15) \times 1 = 101$

Further movement to the left, therefore, still influences an increase. Continuing the movement, 22 tons arrives at D under the peak H

when the 7 tons load has passed 2 feet beyond C. As this position creates a new maximum value, again move the loads slightly to the left to ascertain if a further increase will take place.

$$\text{Relative decrease} = (7 \times 3) + (18 + 19 + 20 + 22) \times 1 = 100$$
$$\text{,, increase} = (21 + 13 + 15 + 15 + 15) \times 1 = 79$$

Hence, the stress in CD will *decrease* if the 22 tons load passes to the left of D. Ruling in the ordinates for this position (22 tons at D) and taking off their values to the scale of EF = 1·25 tons, the maximum stress in CD = (7 × 0·75) + (18 × 0·9) + (19 × 0·97) + (20 × 1·04) + (22 × 1·11) + (21 × 1·04) + (13 × 0·93) + (15 × 0·78) + (15 × 0·68) + (15 × 0·58) = 147·41 tons tension. Fig. 226 shows the position of the loads and their ordinates.

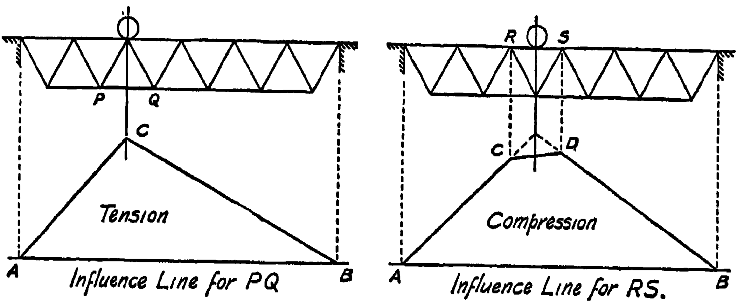


FIG. 227.

(*b*) **Warren Girder—Through Bridge—Influence Line for an Upper Chord Member, say KL, Fig. 226.**

The maximum stress in KL occurs simultaneously with maximum bending moment at section XY. Hence, place unit load at Y. The 1-ton load is actually applied to the main girder at Y, since at this

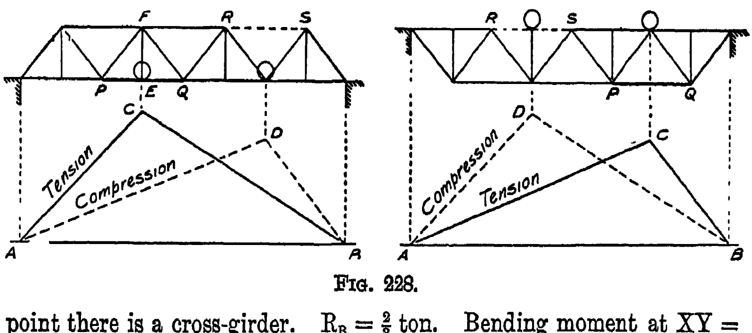


FIG. 228.

point there is a cross-girder. $R_B = \frac{2}{3}$ ton. Bending moment at XY = $\frac{2}{3} \times 40$ ft.-tons, and stress in KL due to unit load at Y = $\frac{2}{3} \times 40 \div 18$ = 1·48 tons compression. Making MN = 1·48 tons, AMB (shown dotted) is the influence line for KL. Its ordinates represent compressive stress and those of AGHB tension. The relative slopes of MA and MB are obviously 1 to 2. The application of this influence line introduces no new feature.

(*c*) **Warren Girder—Deck Bridge.**—In Warren girders used for deck bridges, with the load imposed on the upper chord joints, the above cases (*a*) and (*b*) become reversed, and the influence lines for lower and upper chord members are of the types shown in Fig. 227.

(*d*) **Warren Girder with Verticals—Through Bridge,** Fig. 228.—The unit load at E is here applied directly to the main girder in the vertical plane EF by means of the suspender EF which transfers the load to F. The influence line for PQ is ACB having only the one peak C. The influence line for RS is ADB.

(*e*) **Warren Girder with Verticals—Deck Bridge,** Fig. 228.—

ADB is the influence line for RS, its ordinates denoting compression,
and ACB " " PQ, " " tension.

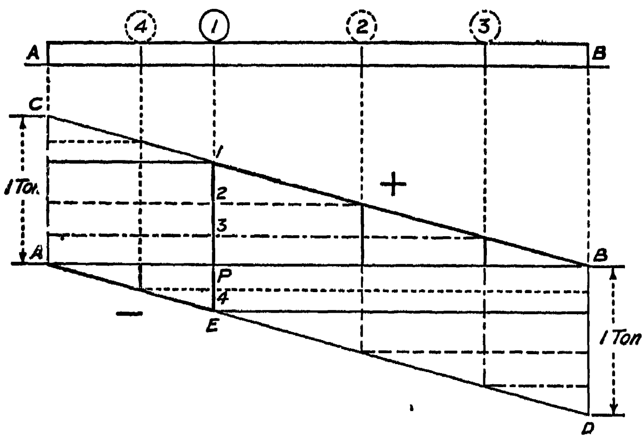


Fig. 229.

Influence Lines of Shear Stress.—Influence lines of shear stress are required for estimating the maximum vertical shear stress in the webs of plate girders, or the maximum stresses in the web members of braced girders due to particular positions of the rolling load.

Influence Line of Shear in a Girder with Solid Web.—Consider the span AB in Fig. 229.

A load of 1 ton at position 1 causes a positive shear = P1 at section P. A second load of 1 ton at position 2 causes a positive shear = P2 at P; and a third load of 1 ton at position 3 a positive shear = P3 at P. If all three loads be on the span together the total positive shear at P = P1 + P2 + P3 = the sum of the ordinates under the loads. These ordinates are cut off by the line BC which envelops the individual shear force diagrams due to the three loads, AC being = 1 ton. If a fourth 1-ton load be placed at position 4 to the left of P, it will cause a negative shear = P4 at P. P4 = the intercept under the load 4 cut off by AD, the enveloping line of the negative portions of the shear force diagrams. The diagram AE1B, shown by heavy lines, is the influence line of shear for the section P. With all four loads acting

the total shear at P = P1 + P2 + P3 − P4. For other than unit loads the ordinates will be multiplied by their respective loads.

Position of Loads for Maximum Shear at any Section.—In Fig. 230 let AB = 50 feet, and P be the section 20 feet from A. The positive shear at section P due to the group of loads advancing from B to A evidently continues to increase until the leading load arrives at P, since the ordinates of *all* the loads are increasing up to this point. As the leading load of 6 tons passes P, its ordinate changes from +PD to −CP = a total fall of 1 ton, which causes a sudden reduction of the positive shear = 6 × 1 tons.

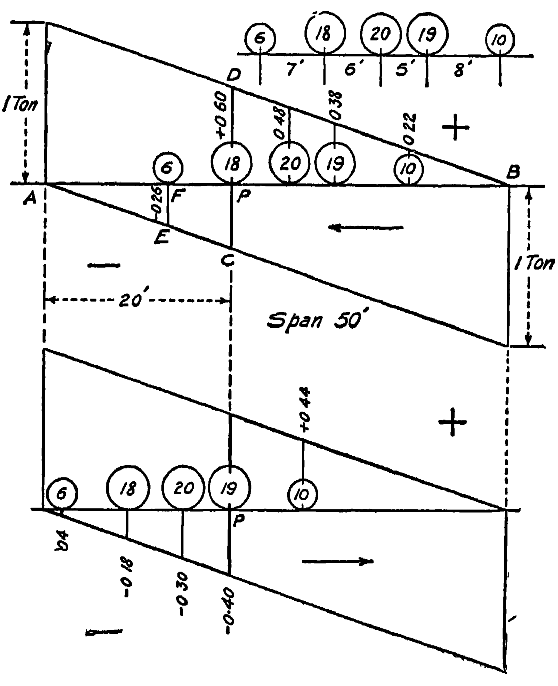


FIG. 230.

As the motion continues until the 18 tons load arrives at P the 6 tons load has moved 7 feet to the left and has *reduced* its *negative* ordinate from PC to EF, equivalent to an *increase* of positive shear $= \frac{7}{50} \times 6$ tons. The net effect of the movement of the 6 tons load from P to E is therefore to decrease the shear at section P by $-6 + (\frac{7}{50} \times 6)$ tons. In the same time the other loads have all moved 7 feet on the positive side of the diagram, thus causing an increase in positive shear at $P = (18 + 20 + 19 + 10) \times \frac{7}{50}$ tons. The total change of shear at P for the 7 feet advance is therefore $(6 + 18 + 20 + 19 + 10) \times \frac{7}{50} - 6$ tons $= + 4{\cdot}22$ tons, which being positive, indicates that an increased positive shear occurs when the

18 tons load is at P than when the 6 tons load is at P. Continuing the motion a further 6 feet to the left until 20 tons arrives at P, the further change in shear at $P = (6 + 18 + 20 + 19 + 10) \times \frac{6}{50} - 18 = -9{\cdot}24$ tons, which being negative, indicates a less positive shear for the 20 tons load at P than for the 18 tons load at P. Hence the position for maximum positive shear at P is when the 18 tons load arrives at P. Placing the loads in this position and scaling off the ordinates,

Maximum positive shear at $P = (-6 \times 0{\cdot}26) + (18 \times 0{\cdot}6) + (20 \times 0{\cdot}48) + (19 \times 0{\cdot}38) + (10 \times 0{\cdot}22) = 28{\cdot}26$ tons.

For maximum negative shear at P if the loads advance from A with 10 tons leading, the negative shear increases until 10 tons arrives at P. The 6 tons load is off the span 6 feet behind A. A further movement of 8 feet towards B brings 19 tons to P, the 10 tons load has passed to the positive side of the diagram and the 6 tons load has advanced 2 feet on to the span. Hence,

$$\text{Change in shear} = \left(-\tfrac{2}{50} \times 6\right) - (18 + 20 + 19 + 10) \times \tfrac{8}{50} + (10 \times 1) = -0{\cdot}96 \text{ ton},$$

that is, an increase of negative shear takes place. The negative shear for this position is obviously the maximum, since if 19 tons passes P a large increase of positive shear takes place. Hence,

Maximum negative shear at $P = (-6 \times 0{\cdot}04) - (18 \times 0{\cdot}18) - (20 \times 0{\cdot}3) - (19 \times 0{\cdot}4) + (10 \times 0{\cdot}44) = -12{\cdot}68$ tons.

Influence Lines of Shear Stress in Braced Girders.

1. Parallel Girder—Influence Line of Shear Stress in a Diagonal Tie.—In Fig. 231 consider the diagonal PQ. Let a load of 1 ton roll over the span from A to B. When the load reaches L, $R_A = \frac{5}{6}$ ton and vertical shear in panel LQ to right of the load $= +\frac{5}{6} - 1 = -\frac{1}{6}$ ton. But this is the vertical component of the stress in PQ. For a member sloping in the direction PQ a negative shear causes compression. Hence, for 1 ton at L, direct compression in $PQ = \frac{1}{6} \text{ ton} \times \frac{PQ}{PL} = \frac{1}{6} \text{ ton} \times \sqrt{2}$. Make $EC = \frac{1}{6} \times \sqrt{2}$ ton.

When the load is just entering the span at A there is no shear in the panel LQ. The shear stress in PQ therefore increases from zero with load at A to EC (compression) with load at L.

Now move the 1 ton load across the panel LQ to Q. $R_A = \frac{2}{3}$ ton and vertical shear in panel LQ to left of load $= +\frac{2}{3}$ ton. But a positive shear in this panel causes tensile stress in PQ the vertical component of which is $\frac{2}{3}$ ton. Hence, direct tension in $PQ = \frac{2}{3} \text{ ton} \times \frac{PQ}{PL} = \frac{2}{3} \text{ ton} \times \sqrt{2}$. Make $FD = \frac{2}{3} \times \sqrt{2}$ ton to the same scale as used for EC. Join CD and DB. ACDB is the influence line of shear stress for PQ. The diagram is most quickly constructed by making $AG = BH = 1 \text{ ton} \times \frac{PQ}{PL}$, joining GB and AH and projecting the panel points L and Q to C and D.

By similar triangles EC $= \frac{1}{6}$BH and DF $= \frac{2}{3}$AG. As the load moves from E to F the stress in PQ changes first from EC tons compression to zero at Z and then increases from zero to FD tons tension when the load arrives at F. From F to B the tensile stress uniformly diminishes to zero, since with load at B there is no shear in panel LQ.

In order to examine the influence of a group of moving loads on the stress in PQ, the relative slopes of AC, CD, and DB are required. These are—

Slope AC = EC vertical to AE horizontal $= \frac{1}{6}$ to $1 = \frac{1}{6}$
„ CD = (EC + FD) vert. to EF hor. $= (\frac{1}{6} + \frac{2}{3})$ to $1 = \frac{5}{6}$
„ DB = FD vertical to FB horizontal $= \frac{4}{6}$ to $4 = \frac{1}{6}$

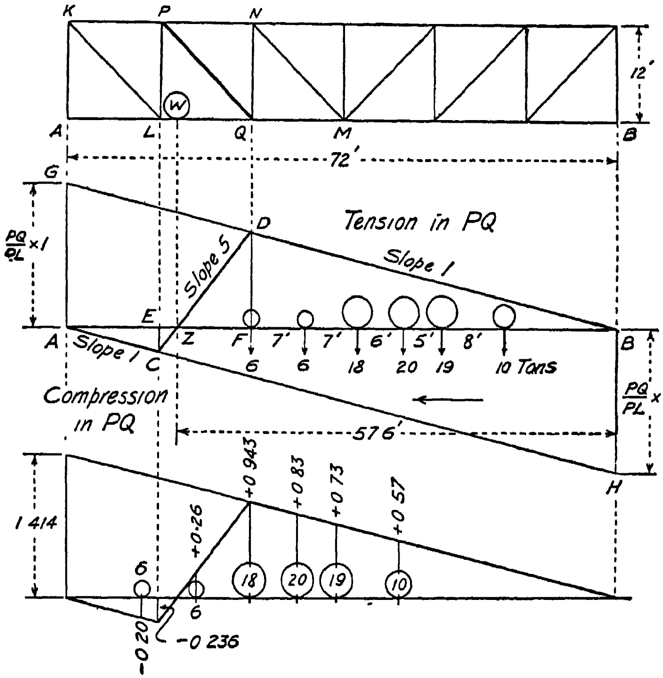


FIG. 231.

The relative slopes are therefore as 1 : 5 : 1, or generally if n = number of panels between A and B, as $1 : n - 1 : 1$

Position of Loads for Maximum Tension in PQ.—Let the given loads, Fig. 231, advance from right to left. Leading 6 tons at F. Move to left—

Relative increase = (6 + 18 + 20 + 19 + 10) × 1 = 73
„ decrease = 6 × 5 = 30

Second 6 tons at F. Move to left—

Relative increase = (18 + 20 + 19 + 10) × 1 = 67
„ decrease = (6 + 6) × 5 = 60

The tension in PQ therefore continues to increase after the second 6 tons load has passed F. Before the 18 tons load arrives at F, the leading 6 tons load reaches E. This load, which so long as it was moving under DC, was influencing a rapid decrease of stress in PQ, will, after passing E, contribute towards an *increase* of stress, since, whilst still *causing* compression in PQ, its *compression* ordinate will now be *decreasing*. Placing the leading 6 tons at E and moving to the left—

$$\text{Relative increase} = (6 + 18 + 20 + 19 + 10) \times 1 = 73$$
$$\text{,, decrease} = 6 \times 5 = 30$$

A further movement of 2 feet brings 18 tons to F. Let 18 tons pass to the left of F.

$$\text{Relative increase} = (6 + 20 + 19 + 10) \times 1 = 55$$
$$\text{,, decrease} = (6 + 18) \times 5 = 120$$

Evidently a maximum value occurs when 18 tons is at F. Other maximum values may occur as other loads from the rear arrive at F, according as heavier loads have passed E, whilst lighter loads may be moving under DC; but the greatest maximum may readily be identified by summing up the ordinates if doubt exists. Moreover, as more of the loads pass to the left compression side of the diagram, the compression caused by them neutralises an increasing amount of the tension caused by the loads on the tension side. In this case no greater maximum occurs for a further movement of the loads, and with 18 tons at F the respective ordinates have the values in Fig. 231, and

$$\text{Maximum tension in PQ} = (10 \times 0{\cdot}57) + (19 \times 0{\cdot}73) + (20 \times 0{\cdot}83) + (18 \times 0{\cdot}943) + (6 \times 0{\cdot}26) - (6 \times 0{\cdot}20) = 54{\cdot}70 \text{ tons.}$$

The maximum compression in PQ is obtained in a similar manner by advancing the loads in the same order, from left to right, over the compression side of the diagram. A brief inspection will show that the maximum compression occurs when the 10 tons load is at E and 19 tons 4 feet from A, its value being $= -(0{\cdot}236 \times 10) - (0{\cdot}08 \times 19) = -3{\cdot}88$ tons, the minus sign indicating compression.

Position of Uniformly Distributed Load for Maximum Stress in a Diagonal.—Referring to Fig. 231 a uniform load advancing from B must cover the span from B to Z in order to cause the maximum tension in PQ. The value of the maximum tension then = area ZDB × load intensity. If the head of the load passes Z, the load overlaps a portion of the negative area ACZ and the stress in PQ is reduced, being = area ZDB − negative area overlapped by load. The position of Z is readily found thus, EZ : ZF : : EC : FD or as 1 : 4. ∴ EZ = $\frac{1}{5}$ EF = $\frac{1}{5} \times 12$ ft. = $2\frac{2}{5}$ feet. Hence, ZB = 57·6 ft. Area ZDB = $\frac{1}{2} \times 57{\cdot}6 \times 0{\cdot}943 = 27{\cdot}158$. Suppose the load to be 2 tons per foot run, then maximum tension in PQ = $27{\cdot}158 \times 2 = 54{\cdot}31$ tons.

Incidentally it may be noticed that a distributed load of 2 tons per foot run creates (in this diagonal) almost exactly the same maximum tension as the above axle loads. The maximum compression in PQ due to a distributed load occurs when the load covers AZ, and for a load of

2 tons per foot run = area ACZ $\times 2 = \frac{1}{2} \times 14{\cdot}4 \times 0{\cdot}236 \times 2 = 3{\cdot}4$ tons.

Position of Zero Point Z.—In Fig. 232 the influence lines for all the diagonals are shown. The points Z_0, Z_1, Z_2, Z_3, Z_4, and Z_5 are the points at which the head of a distributed load advancing from B must arrive to create the maximum tension in the corresponding diagonals.

Points Z_1, Z_2, etc., are also the points where a rolling concentrated load causes no stress in the diagonal concerned. It will be seen that reversal of stress due to rolling load takes place in all but the two end diagonals. In a girder as built, the diagonals on the right of the centre are of course reversed and the ordinates of their influence lines also reverse their significance, the lower side of the diagram becoming the tension side and the upper the compression side.

Referring to Fig. 231, EZ : ZF :: ZC : ZD or as AZ : ZB. That is, the point Z divides the panel EF in the same ratio as it divides the

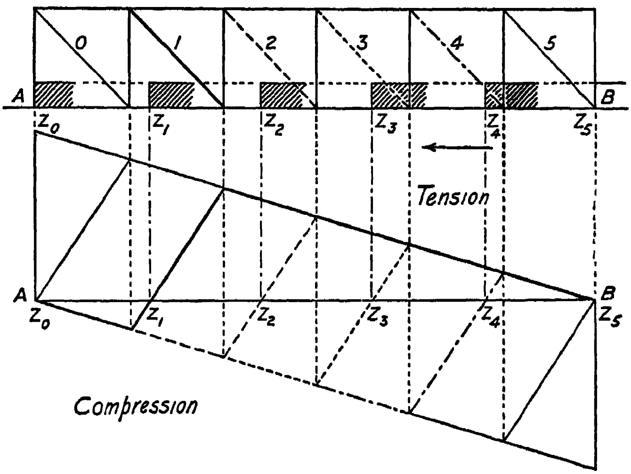


FIG 232.

span AB. A concentrated load W at Z rests directly on the longitudinal rail bearer EF and is transferred by the cross girders at E and F to the main girder, as two loads in the ratio of ZF and ZE respectively. Hence cross girder load at E $= W \times \frac{ZF}{EF}$ and $R_A = W \times \frac{ZB}{AB}$. But $\frac{ZF}{EF} = \frac{ZB}{AB}$, $\therefore$ vertical shear in panel EF $= R_A -$ load at E $= 0$, for this position of the load.

In Fig. 232 the positions Z_1, Z_2, Z_3, etc., of the head of a distributed rolling load for maximum stresses in the diagonals should be compared with the positions assumed in the general method for rolling load stresses given on p. 245, where the maximum stress in the diagonals was assumed to occur when the head of the equivalent distributed load

reached the centre of the panel in every case. The result is sufficiently closely correct in practice, but the influence line method is, of course, exact. The maximum tension in PQ by the approximate method is 56·56 tons as against 54·7 tons by the exact method. The former approximate method (p. 245) gives exact results for a uniform load extending from B to Q (Fig. 231) and headed by a concentrated load at Q equal to half a panel length of distributed load. The results of the two methods coincide for the diagonals in the end panels.

2. Parallel Girder—Influence Line of Shear Stress in a Vertical Strut.—At any instant the vertical component of the tension in PQ, Fig. 231, equals the vertical compression in PL. The influence line for PL is therefore similar to that for PQ, but having end ordinates AG and BH = 1 ton instead of $\frac{PQ}{PL} \times 1$ ton.

Also the area ZDB becomes the compression area and ACZ the tension area. The influence line for MN with the same modifications will also apply to NQ, and that for LK to KA.

3. Girder of Variable Depth—Influence Line of Shear Stress in a Diagonal Tie.—In Fig. 233 let AB be a girder of 90 feet span, 15 feet panel width, and having depths as figured. To draw the influence line for PQ.

Place 1 ton at L, the position for maximum compression in PQ. To find the resulting stress in PQ cut the panel LQ by a section X—X and take moments about C, the point of intersection of NP and QL produced, of the forces to the left of section X—X. Point C falls 37·5 feet from L. This distance is preferably calculated thus—

In similar triangles CLP and CQN, $\frac{CL}{CQ} = \frac{PL}{NQ} = \frac{10}{14}$. But CQ = CL + 15′, $\therefore \frac{CL}{CL + 15} = \frac{10}{14}$, whence CL = 37·5 feet. CD, the perpendicular distance of C from QP produced, is also required.

From similar triangles CDQ and PLQ, $\frac{CD}{CQ} = \frac{PL}{PQ}$, *i.e.* $\frac{CD}{52 \cdot 5} = \frac{10}{18 \cdot 027}$, whence CD = 29·12 feet.

Take moments about C. The stresses in PN and LQ have no moment about C. The other forces to left of X—X are R_A acting upwards and tending to cause tension in PQ, the 1-ton load at L acting downwards and tending to cause compression in PQ, and the stress in PQ acting at the leverage of 29·12 feet about C. Call R_A negative and the load at L positive.

$-(R_A \times 22 \cdot 5) + (1 \times 37 \cdot 5) =$ stress in PQ $\times$ 29·12
$R_A = \frac{5}{6}$ ton for 1 ton at L.
$\therefore -18 \cdot 75 + 37 \cdot 5 =$ stress in PQ $\times$ 29·12
whence stress in PQ = + 0·644 ton, the plus sign denoting compression.

On the influence diagram make GE = 0·644 ton.

Now move the 1-ton load to Q, the position for maximum tension

in PQ. $R_A = \frac{2}{3}$ ton and there is no load to the left of X—X. Take moments about C

$$-(R_A \times 22\cdot5) = \text{stress in PQ} \times 29\cdot12$$
$$-(\tfrac{2}{3} \times 22\cdot5) = \quad ,, \quad PQ \times 29\cdot12$$

whence stress in PQ $= -0\cdot515$ ton, the minus sign denoting tension. On the influence diagram make HF $= 0\cdot515$ ton. AEFB is the

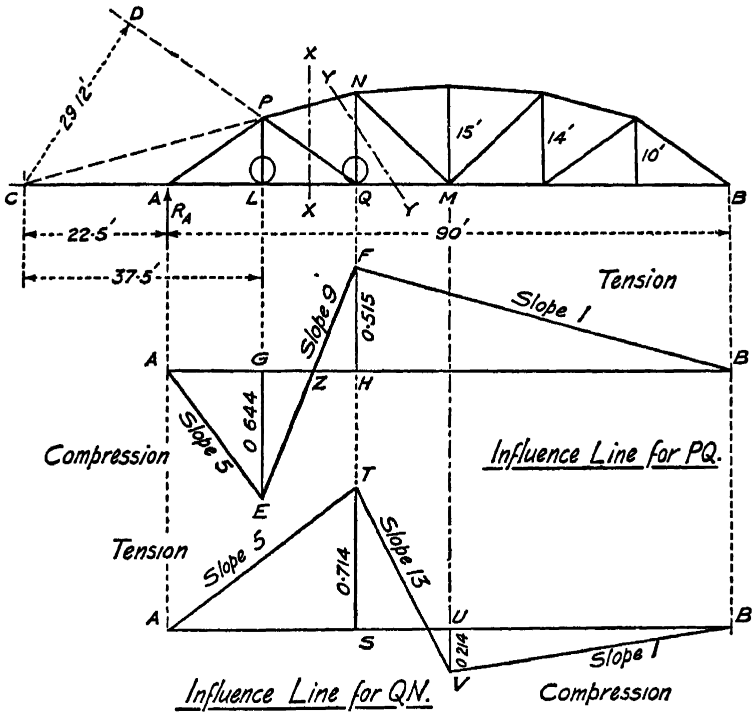


FIG 233.

influence line for PQ. For maximum tension in PQ a distributed load must cover the length BZ, and the magnitude of the tension = area ZFB × load intensity For maximum compression in PQ the length AZ must be covered and maximum compression = area ZEA × load intensity. For a group of concentrated loads the diagram is used in the same manner as the preceding one for a parallel girder, having regard to the relative slopes of AE, EF, and FB.

In this example the relative slopes are

$$\left.\begin{array}{ll} \text{AE, } 0\cdot644 \text{ vertl. to 1 horl.} & = 0\cdot644 \\ \text{EF, } (0\cdot644 + 0\cdot515) \text{ vertl. to 1 horl.} & = 1\cdot159 \\ \text{FB, } 0\cdot515 \text{ vertl. to 4 horl.} & = 0\cdot129 \end{array}\right\} \begin{array}{c}\text{or} \\ \text{as}\end{array} \left\{\begin{array}{l} 5 \\ 9 \\ 1 \end{array}\right.$$

It should be noted that for a girder of variable depth AE and FB are not parallel.

4. Girder of Variable Depth—Influence Line of Shear Stress in a Vertical Member.—Consider the member NQ, Fig. 233. To find the stress in NQ cut NP, NQ, and QM by section Y—Y and produce NP and MQ to meet at C, which is also the centre of moments for NQ. Place 1 ton at Q, the position for maximum tension in NQ. $R_A = \frac{2}{3}$ ton.

Take moments about C of the forces to left of Y—Y.

$$+ (R_A \times 22\cdot5) - (1 \times 52\cdot5) = \text{stress in NQ} \times CQ$$
$$+ (\tfrac{2}{3} \times 22\cdot5) - 52\cdot5 \quad = \quad ,, \quad NQ \times 52\cdot5$$

whence stress in NQ $= -0\cdot714$ ton (tension).

Now move the 1-ton load to M, the position for maximum compression in NQ. $R_A = \frac{1}{2}$ ton and there is no load to left of Y—Y.

Take moments about C.

$$+ (R_A \times 22\cdot5) = \text{stress in NQ} \times CQ$$
$$+ (\tfrac{1}{2} \times 22\cdot5) \;=\; ,, \quad NQ \times 52\cdot5$$

whence stress in NQ $= +0\cdot2142$ ton (compression).

In the lower diagram, make ST $= 0\cdot714$ ton and UV $= 0\cdot214$ ton ATVB is the influence line for NQ.

The relative slopes of AT, TV, and VB are,

$$\left.\begin{array}{lll} \text{AT,} & 0\cdot714 \text{ vertl. to 2 horl.} & = 0\cdot357 \\ \text{TV,} & (0\cdot714 + 0\cdot214) \text{ vertl. to 1 horl.} & = 0\cdot928 \\ \text{VB,} & 0\cdot2142 \text{ vertl. to 3 horl.} & = 0\cdot0714 \end{array}\right\} \begin{array}{c}\text{or}\\ \text{as}\end{array} \left\{\begin{array}{c} 5 \\ 13 \\ 1 \end{array}\right.$$

CHAPTER IX.

DEFLECTION.

In all structures changes of length of the separate members are produced by the compressive or tensile stresses in the members, and although the change in length of any individual member is comparatively very small, yet the compounded elongations and contractions of the successive members produces an appreciable displacement of certain points in the structure. The vertical component of such displacement is termed the deflection of that point. The importance of the magnitude of the deflection will vary according to the class of structure under consideration. For bridges and girders in general, it is usual to specify a limiting ratio of deflection to span to ensure that no serious changes of length take place in any of the members, and the deflection under test loads is often measured for that purpose; but such factors as the relative stiffness of the connections, workmanship, etc., may so alter the theoretical distribution of stress in the members that the deflection can in nowise be assumed as a measure of the strength of the structure, or even an evidence that no member of the structure is being unduly stressed.

However, in many structures such as the arms of swing-bridges, cantilevers, arches, etc., that are being erected by building out from the ends, it is of great importance to estimate to a close degree of accuracy the deflection that will occur, to ensure the seatings being placed at the correct levels, the overhanging ends meeting accurately, etc.

Live loads produce greater deflections than static loads of the same magnitude owing to the dynamic action of such loads. The deflection produced by a suddenly applied load to, say, a crane girder, would be double the deflection produced by a similar static load, but would be of only momentary duration, and as the vertical vibration ceased such deflection would be reduced to the magnitude of the static deflection. The increase in deflection produced by the live loads on bridges will vary according to the span, velocity of the moving loads, etc., and it is impossible to establish an exact law for these conditions, but by making due allowance for impact when calculating the stresses in the members, a close approximation to the deflection will be obtained.

Deflection due to Static Loading.—Providing the elastic limit is not exceeded, the change in length in any member will be—

$$l = \frac{Ls}{E}$$

where L = the original length of the member
l = the change in length of the member
s = the intensity of stress
E = the modulus of elasticity of the material.

Or if S_1 = the total stress in the member
and A = the sectional area of the member

then $$l = \frac{LS_1}{EA}$$

l, L and A, and S_1 and E, being measured in the same units.

Consider the action of the force P applied gradually to the girder in Fig. 234. To produce the changes in lengths of the members forming the girder a certain amount of work is performed. If Δ be the deflection of the point of application of P, the work performed during that deflection $= \frac{1}{2}\Delta P$. As this is the only external work done, it must be equal to the total internal work done in the girder. The work done in straining any member is equal to half the product of the stress in the member, and the change in length $= \frac{S_1 l}{2}$.

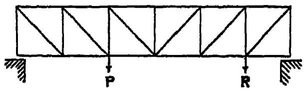


FIG. 234.

But $$l = \frac{LS_1}{EA}$$

Therefore the internal work performed on any member

$$= \frac{S_1^2 L}{2EA}$$

and the total internal work is the sum of the work performed on each member.

Therefore $$\tfrac{1}{2}P\Delta = \Sigma \frac{S_1^2 L}{2EA}$$

or $$\Delta = \frac{1}{PE} \Sigma \frac{S_1^2 L}{A}$$

Calculating $\frac{S_1^2 L}{A}$ for each member and dividing the sum of such terms by the product of P and E, the deflection of the loaded joint is obtained.

Let an additional load R be placed anywhere on the girder, and the stress in any bar which was stressed S_1 under the action of P only, be now equal to S_2. The total work performed on such bar

$$= \tfrac{1}{2}S_2(l + l_1)$$

where l = the change in length due to the load P
l_1 = " " " R

The work performed on the bar by P therefore

$$= \tfrac{1}{2}S_2 l$$

$$= \tfrac{1}{2}S_2\left(\frac{LS_1}{EA}\right)$$

The total internal work performed by P is the sum of the work done in the members

$$= \Sigma \frac{S_1S_2L}{2EA}$$

Therefore $$\tfrac{1}{2}P\Delta = \Sigma \frac{S_1S_2L}{2EA}$$

and $$\Delta = \frac{1}{PE}\Sigma\frac{S_1S_2L}{A}$$

where Δ is the deflection of the point of application of P.

Since the load R has been assumed anywhere on the girder, the above expression is applicable to any system of loading, but attention must be paid to the signs of S_1 and S_2, and the *algebraic* sum of the terms $\frac{S_1S_2L}{A}$ taken. If instead of considering the work performed by the whole force P, the work performed by a unit portion of such load be considered, the deflection will be obtained from the above expression by substituting for S_1 the stress in the members due to the unit load and replacing P by unity. Let S be the stress in any member due to a unit portion of the load P.

Then $$\Delta = \frac{1}{E}\Sigma\frac{SS_2L}{A}$$

The deflection of any other point in the girder may be obtained by substituting for S the stress in the members due to unit load at such point.

EXAMPLE 32.—*To find the deflection at the middle of the span of a braced girder.*

Let Fig. 235 represent the girder and the static loads at the panel points.

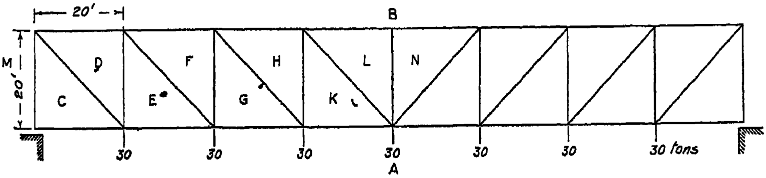


FIG. 235.

The stresses in columns 1 and 2 of the following table have been calculated by the methods described in Chapter VII., and E assumed as 14,000 tons. In practice the sectional areas of the members are taken from the completed design.

Member.	Stress, S_2.	Stress, S	Length, L	Sectional area, A.	$\frac{S_2SL}{A}$
	tons	tons.	ft	sq. in	
AC	− 0	0	20	15	+ 0
AE	−105	$-\frac{1}{2}$	20	18	58·33
AG	−180	−1	20	30	120·0
AK	−225	$-1\frac{1}{2}$	20	38	177·6
BD	+105	$+\frac{1}{2}$	20	26	40·4
BF	+180	+1	20	45	80·0
BH	+225	$+1\frac{1}{2}$	20	56	120 5
BL	+240	+2	20	60	160·0
MC	+105	$+\frac{1}{2}$	20	25	42·0
CD	$-105\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	$20\sqrt{2}$	24	$87\ 5\sqrt{2}$
DE	+ 75	$+\frac{1}{2}$	20	20	37·5
EF	$-75\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	$20\sqrt{2}$	17	$88{\cdot}2\sqrt{2}$
FG	+ 45	$+\frac{1}{2}$	20	12	37·5
GH	$-45\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	$20\sqrt{2}$	12	$75\sqrt{2}$
HK	+ 15	$+\frac{1}{2}$	20	5	30
KL	$-15\sqrt{2}$	$-\frac{1}{2}\sqrt{2}$	$20\sqrt{2}$	6	$50\sqrt{2}$
LN	0	0	20	5	0
		Total for the half truss . . .			1,325
		" whole " . .			2,650
		Multiplying by 12 to reduce L to inches			12
				$\Sigma\frac{S_2SL}{A}$ =	31,800

$$\Delta = \frac{1}{E}\Sigma\frac{S_2SL}{A} = \frac{31800}{14000} = 2{\cdot}27 \text{ in.}$$

Horizontal Displacement.—The horizontal displacement of any point in a structure may be found by a similar method. If a unit horizontal load be applied at such point, the work performed by this load will be half the product of the displacement and unity. The total internal work will

$$= \Sigma\frac{S_3S_2L}{2EA}$$

where S_3 = the stress in the members due to the horizontal unit load.

Let H = the horizontal displacement

Then $$\tfrac{1}{2}H = \frac{1}{2E}\Sigma\frac{S_3S_2L}{A}$$

or $$H = \frac{1}{E}\Sigma\frac{S_3S_2L}{A}$$

EXAMPLE 33.—*To find the total displacement of the left-hand end P of the cantilever in Fig.* 186. The outline of the girder is reproduced in Fig. 236.

In the following table suitable sectional areas for the members have been inserted. In a bridge of this type, the superstructure would be securely fixed to one of the piers at M or N and allowed to slide horizontally on the other. It will be assumed in this example that N is the fixed point and horizontal movement may take place at M. This assumption in no way affects the vertical deflection of the point P, but has a direct bearing on the horizontal displacement. If M be fixed and

N capable of horizontal movement, the horizontal displacement of P includes the contraction of the member MN, but by fixing N the contraction of MN does not affect the horizontal displacement of P. It will be noticed that the stress in the members DT and ET due to

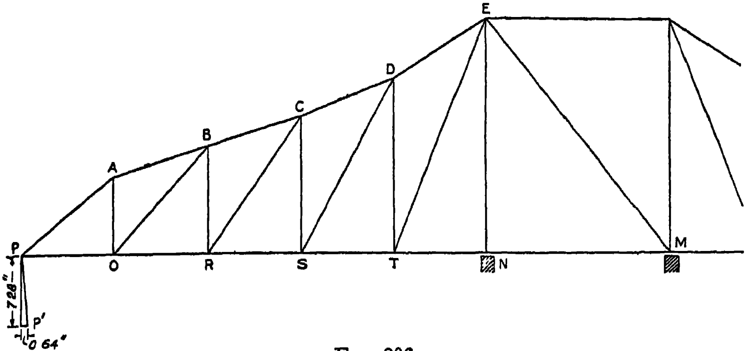


FIG. 236.

the vertical unit load at P is opposite in sign to the actual stress in those members, and therefore the values of $\frac{S_2SL}{A}$ will be negative and must be subtracted from the sum of the positive values in the other members. The unit horizontal load creates stresses in the members of the lower boom only, and the values of $\frac{S_3S_2L}{A}$ for all other members will be zero.

Member	Sectional area, A	Length, L.	Total stress, S_2	Unit load stress, S.	$\frac{S_2SL}{A}$	Unit load stress, S_3	$\frac{S_3S_2L}{A}$
	sq in.	ft	tons	tons		ton.	
NT	180	50	+548 8	+2·08	+ 317·1	1	+152·4
TS	153	50	+460	+2 22	+ 333·7	1	+150 3
SR	110	50	+325 7	+2 14	+ 316 8	1	+148 1
RO	63	50	+179 1	+1 81	+ 257·3	1	+142·1
OP	19	50	+ 58 8	+1 25	+ 193 4	1	+154 7
ED	120	58 3	−536·3	−2 59	+ 674 8	—	—
DC	81	53 9	−351 1	−2 31	+ 539 7	—	—
CB	43	52·2	−188	−1 89	+ 431·3	—	—
BA	15	52 2	− 61·7	−1 305	+ 280 2	—	—
AP	17	64	− 75 3	−1 88	+ 532 9	—	—
EN	280	120	+836 7	+3 5	+1255·1	—	—
DT	42	90	+122·1	−0 33	− 86 3	—	—
CS	45	70	+152 7	+0·14	+ 33 2	—	—
BR	30	55	+118 2	+0·46	+ 99·7	—	—
AO	12	40	+ 48 3	+0 61	+ 98 2	—	—
ET	39	130	−230·8	+0 36	− 276·9	—	—
DS	48	103	−276 6	−0·16	+ 95·0	—	—
CR	42	86	−252 1	−0 57	+ 294 2	—	—
BO	30	74·3	−178·7	−0·83	+ 367·3	—	—
EM	100	156 2	−406·9	−3 25	+2065·6	—	—
NM	180	100	+548 8	+2·2	+ 670·7	—	—
					+8493·0		+747 6

Vertical displacement of P

$$= \Delta = \frac{8493 \cdot 0 \times 12}{14{,}000}$$

$$= 7 \cdot 28 \text{ in.}$$

Horizontal displacement of P

$$= \mathrm{H} = \frac{747 \cdot 6 \times 12}{14{,}000}$$

$$= 0 \cdot 64 \text{ in.}$$

Therefore when the bridge is loaded so as to produce the stresses S_2 in the members, the point P would be displaced to P_1, as shown in Fig. 236, the scale for the displacements being greatly exaggerated.

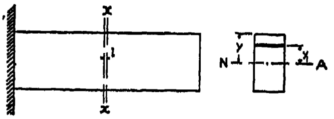


Fig. 237.

Beams with Solid Webs.—The deflection of beams having solid webs may be found by the application of the foregoing principles. Consider the work done on any section x–x, Fig. 237, by a unit load placed at the point at which the deflection is required.

Let s = the skin stress due to any system of loading,
s_1 = the skin stress due to the unit load.

Then the stress on an elementary strip situated at a distance y_1 from the neutral axis and of area a

$$= as\frac{y_1}{y} \text{ due to the total loads}$$

$$= as_1\frac{y_1}{y} \quad \text{,,} \quad \text{unit load}$$

Let l = the length of the strip.
Then the extension due to the unit load

$$= \frac{as_1\frac{y_1}{y}l}{a\mathrm{E}}$$

The work performed by the unit load on the strip

$$= \tfrac{1}{2}\left(as\frac{y_1}{y} \times s_1\frac{y_1 l}{y\mathrm{E}}\right)$$

$$= \frac{ss_1 a y_1^2 l}{2y^2\mathrm{E}}$$

The work performed on the whole cross-section

$$= \frac{1}{2\mathrm{E}}\Sigma\frac{ss_1 a y_1^2 l}{y^2}$$

But $\Sigma a y_1^2$ = the moment of inertia of the section = I.
Let M = the bending moment at the section due to the total loads.
m = ,, ,, ,, unit load.

Then
$$s = \frac{My}{I}$$
$$s_1 = \frac{my}{I}$$

Substituting these values in the above expression, the work performed on the section

$$= \frac{1}{2E} \Sigma \frac{Mml}{I}$$

The work performed by the unit load on the total length of the beam will be obtained by substituting dx for l and integrating. This internal work must equal the external work, or

$$\tfrac{1}{2}\Delta = \frac{1}{2E}\int \frac{Mm}{I} . dx$$
$$\therefore \Delta = \frac{1}{E}\int \frac{Mm}{I} dx$$

For beams having a constant cross-section throughout their length I will be constant, and

$$\Delta = \frac{1}{EI}\int Mmdx$$

EXAMPLE 34.—*To find the deflection at the end of a cantilever of constant cross-section when supporting a load P at its outward end.*

Let l = the length of the beam in inches. Then at any section distant x from P

$$M = -Px$$
$$m = -x$$
$$\Delta = \frac{1}{EI}\int_0^l Px . x . dx$$
$$= \frac{1}{EI} \frac{Pl^3}{3}$$

The following general formulæ for beams of constant cross-section may be deduced in a similar manner.

TABLE 27 —DEFLECTION OF BEAMS OF CONSTANT CROSS-SECTION.

	Maximum deflection
Cantilever	
Single concentrated load at the end . . .	$\Delta = \frac{Wl^3}{3EI}$
Uniformly distributed load over the entire length.	$\Delta = \frac{Wl^3}{8EI}$
Beam simply supported at the ends.	
Single concentrated load at a distance b from one end	When $b = \frac{l}{2}$, $\Delta = \frac{Wl^3}{48EI}$
	When $b > \frac{l}{2}$, $\Delta = \frac{Wb(l-b)(2l-b)}{9EIl}\sqrt{\frac{b(2l-b)}{3}}$
Uniformly distributed load over the entire length. .	$\Delta = \frac{5Wl^3}{384EI}$

where Δ = deflection in inches.
W = total load in tons.
l = span in inches.
E = modulus of elasticity in tons per square inch.
I = moment of inertia.

Beams of varying Cross-section.—If the moment of inertia of the cross-section of a beam is not constant throughout the entire length of the beam the deflection will be

$$\Delta = \frac{1}{E}\int \frac{Mm}{I} \, . \, dx$$

For the plate girder of Fig. 238, in which the moment of inertia of the central length is I, and for the remaining portions I_1

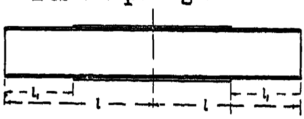


FIG. 238.

$$\Delta E = 2\int_{l_1}^{l} \frac{Mm}{I} dx + 2\int_0^{l_1} \frac{Mm}{I_1} dx$$

To find the deflection at the centre if the girder carries a load of w tons per inch run. For any section distant x from one support

$$M = wlx - \tfrac{1}{2}wx^2$$
$$m = \tfrac{1}{2}x$$

$$\therefore \Delta E = 2\int_{l_1}^{l} \frac{w\left(\frac{lx^2}{2} - \frac{x^3}{4}\right)}{I} dx + 2\int_0^{l_1} \frac{w\left(\frac{lx^2}{2} - \frac{x^3}{4}\right)}{I_1} dx$$

and $$\Delta = \frac{wl}{3E}\left\{\frac{l^3 - l_1^3}{I} + \frac{l_1^3}{I_1}\right\} - \frac{w}{8E}\left\{\frac{l^4 - l_1^4}{I} + \frac{l_1^4}{I_1}\right\} \checkmark$$

CHAPTER X.

ROOFS.

Occasionally roofs are used as additional floors to buildings, or as water tanks, roof gardens, etc., in which case the upper surface requires to be horizontal. Such roofs—generally termed *flat roofs*—are supported by systems of parallel flanged beams whose design is similar to that of the floor beams already described in Chapter IV. and need not be repeated here.

In the majority of roofs, however, the weather-resisting surfaces are inclined at a considerable angle to the horizontal, and it is the design of this type of roof it is now proposed to consider. The particular feature of this system of roofing is the specially shaped girder—known as the *principal* or *truss*—used to furnish the primary support between the walls or columns of the building. The principals are spaced at intervals along the length of the building and support a system of secondary members called *purlins*, which span the spaces between the principals and in turn carry the weatherproof covering.

Slope.—In very large spans the rafters of the principals are sometimes curved, but for small and medium spans they are usually made straight from the eaves to the ridge of the roof, and the covering thus lies on two inclined planes forming an inverted V shape. The inclination given to the slopes must be sufficient to throw off the rain without allowing leakage at the joints of the covering material. Thus the minimum slope is dependent to a great extent on the characteristics of the covering. For architectural effect very steep slopes may be adopted, but roofs so formed present large surfaces to the wind pressure, thereby necessitating specially strong principals, and require a proportionately larger amount of covering material. The proportions of rise to span most commonly employed vary between 1 to 4 and 1 to 3.

Roof Coverings.—The following are the most frequently used materials :—

Slates form probably the most durable of all roof coverings and are not subject to the effects of heat, rain, or other atmospheric conditions, but their heavy weight necessitates very strong structural supports. They are secured by slating nails to battens or close-jointed boarding laid on the common rafters or purlins. The minimum pitch of roof required is about 1 to 4.

Tiles, often used as substitutes for slates, are manufactured from clay, asbestos, and various compositions, many forms being on the

market. Each material has special qualities claimed for it by the manufacturers, to whose catalogues the reader is referred. Generally tiles do not possess to the same degree the durability of slates, but they form a somewhat cheaper covering. The method of laying and the minimum slope are similar to those of slates.

Galvanized corrugated iron sheeting is used only on the lightest types of structures. Being manufactured in sheets of considerable size and easily fixed to the purlins by screws or hook bolts, the cost of material and labour is very low. Its great disadvantage is its comparatively short life; the rate of decay through atmospheric causes is very rapid at the bolt holes and throughout the sheet when the galvanizing has worn off. Painting adds little to its life. It can be used on all slopes of 1 to 4 and upwards.

Corrugated asbestos and other composition sheets are used as substitutes for galvanized iron and have the advantage of being immune from the rusting and galvanic action which causes the decay of the latter. They can be fixed in the same manner and used in all situations suitable to iron sheets.

Felt tacked on to close-jointed boarding forms a very cheap type of covering, easily erected but not durable, is highly inflammable and easily damaged. In many localities the bye-laws prohibit its use on all except very small or temporary structures. It is laid with simple lap joints which are coated with mastic to render them waterproof. It is suitable for curved and low-pitched roofs.

Lead, formerly used to a great extent on large flat roofs, is now seldom adopted owing to its weight and cost. It is very durable, but unsuitable for use on any but very low-pitched roofs owing to its tendency to creep down the slopes. It is laid on boarding in sheets running down the slope and the joints are formed by side laps over timber rolls.

Zinc is another form of metal covering occasionally used, but whilst lighter and cheaper than lead, it does not possess such durable qualities, as it is very subject to decay and corrosion from atmospheric causes. It is laid in a similar manner to lead but on somewhat steeper slopes.

Glass used in the slopes for lighting purposes is manufactured in sheets of varying size and thickness, the rough rolled plate quality of $\frac{1}{4}$ to $\frac{3}{8}$ inch in thickness being generally employed. Special glazing bars, made in timber or metal, are required to support it and to ensure weather tightness in the vertical joints. Lead flashing is used as covering to the horizontal joints between the glass and other materials in the slope.

Purlins and Common Rafters.—As the roof covering does not possess the requisite strength to support itself between the principals, purlins, and in some cases common rafters (see Figs. 240, etc.), are introduced for this purpose. Common rafters act as secondary beams spanning the spaces between the purlins and are formed of deals spaced at about 12 inch centres. The required section of these rafters is determined by the principles of beam design described in Chapter IV.

Where the principals are spaced at short intervals, say up to 14 feet, the purlins can usually be made of solid web sections of either timber or

steel, but for larger spans built-up sections in the form of plate or lattice girders or trussed members may be required (see page 313 for example of trussed purlin).

The purlin loads consist of the weight of the covering and purlin, together with the pressure exerted by the wind on one or other of the slopes. The dead load can be closely computed from the unit weights of materials given in Chapter II. The wind pressure is much less definite and as already explained in Chapter II. cannot, with any degree of accuracy, be stated for any given case. Empirical formulæ have been devised from the results of observations on wind pressure, and it is common practice to make use of these when estimating the wind loads. The wind may impinge on the roof slope at any angle, but it is then guided up or along the slope and during the change of direction creates two pressures, one being normal and the other parallel to the slope. The force on the roof members produced by the latter is very

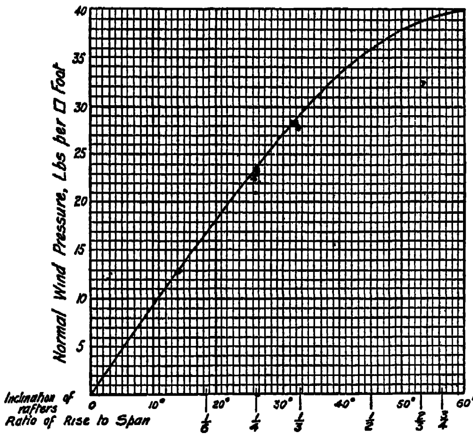


FIG. 239.

small, being only the frictional resistance offered by the slope, and may be neglected when computing the stresses in the purlins. The normal pressure is resisted directly by the purlins and forms the live load to be considered when designing those members. In the following formula the normal pressure is dependent on the horizontal pressure, which will vary according to the exposed or sheltered situation of the structure. For all ordinary situations 40 lbs. per square foot is a very commonly adopted horizontal pressure and has been found in practice to give satisfactory results.

Hutton's Formula.

Let P_N = normal pressure on the slope.
P = horizontal wind pressure.
i = inclination of the slope.

$$\text{Then } P_N = P(\sin i)^{1 \cdot 84 \cos i - 1}.$$

Assuming $P = 40$ lbs. per square foot, the curve of normal pressures has been plotted—Fig. 239.

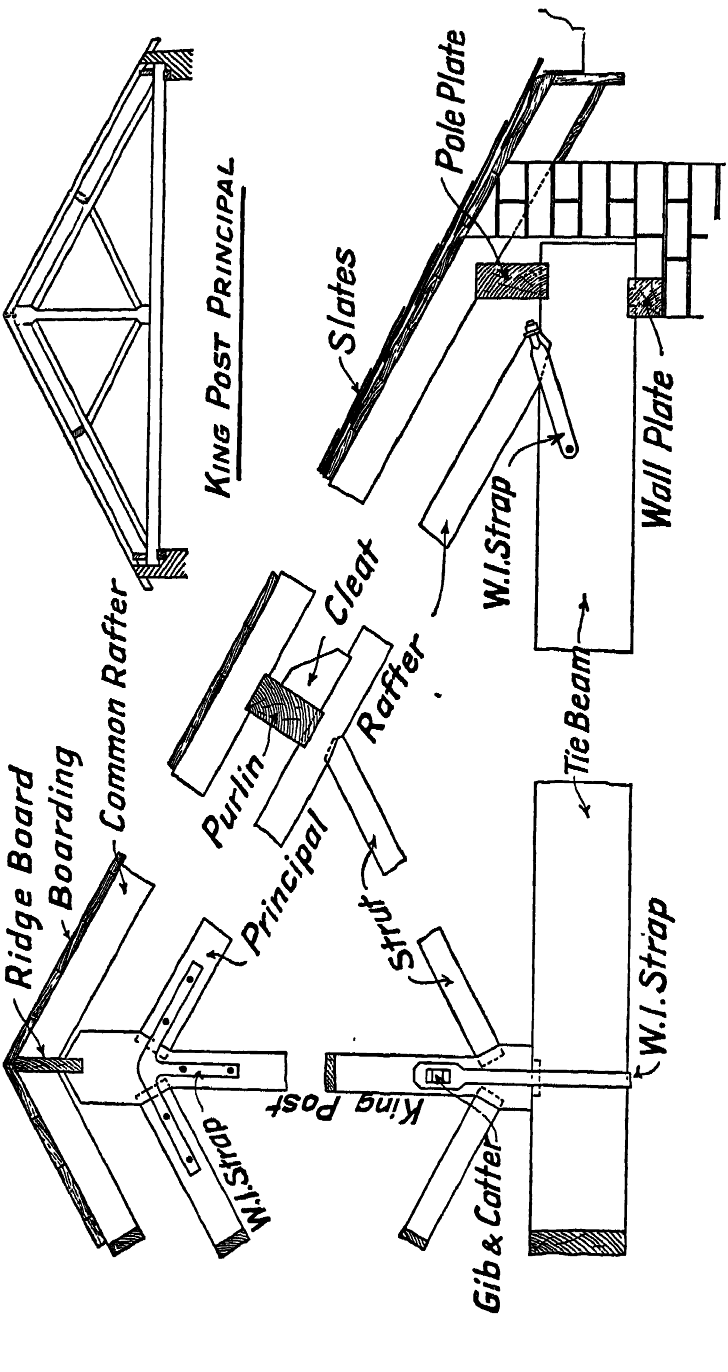

King Post Principal
Ridge Board
Boarding
Common Rafter
Principal
W.I. Strap
King Post
Strut
Gib & Cotter
W.I. Strap
Purlin
Cleat
Rafter
Tie Beam
Slates
Pole Plate
W.I. Strap
Wall Plate

Fig. 240.

NOTE.—A dead load due to the weight of snow lying on the leeward slope may be included in the dead load, but the purlins will be most severely stressed when under the action of the wind blowing on the slope under which they are situated.

The purlins act as beams and owing to the frequency of the joints may be considered as having free ends. The methods of calculating the stresses have been described in former chapters when dealing with beams and plate and lattice girders. An example of the calculation of the strength of a trussed purlin is given on page 313.

Principals may assume a variety of forms and be constructed wholly of timber, wrought-iron or steel, or consist of a combination of members made from the different materials.

Timber principals usually take one or other of the forms shown in Figs. 240 and 241. The former, known as a *King Post Truss*, is suitable for spans of from 20 feet to 30 feet, and the latter—*Queen Post Truss*—for spans up to 50 feet.

The loading tends to spread the rafter shoes, and to prevent this spreading effect from thrusting out the walls a horizontal tie beam is introduced. Apart from the secondary bending effect due to its own weight or that of a ceiling suspended from it, this member is purely in tension. The rafters are tenoned into it and the joints are further strengthened by screwed straps to prevent horizontal shearing of the ends of the tie. The vertical king and queen posts are tension members resisting the vertical components of the thrust in the struts, and also serve as suspenders to the horizontal tie beams to minimise the sag of those members. In king post trusses the horizontal components of the forces in the struts balance each other, and there is no tendency of the foot of the king post to move horizontally. The unbalanced thrusts of the struts in queen post trusses necessitate a strutting piece between the feet of the posts to relieve the lower joints of the posts of the horizontal forces. This strutting piece is spiked to the upper side of the beam. To form the tension joints between the posts and the tie beam wrought iron or steel straps are fixed around the beam and secured to the posts by gibs and cotters to ensure tight joints between the timbers. The heads and feet of the posts are enlarged to provide bearings for the rafters and struts which are tenoned into them. Straps are provided at the junctions of the rafters and posts to prevent opening of the joints if shrinkage of the timber occurs. The straining beam in the queen post truss acts as a strut between the heads of the posts into which it is tenoned. The struts are secured by tenons into the rafters and posts.

The trusses are spaced at not more than 10 feet centres and the shoes rest on wall plates, which distribute the load along the length of the wall to avoid local crushing of the wall under the shoes.

The purlins are checked on to the rafters and prevented from sliding down the slope by timber cleats spiked and notched to the rafters. The pole plates act as additional purlins but are supported directly over the walls by the tie beam on to which they are checked. The common rafters are spaced at about 12 inch centres. They are bevelled to fit against the ridge piece to which they are spiked and

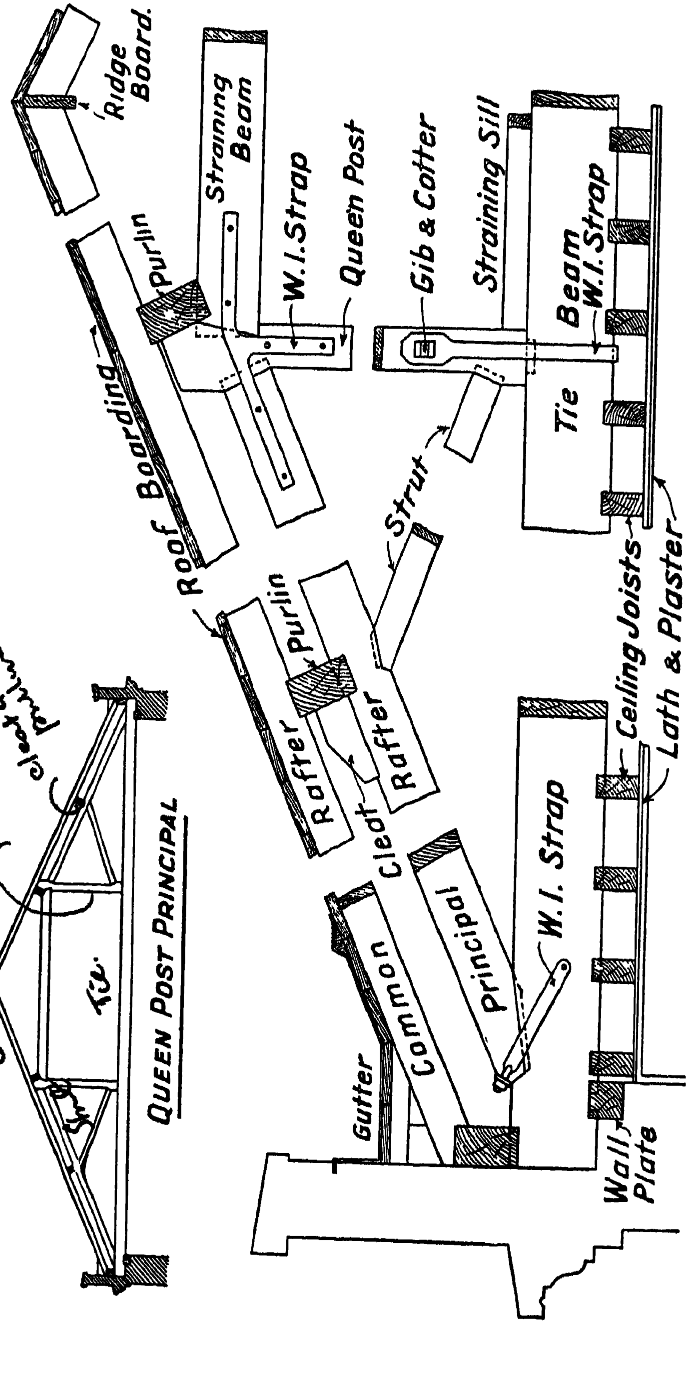

Ridge Board.
Straining Beam
W.I.Strap
Queen Post
Purlin
Gib & Cotter
Straining Sill
Beam
W.I.Strap
Tie
Roof Boarding
Strut
Purlin
Rafter
Rafter
Cleat
Ceiling Joists
Lath & Plaster
Common
Principal
W.I. Strap
Gutter
Wall Plate
QUEEN POST PRINCIPAL
Tie.

FIG 241.

checked on to the purlins and pole plate. The ridge piece is supported

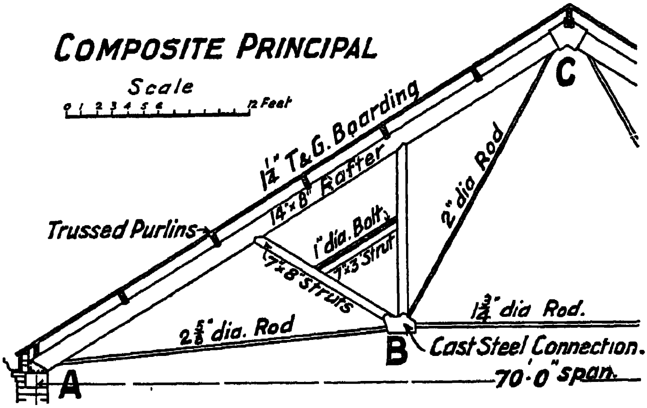


FIG. 242.

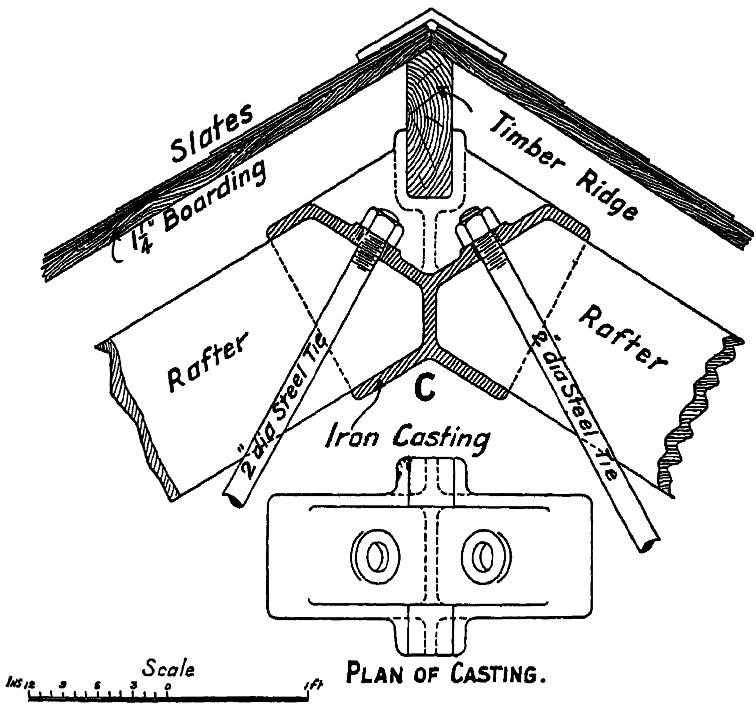


FIG. 242A.

by the common rafters and fits into grooves in the heads of the king posts. Battens or close-jointed boarding are nailed on to the common

rafters and support the slates or other covering materials. Ceilings may be suspended from the tie beams as shown in Fig. 241, the ceiling joists being nailed to the under sides of the ties.

The sections of the different members of these types of trusses have been standardized and may be found in text-books dealing more particularly with building construction.

Composite Principal.—Figs. 242 to 242C show the general elevation and details of a principal of the composite type in which the struts and rafters are made of timber and the ties of steel rods. This arrangement of material obviates the use of the very large timber sections necessary if timber be used in the ties.

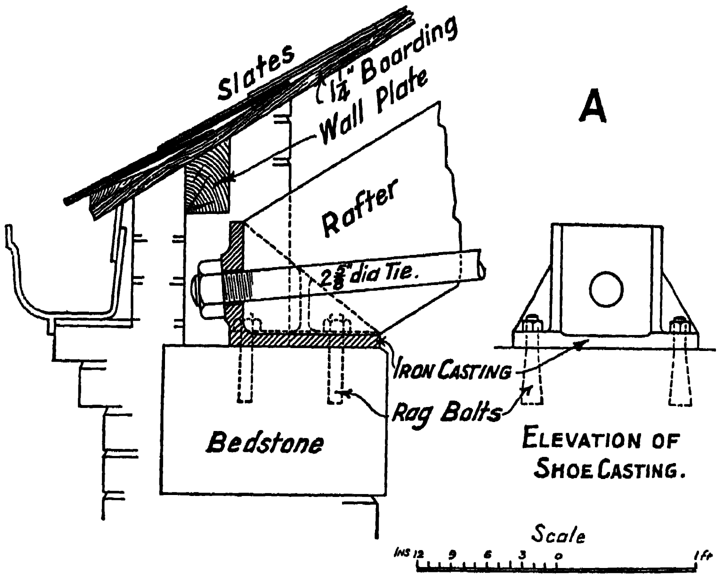


FIG. 242B.

The principal has relatively few members, and for large spans the strut lengths of the rafters necessitate large sections for those members. The connections between the struts and ties are most readily formed by means of castings. At the ridge and shoes iron castings may be used, since the pull on the ties is resisted directly by the rafters and the castings are not subject to any tensional stress. At the internal joints, however, the castings are called upon to resist the pull of the ties, and for this reason steel are preferable to iron castings. A suitable strength for these steel castings is usually arrived at from the results of destructive trial tests. Built-up steel connections composed of bent angles or channels and steel plates riveted together can be used as an alternative to the steel castings.

The stresses in the various members of the principal are found in a

similar manner to that adopted for steel principals explained later. The timbers inserted between the struts add stiffness to those members but take no part of the primary stresses

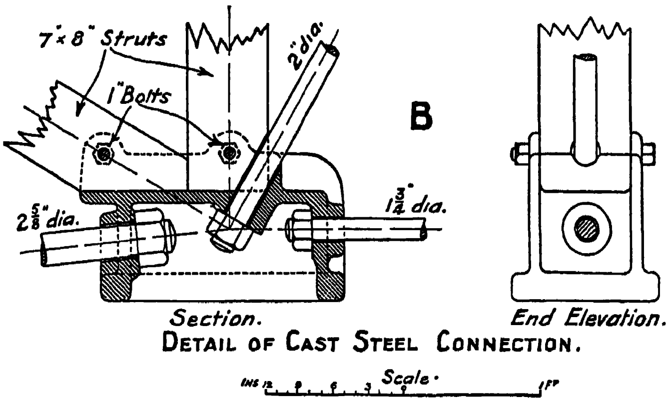


FIG. 242C.

The spacing of the principals in the above example was 26 feet centre to centre, thus necessitating the trussing of the purlins as shown

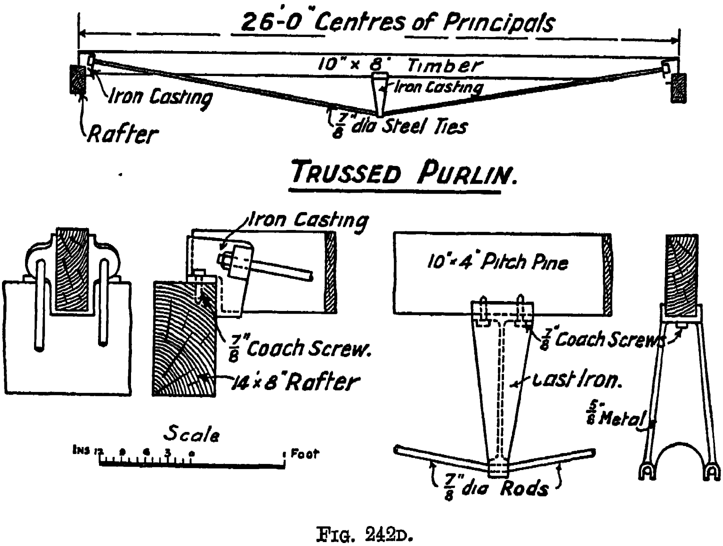


FIG. 242D.

in Fig. 242D. The steel ties of the purlin are connected to the ends of the timber beam by iron castings, and at midspan other iron castings provide the strut resistance and the tie bearings.

The method of calculating the strength of the different members of the purlin is as follows :—

Loads :—	$1\frac{1}{4}''$ boarding . .	= 4·5	lbs. per sq. ft.
	slates	= 8·0	„ „
	normal wind pressure	= 26·5	„ „
		39·0	„ „

Say 40 lbs. per square foot of roof slope.

Load on one purlin (at 7 feet centres)

$$= 40 \text{ lbs.} \times 26 \text{ ft.} \times 7 \text{ ft.} = 7280 \text{ lbs.}$$

weight of purlin (say) = 600 „

7880 „

Say 70 cwts.

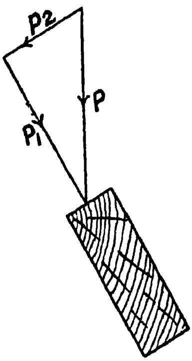


FIG. 242E.

The dead load P acting vertically can be resolved into two components P_1 and P_2, as in Fig. 242E, normal and parallel to the axes of the purlin section. The component P_2 is in this case resisted by the roof boarding acting as suspender from the ridge and balanced there by a similar force from the opposite slope. Thus the purlin is not called upon to resist transverse bending. The component P_1 will be somewhat less than P, and if for simplicity of calculation the whole load P be assumed as acting normal to the slope, the resulting stresses in the purlin will be a little greater than if the more accurate load P_1 be used, but will not materially affect the member sections.

Fig. 242F is the frame and stress diagram of the purlin. The method of drawing these diagrams will be described later when dealing with stresses in the members of principals. The stresses in the ties A1 and A2 = 5·8 tons.

Area of two $\frac{7}{8}''$ diameter rods at the bottom of the screw threads = 0·84 square inch.

Stress in rods $= \dfrac{5\cdot8}{0\cdot84} = 6\cdot9$ tons per sq. in., which gives a factor of safety of about 4.

The shear on the lugs of the end castings

$$= \frac{5\cdot8}{2(2\cdot75'' \times 2\cdot5'')} = 0\cdot4 \text{ ton per sq. in.}$$

which is well within the safe limit of stress.

The proportions of the remaining parts of the end castings have been designed to accord with the practical requirements of the foundry, and the stresses in the material are below theoretical requirements.

The compression on the middle strut = 1·75 tons.

To design this member by the principles described in Chapter V. would result in the sections being too thin for casting, and therefore the thicknesses of metal have been determined by foundry considerations.

The timber beam has, in addition to a direct compressive stress of 116 cwts. as found by the stress diagram, bending stresses due to the distribution of load on it between the panel points.

Maximum bending moment

$$= \frac{\frac{70}{2}\text{ cwts.} \times 13\text{ ft.} \times 12}{8} = 682\cdot5\text{ cwt.-ins.}$$

Maximum stress due to bending

$$= \frac{\text{B Mt.}}{\text{Mod. of section}} = \frac{682\cdot5 \times 6}{4 \times 10 \times 10} = 10\ 2\text{ cwts. per sq. in.}$$

Compression due to direct load

$$= \frac{116\text{ cwts.}}{4'' \times 10''} = 2\cdot9\text{ cwt. per sq. in.}$$

Combined maximum compressive stress

$$= 10\cdot2 + 2\cdot9 = 13\cdot1\text{ cwts. per sq. in.}$$

The beam is pitch pine the ultimate strength of which may be taken as 96 cwts. per sq. in. Therefore the factor of safety $= \frac{96}{13\cdot1} = 7\cdot3$, which may be considered reasonable for this purpose.

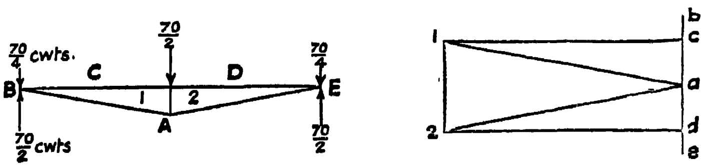


FIG. 242F.

Steel Principals.—A typical example of a principal formed throughout of steel members connected by steel junction plates is shown in Figs. 250 and 251. This type of principal has the following advantages over the foregoing types: high resistance of the material, greater variety of sections from which to select, ease of forming the connections, greater reliability of the material and suitability for all spans. The arrangement and number of members of the frame depends to a great extent on the span to be bridged. Fig. 243 shows alternative frame diagrams for various spans. Very long struts necessitate correspondingly large member sections and unwieldy connections, whilst an undue number of members results in loss of economy owing to the cost of the numerous connections. An economical arrangement will generally be attained if the spacing of the strut connections to the rafters be made from about 8 feet to 10 feet.

For spans up to about 100 feet V principals are generally adopted. For larger spans it is more economical to employ some form of principal with curved rafters, and so avoid the excessive height required by V

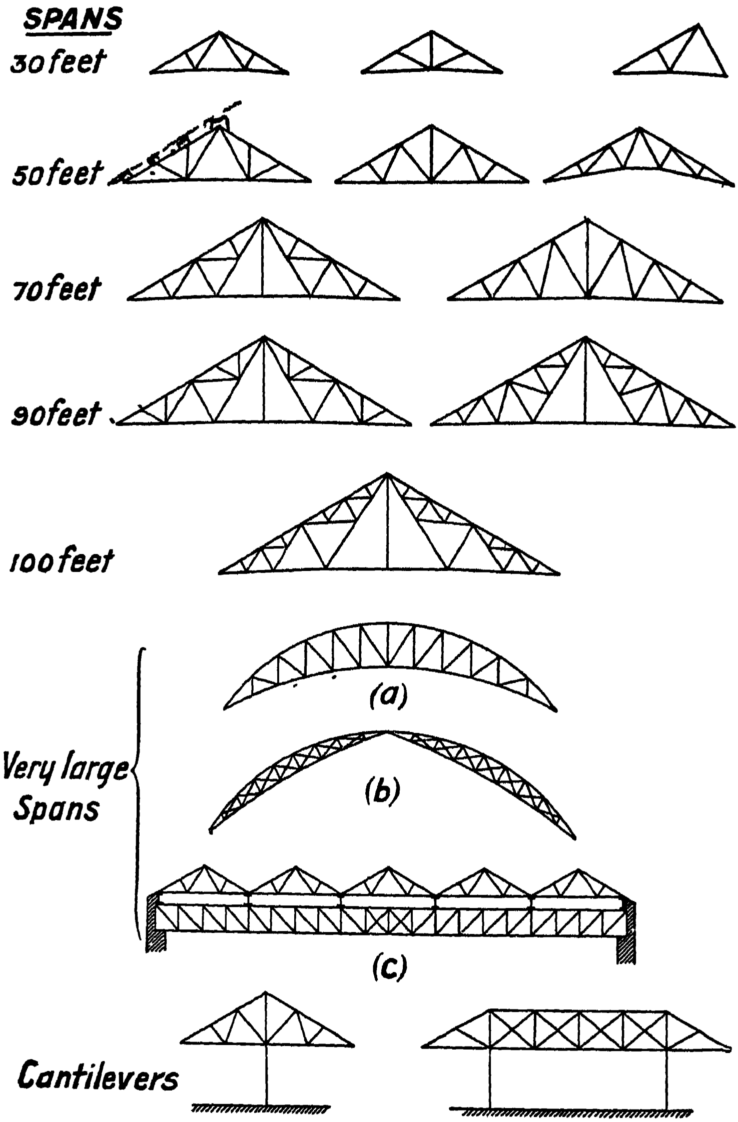

SPANS
30 feet
50 feet
70 feet
90 feet
100 feet
Very large Spans
(a)
(b)
(c)
Cantilevers

FIG. 243.

principals. Two types of curved principals are shown in Fig. 243 at (*a*) and (*b*). The former is similar to a lattice girder, and the reactions will be similar to those of V principals. (*b*) is an arch and requires either ties between the shoes or stiff abutments to resist the outward thrust due to the arch pressures. An alternative method of roofing large spans is shown at (*c*), where one or two systems of girders serve as the primary support between the walls of the building and a series of small span V principals are erected on the upper flanges of the girders. This system is known as *Ridge and Furrow* roofing, and has to a great extent supplanted the different forms of curved rafter principals.

Where conditions permit, principals are usually spaced at 12 feet to 14 feet centres. This ensures member sections and connections of economical proportions and weights of principals that can be readily handled. If larger spacings are compulsory, trussed or built-up purlins will be necessary.

Determination of Stresses in Principals.—Principals are special types of lattice girders differing from those dealt with in Chapter VII. by reason of their shapes and the inclination of the line of action of the live loads. The loading consists of the purlin reactions and the weight of the principal itself. The former will be computed when designing the purlins, and the latter can be estimated from the formulæ given in Chapter II. and allotted equally between the panel points in the rafters. The purlin bearings, wherever possible, should be situated at the junction of the struts and rafters to avoid secondary bending in the rafters.

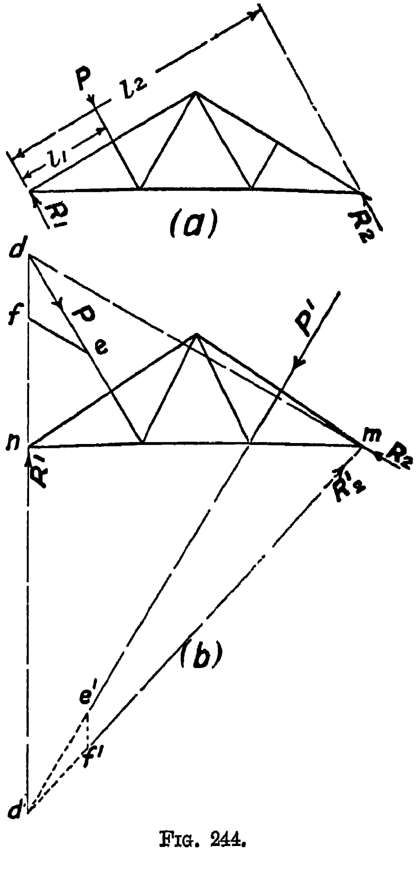


FIG. 244.

Reactions.—Since the dead loads all act vertically the dead load reactions at the shoes must also be vertical. For the ordinary V principals these reactions can be ascertained by taking moments about either shoe, as when computing the reactions of a beam with simple end supports.

The line of action of the wind loads being inclined to the vertical tends to slide the principal horizontally on its bearings and necessitates one or both shoes being secured to the supports to resist such motion. Usually both shoes are bolted down, but for large spans, one shoe should be allowed to slide on its bearing to allow of the principal expanding and contracting during changes of temperature. The reaction at a sliding bearing is necessarily vertical since there is no resistance to horizontal motion, and therefore the whole of the horizontal component of the wind load must be resisted by the fixed shoe.

Where both shoes are fixed the reactions will be parallel to the resultant wind load. In Fig. 244 (*a*) the wind pressure on the left-hand slope has a resultant P acting at the mid-point of the slope.

Taking moments about the left-hand shoe—

$$P \times l_1 = R_2 \times l_2$$

$$\therefore R_2 = \frac{Pl_1}{l_2}$$

and since $$R_1 + R_2 = P$$

$$R_1 = P - R_2$$

Suppose the left-hand shoe be supported on rollers. The reaction R_1 must now be vertical. By the principles of statics the reactions must intersect the force P at the same point. Producing R_1 vertically the only point of intersection is at *d*, which must also be the point of intersection of R_2 and P. Joining *d* to *m* will establish the line of action of R_2. The magnitudes of R_1 and R_2 are found from the triangle of forces *def*. Set off *de* to scale to represent the force P. From *e* draw *ef* parallel to *dm* cutting *dn* at *f*. Then the sides of the triangle *de*, *ef* and *fd* represent the forces P, R_2 and R_1 respectively to the same scale.

When the wind acts on the right-hand slope the same method of procedure may be adopted to find the reactions. The point of intersection of the forces will now be *d'* and *e'f'* and *d'f'* will represent the directions and magnitudes of R_1 and R'_2.

Stress Diagrams.—The stresses in members of principals are most conveniently determined by the following diagrammatic method founded on the fact that if any number of forces acting in one plane and through one point are in equilibrium, they can be represented by the sides of a polygon.

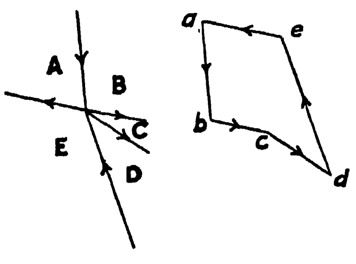


FIG. 245A.

Let the five forces shown in Fig. 245A act in the same plane through one point and be in equilibrium. The force polygon is then constructed by commencing at the point *a* and drawing *ab* parallel to AB and in the direction of action AB, *i.e.* downwards. Make the length *ab* represent to scale the magnitude of the force. From *b* draw

bc parallel to and acting in the same direction as BC. Make the length *bc* to the same scale as *ab* equal to the force BC. Drawing *cd*, *de* and *ea* likewise parallel to and in the directions of action of the forces CD, DE and EA respectively, it is found that the closing line *ea* terminates at the point *a*, thus forming a closed polygon.

At each joint in the frame of a roof principal there are a number of forces acting which are in equilibrium and which can be represented by a polygon of forces constructed in the above manner. Consider the left-hand shoe of the frame shown in Fig. 245 when the principal is

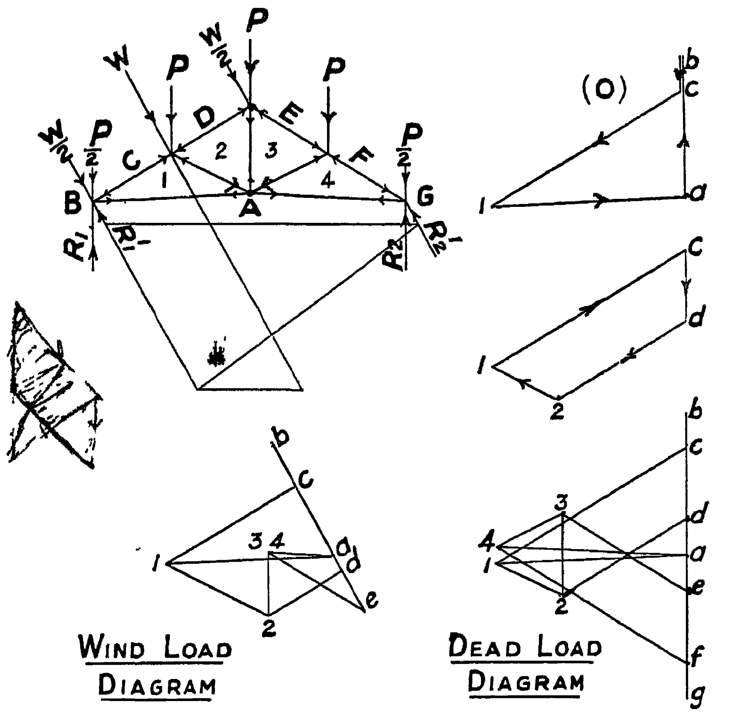


FIG. 245.

subject to dead load only. At this joint there are four forces acting, viz. the reaction R_1, the panel load $\frac{P}{2}$, the force in the rafter C1, and the force in the tie 1A, of which the magnitudes of R_1 and $\frac{P}{2}$ will have been previously ascertained when computing the dead loads and reactions. The polygon (O) has been constructed to represent these forces by commencing at *a* and drawing *ab* parallel to R_1, making its length to scale represent the magnitude of the reaction. This force

acts upward from *a* to *b*. Taking the forces in clockwise order the next force is the panel load $\frac{P}{2}$ which is represented to the same scale as *ab* by *bc* acting from *b* downwards and coinciding with the line *ab* (drawn at the side of *ab* for clearness). The force C1 is represented by *c*1 passing through *c* and 1A by 1*a* passing through *a*, each parallel to the force it represents. From the polygon so formed the magnitudes and directions of action of the unknown forces C1 and 1A may be determined. Since *ab* acts upwards and *bc* downwards then, following the remaining sides in order, *c*1 must act in the direction *c* to 1 and 1*a* in the direction 1 to *a*. Transferring these directions on to the frame diagram, it is seen that the force in C1 acts towards the joint, whilst 1A acts away from the joint, thus indicating that the rafter thrusts on to the joint and is in compression whilst the tie pulling away from the joint is in tension.

It will be observed that if the magnitudes of two forces only in a system of forces acting at a point be unknown they can be determined by the foregoing method. In most forms of principals it is possible by commencing at either end joint to proceed along the joints in such order that not more than two unknown forces are encountered at any one joint. In certain types of principals where this is impossible other methods, to be discussed later, must be employed.

Having ascertained the stresses in C1 and 1A the next joint to be considered is CD21—described by the surrounding letters and figures. The known forces are C1 acting towards the joint since the rafter is in compression, and the load P acting vertically downwards, whilst the unknowns are D2 and 21. By repeating the construction of the previous force polygon another polygon *cd*21 is obtained from which the magnitudes and directions of the forces in D2 and 21 are determined. Taking the remaining joints in the following order, DE32, A1234, 3EF4 and A4FG, for each a polygon can be constructed and the forces in the remaining members ascertained. For convenience in drawing, the polygons for the different joints may be incorporated in a single diagram as shown in the dead and live load diagrams. The closing line of these diagrams serves as a check on the accuracy of the drawing, since it must pass through two previously established points, viz. points 4 and *a*. The method of drawing the wind load diagram is similar to that for the dead load. In the figure the stresses produced when the wind acts on the left-hand slope are shown. It will be noticed that the points 3 and 4 on the diagram coincide, thus indicating that the wind pressure on the left-hand slope produces no stress in the member 3–4. When the wind changes its direction and acts on the right-hand slope the magnitude of the wind stresses in the various members will undergo a change, and a second wind load diagram might be constructed for the changed loading conditions. In the present example this is unnecessary because of the symmetry of the frame and end conditions. The maximum wind stresses in the members of the left-hand half of the frame occur when the wind acts on the left-hand slope, and similar stresses are produced in corresponding members in the right-hand half of the frame when the wind acts on the right-hand slope. The total stress in

each member varies from a minimum when the dead load only is acting, to a maximum when the wind acts on one or other of the slopes. It is convenient for purposes of design to tabulate the forces in the members due to the various loading conditions in the manner shown on page 328.

Design of Members of Principals.—The axis through the centre of gravity of the section of each member, wherever possible, should lie on the frame diagram line so that all member axes at a joint pass through the same point, thus avoiding secondary bending stresses. The rivets, bolts or pins connecting any member at a joint should, for the same reason, be disposed so that their centre of gravity also lies as near as possible on the frame diagram line. In selecting the sections of the different members attention should be paid to the facilities offered by the different forms for connections, and in the case of the rafters the sections adopted should provide suitable bearings for the purlins.

Struts.—The rafters are usually made of single angles or tees in light principals and of double angles or double channels in the heavier types of principals. Although the stress in the rafters varies from a minimum at the ridge to a maximum at the shoes, the section of the member is made continuous, as the cost of the additional joints, necessitated if a varying section be used, would absorb any saving there would be in cost of material if the sections be proportioned to the stress. The internal struts are generally formed of single or double angles, but double flats or channels, spaced by distance pieces and prevented from buckling outwards by bolts (see Fig. 129), can be adopted as alternatives.

Where either single or double angle sections are used there is always an unavoidable amount of eccentric loading due to the impossibility of placing the rivets at the connections coincident with the axis of the member. To minimise this eccentricity the rivets should be driven as near the axis of the member as practical considerations will allow. The ends of all struts should be fixed by at least two rivets, even where theory demands only one, to obtain a certain amount of fixation of the ends.

A series of tests[1] has recently been carried out on single angle struts fixed in a manner similar to those employed in roof principals, and the following conclusions were drawn from the results obtained :—

(*a*) The resistance varies with the method of fixing the ends. Where one rivet only is used at either end the resistance is approximately equal to that of a round ended strut. If two or more rivets are used at either end a greater degree of fixation is attained with a consequently increased resistance of the strut. The authors of the paper referred to suggest that a "fixation factor f" be used when calculating the resistance of such struts. The values of f arrived at were :—

For a single rivet at either end . . . . . $f = 1{\cdot}1$
For two or more rivets at either end . . . $f = 1{\cdot}3$

In the tests the rivets were placed closer to the root of the angle than

[1] No. 218, Technological Papers, Bureau of Standards, Department of Commerce, Washington, U.S.A.

is usual in ordinary practice, and for this reason it would probably be safer to reduce the values of f to 1 and 1·25 respectively.

This factor is to be used in conjunction with formulæ for determining the resistance of struts having "rounded ends"—see formula following.

(*b*) "It is believed that these values of end fixation factor are of importance in the design of structures where the end conditions approximate to those used in the tests, no matter what formula the designer prefers to use."

(*c*) "Formulæ of the Rankine-Gordon type represent the results fairly well for values of the slenderness ratio $\left(\frac{l}{r}\right)$ up to about 150, but for longer columns the results are evidently best represented by the Euler formula."

Roof principal struts seldom exceed the ratio 150, so it would appear

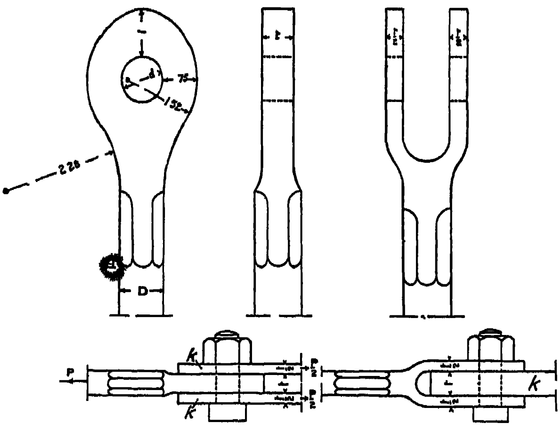


FIG. 246.

admissible to use Rankine's formula, page 152, after inserting the fixation factors.

Then $$p = \frac{6{\cdot}75}{1 + \frac{1}{8{,}000}\left(\frac{l}{fr}\right)^2}$$

The above formula applies more particularly to single angle struts, but if used for double-angle members a rather larger factor of safety will result, since the eccentricity of loading is relatively smaller for such members.

Struts formed of double sections should have intermediate connecting rivets between the panel points to ensure that each of the angles, etc., has an equal unit area resistance to that of the combined member.

Ties.—The tension members may be made of single or double flat bars, angles or round rods. Flat bars are readily connected to the

junction plates, but lack lateral stiffness and are for this reason often supplanted by angles in the lower ties. Round bars require eyes or forks at the ends for connection purposes, and are thereby specially liable to defective workmanship. They also possess little resistance to bending, and therefore are unsuitable for use in long horizontal members. Single flats when riveted to single junction plates and single or double angle ties are always subject to eccentric loading—see Example 35—which must be taken into consideration in designing such members.

The minimum sectional area of a tie is determined by dividing the maximum load on the member by the allowable working stress. As this minimum area occurs at a section in which rivet holes are situated, the gross area of the tie will be the minimum area required plus the area of the number of rivet holes in the weakest section. Where round bars with screwed couplings are used the area at the bottom of the thread must be used as the sectional area of the tie. The proportions of eyes and forks forged on the ends of tie rods have been more or less standardized by different authorities from results of destructive tests. Fig. 246 shows a commonly adopted standard of proportions, the dimensions of the eyes being given in terms of the diameter of the tie rod.

The diameter d_1 of the pin and the thickness t of the eye or fork may be determined as follows:—

Let P = pull in the tie in tons. D = diam. of tie-rod in ins.
f_s = safe shearing stress on the pin in tons per sq. in.
f_b = " bearing " " "
d_1 = diameter of pin.

Assuming that the double shearing resistance of the pin is $1\frac{3}{4}$ times that of its single shearing resistance, and since the shearing resistance must equal the pull,

$$P = 1\tfrac{3}{4}\left(\frac{\pi}{4}d_1^2 f_s\right)$$

or

$$d_1 = \sqrt{\frac{16P}{7\pi f_s}} = 0{\cdot}85\sqrt{\frac{P}{f_s}}$$

As the bearing resistance of the pin must also equal the pull.

$$P = d_1 t f_b$$

or

$$t = \frac{P}{d_1 f_b}$$

Assuming $f_s = 5$ tons per square inch
$f_b = 8$ " "
$f_t = 6$ " "

where f_t = intensity of stress in the tie,

then

$$P = \frac{\pi}{4}D^2 f_t$$

and

$$d_1 = \sqrt{\frac{16}{7\pi f_s} \times \frac{\pi}{4}D^2 f_t}$$
$$= 0{\cdot}83D$$

From above
$$t = \frac{P}{d_1 f_b} = \frac{\frac{\pi}{4} D^2 f_t}{d_1 f_b}$$
$$= 0 7D$$

The minimum area of the connecting plates k
$$= (b - d_1)t_t$$
where b is the width of the plate.

For the resistance of the plate in tension to equal the pull in the tie
$$P = (b - d_1)tf_t$$
$$\therefore b = \frac{P}{tf_t} + d_1$$

Connection plates may be used either singly or in pairs. Where the principal members are formed of double sections, single plates will usually be the more convenient; but if single section members are employed, double connecting plates enable the rivets to be used in double shear, thus reducing the number of rivets required. The plates should not be less than $\frac{3}{8}$ of an inch in thickness and generally will be proportioned to give a bearing resistance to the rivets equal to their shearing resistance. At any section of the plate its sectional area must be capable of transmitting the forces imparted to it by the rivets.

The calculation of the required number of rivets or bolts in the connections follows the same principles as those described in Chapter IV. for beam connections.

EXAMPLE 35.—*Design of a steel roof principal and purlins for a span of 70 feet and a spacing of principals of 12 feet. The covering to consist of slates laid on boarding and common rafters. One shoe of the principal to be fixed and the other free to slide horizontally.*

The members to be proportioned on the following allowable working stresses:—

Purlins	6 tons per square inch.
Tension members . .	6 " "
Compression members	safe stresses as determined by the Rankine formula on p. 152.
Shear	5 tons per square inch for single shear; double shear $1\frac{1}{2}$ times single shear.
Bearing	8 tons per square inch.

A suitable type of principal for such a span is shown diagrammatically in Fig. 247. Making the rise $\frac{1}{4}$ of the span the spacing of the purlins is found to be 9·8 feet and the area of slope supported by the intermediate purlins = 117·6 sq. ft.

Dead Load.—From the weights of materials given in Chapter II. the dead loads are:—

Covering	16 lbs. per sq. ft.
Approximate weight of principal . . .	46 cwts.
Snow	37·5 cwts. per principal.

Wind Load.—From Fig. 239 the normal wind load for this slope = 23·5 lbs. per sq. ft.

Purlins.—Load on intermediate purlins :—

$$\text{Covering} = \frac{16 \times 117{\cdot}6}{112} = 16{\cdot}8 \text{ cwts.}$$

$$\text{Snow} = \tfrac{1}{8} \times 37{\cdot}5 = 4{\cdot}7 \quad ,,$$

$$\text{Total} \quad . \quad 21{\cdot}5 \quad ,,$$

(This load acts vertically.)

$$\text{Normal wind load} = \frac{23{\cdot}5 \times 117{\cdot}6}{112} = 24{\cdot}7 \text{ cwts.}$$

Component of the dead load acting normally to the slope

$$= 21{\cdot}5 \times 0{\cdot}88 = 18{\cdot}9 \text{ cwts.}$$

Component of the dead load acting parallel to the slope

$$= 21{\cdot}5 \times 0{\cdot}44 = 9{\cdot}5 \text{ cwts.}$$

Maximum bending moment on the purlin due to the combined loads acting normally to the slope

$$= \frac{(24{\cdot}7 + 18{\cdot}9) \times 12 \text{ ft.} \times 12}{20 \times 8} = 39{\cdot}24 \text{ in.-tons.}$$

Maximum bending moment due to the dead load component acting parallel to the slope

$$= \frac{9{\cdot}5 \text{ cwts.} \times 12 \text{ ft.} \times 12}{20 \times 8} = 8{\cdot}55 \text{ in.-tons.}$$

Various forms of steel or timber purlins might be adopted and a consideration of their relative cost and availability would probably decide what form to use in practice. Here a steel channel will be adopted as furnishing a ready means of connection to the principal and common rafters. The depth of the purlin should not be less than $\frac{1}{20}$ of the span or excessive deflection is probable.

Try an $8'' \times 3\frac{1}{2}'' \times 22{\cdot}7$ lbs. channel.

Modulus of section about axis parallel to slope = 15·94
,, ,, ,, normal ,, = 2·84

$$\text{Maximum stress} = \frac{39{\cdot}24}{15{\cdot}94} + \frac{8{\cdot}55}{2{\cdot}84} = 5{\cdot}5 \text{ tons per square inch.}$$

This stress is near enough to the maximum specified stress for practical purposes, and the section may be adopted.

The channels can be obtained in 24-feet lengths, so joints will occur on alternate principals, and joints in neighbouring purlins can also be arranged so that they do not occur on the same principal. If a double-angle rafter be used the channels can be fixed directly to the rafters by $\frac{5}{8}''$ bolts, and to provide fish plates at the joints and additional lateral stiffness $5'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$ angle cleats riveted to the rafters and bolted to the purlins can be employed.

The purlins at the shoes have only half the load of the intermediate purlins and a single angle 7 × 4 × ½ will be found to supply the necessary strength The angle could be fixed to the rafters by angle cleats which would also act as fish plates at the purlin joints However there would be little saving by adopting the angle so to avoid multiplicity of sections this purlin will be made of a channel similar to the intermediate purlins

A timber rid e will foim a suitable beaiing for the common rafters and can be connected to the principal rafters by splayed an le cleats The dead load in this case will act axially to the ridge whilst the wind load will be inclined to the ridge axes The iidge cannot deflect laterally without strainin the common rafters and purlins on the leeward slope The resistance offered by these members is more than sufficient to take the wind load on the ridge since at such time they are not called upon to resist any other wind load The ridge may theie fore be designed to iesist the dead load forces only

The bending moment produced by the dead load

$$= \frac{21\ 5 \times 12 \times 12}{8} = 387 \text{ in cwts}$$

The depth of the ridge will require to be at least 8 to fit between the common rafters and the principal and to provide bearings for the common rafters the width should not be less than 4

Using pitch pine the ultimate moment of resistance

$$= \frac{96 \text{ cwts} \times 8 \times 8 \times 4}{6} = 4096 \text{ in cwts}$$

$$\text{Factor of safety} = \frac{4096}{387} = 10\ 6$$

which is more than ample but for the piactical reasons stated above cannot be reduced

The detailed design is shown in Figs 250 and 251

Load on Principal —Dead load on each intermediate joint—

Coveiing and snow load	=	21 5 cwts
Weight of purlin	=	2 0
Proportion of weight of principal =	=	5 75
Total	=	29 25
		say 30 cwts

Wind load on each intermediate joint = 24 7 say 25 cwts

Stress Diagrams —The stress diagrams for the dead load and for the wind acting on either slope the left hand shoe being supported on a sliding bearing are drawn in Fig 247 When drawing the diagrams a difficulty is met with when the joint D–E–5–4–3–2 is reached Here there are three unknown forces and the usual procedure provides for only two. The points 4 5 and 6 on the dead load diagram may be obtained in the following manner Select on the line *e*–5 any point 5 and draw lines 5′–6′ 5–4, and 6–4 parallel to 5–6 5–4 and 6–7 on the principal

so obtaining the point 4′. The correct position of the point 4 is on the line 3–4. By moving the triangle 5′–4′–6′ in a direction parallel to e–5′ until the apex 4′ rests on the line 3–4, the correct positions of 4, 5, and 6 are obtained. It will be noticed that in all three diagrams the stress lines 1–2 and 5–6, and 8–9, 12–13 are in the same straight

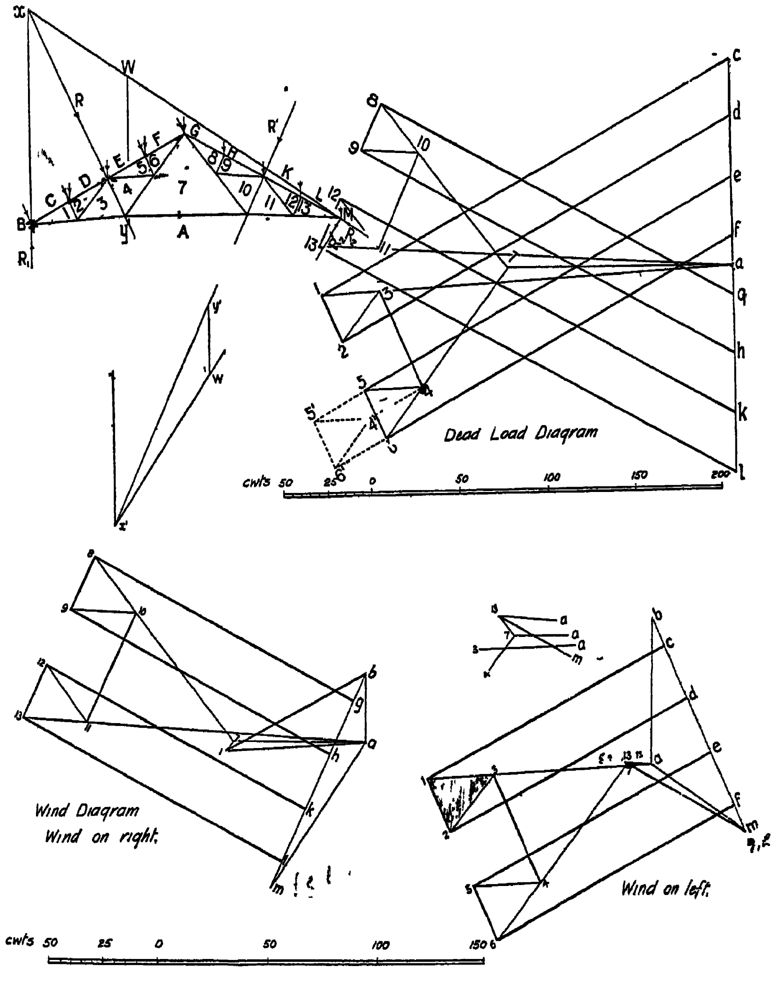


FIG. 247.

line. This is due to the joints C–D–2–1 and E–F–6–5 being equally loaded, and in all such cases the points 5 and 6 may be obtained by producing the stress line 1–2 until it cuts the lines e–5 and f–6.

The stresses in the members as scaled from the diagrams are given in the following table.

TABLE OF STRESSES.

Member.	Stresses.				Length of struts.
	Dead load.	Wind on left.	Wind on right.	Total maximum stress.	
	cwts.	cwts.	cwts.	cwts	ft
C–1	263	123·2	70·4	386·2	9·8
D–2	250	123·2	70·4	373·2	,,
E–5	237·5	123·2	70·4	360·7	,,
F–6	223	123·2	70·4	346·2	,,
G–8	223	62·8	132·8	355·8	,,
H–9	237·5	62·8	132·8	370·8	,,
K–12	250	62·8	132·8	382·8	,,
L–13	263	62·8	132·8	395·8	,,
1–2	25·6	26	0	51·6	4·3
3–4	54	49·6	0	103·6	8·6
5–6	25·6	26	0	51·6	4·3
8–9	25·6	0	24·4	50	4·3
10–11	54	0	50	104	8·6
12–13	25·6	0	24·4	50	4·3
A–1	236	104·8	64	340·8	—
A–3	202·5	73·6	64	276·1	—
A–7	129	11·2	60	189	—
A–11	202·5	10·8	128	330·5	—
A–13	236	10·8	156·8	392·8	—
2–3	33·5	30·4	0	63·9	—
4–5	33·5	30·4	0	63·9	—
9–10	33·5	0	30	63·5	—
11–12	33·5	0	30	63·5	—
4–7	77·5	63·2	5·2	140·7	—
6–7	110·5	94	5·2	204·5	—
8–7	110·5	0·8	96·8	207·3	—
10–7	77·5	0·8	69·2	146·7	—

The total stress in any member varies according to the direction of the wind, the maximum stress being the sum of the dead load stress, and the larger stress produced by the wind acting on either slope. Although not subject to quite the same stresses, corresponding members in the principal have been made of the same section for symmetry.

Principal Rafters.—In selecting the type of section for these members consideration must be paid to the facilities offered for the connection of the purlins and truss bracing. A double-angle section will most probably supply the necessary strut resistance and also afford suitable bearings for the purlins. If double sections be used for all internal members the connecting rivets will be in double shear with single junction plates only, and there will be no unduly large connections. In this example single junction plates will be adopted and made ½ inch thick throughout, to keep the spacing of the double section members constant and to avoid packings wherever possible.

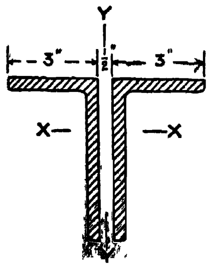


FIG. 2

Maximum load on the rafters = 19·79 tons.
Strut length = 9·8 feet.
As a trial section take two 5″ × 3″ × 0·3″ angles, Fig. ~~428.~~
Radius of gyration about XX = 1·6″
" " " YY = 1·23″
Using formula given on page 322. Sectional area = 4·6.

$$\text{safe load} = \frac{6{\cdot}75 \times 4{\cdot}6}{1 + \dfrac{1}{8000}\left(\dfrac{117{\cdot}6}{1{\cdot}25 \times 1{\cdot}23}\right)^2}$$

$$= 18 \text{ tons.}$$

This is less than the maximum load on the strut. Two 5″ × 3″ × $\frac{3}{8}$″ angles have a safe load = 22·8 tons, which is rather more than is theoretically required; but little loss of economy will result from their adoption.

Using $\frac{7}{8}$″ dia. rivets in the connections, the resistance of one rivet

in double shear = 0·6 sq. in. × 5 tons × $1\frac{1}{2}$ = 4·5 tons
in bearing in $\frac{1}{2}$″ plate = $\frac{7}{8}$″ × $\frac{1}{2}$″ × 8 tons = 3·5 tons

The bearing resistance being the lesser will be used in calculating the number of rivets required. At the shoe the load = 19·79 tons.

$$\text{Rivets required} = \frac{19{\cdot}79}{3{\cdot}5} = 6$$

$$\text{At the ridge the load} = 17{\cdot}8 \text{ tons}$$

$$\text{Rivets required} = \frac{17{\cdot}8}{3{\cdot}5} = 5$$

The rafter angles should be riveted together at intermediate positions between the member junctions to reduce the strut length of the single angles, so that the safe unit load on the single angle will be, at least, equal to that on the double-angle section.

Let r_1 = least radius of gyration of the double angles = 1·26″
r = " " " single angles = 0·65″
l_1 = length of strut of double angles = 9·8 feet.
l = " " single angle.

Then
$$\frac{P}{1 + a\left(\dfrac{l_1}{fr_1}\right)^2} = \frac{P}{1 + a\left(\dfrac{l}{fr}\right)^2}$$

from which
$$l = \frac{l_1 r}{r_1}$$
$$= \frac{9{\cdot}8 \times 0{\cdot}65}{1{\cdot}26}$$
$$= 5{\cdot}06 \text{ feet.}$$

Theoretically one intermediate rivet only is necessary, but in practice it is usual to space such rivets at about 3 feet centres, so two intermediate rivets will be used.

Struts 3–4 *and* 10–11.
Load = 5.2 tons. Strut length = 8·6 feet.
Try one $4'' \times 3'' \times \frac{3}{8}''$ angle.
Least radius of gyration = 0·64″.

$$\text{Safe load} = \frac{6{\cdot}75 \times 2{\cdot}48}{1 + \frac{1}{8000}\left(\frac{103{\cdot}2}{1{\cdot}25 \times 0{\cdot}64}\right)^2} = 5{\cdot}4 \text{ tons.}$$

This section is suitable.
Using $\frac{7}{8}''$ dia. rivets, resistance of one rivet
in single shear = 0·6 sq. in. × 5 tons = 3 tons
in bearing = $\frac{7}{8}'' \times \frac{3}{8}'' \times 8$ tons = 2·6 tons.

$$\text{Number of rivets required} = \frac{5{\cdot}2}{2{\cdot}6} = 2$$

Struts 1–2 *and* 5–6.
Load = 2·6 tons. Strut length = 4·3 feet
To provide the necessary width for riveting, one table of the angle should not be less than $2\frac{1}{2}$ ins. Try one $2\frac{1}{2}'' \times 2'' \times \frac{3}{8}''$ angle.
Least radius of gyration = 0·42 in.

$$\text{Safe load} = \frac{6{\cdot}75 \times 1{\cdot}54}{1 + \frac{1}{8000}\left(\frac{51{\cdot}6}{1{\cdot}25 \times 0{\cdot}42}\right)^2} = 4{\cdot}4 \text{ tons}$$

This is in excess of requirements, but for practical reasons a smaller angle is undesirable. Using $\frac{3}{4}$ in. dia. rivets, resistance of one rivet
in single shear = 0·44 sq. in. × 5 tons = 2·2 tons
in bearing = $\frac{3}{4}$ in. × $\frac{3}{8}$ in. × 8 tons = 2·2 tons.

$$\text{Number of rivets required} = \frac{2{\cdot}6}{2{\cdot}2} = 2.$$

Ties A-13 *and A*-11. The difference of loading on these members $= \frac{392{\cdot}8 - 330{\cdot}5}{20} = 3{\cdot}2$ tons, and the difference of sectional area required $= \frac{3{\cdot}2}{6} = 0{\cdot}53$ sq. in. If two angles or two flats be used for each member the difference of area of each flat or angle will be only 0·27 sq. in. in the different members. The saving of this small difference would not repay the cost of the additional connection rivets and plates necessary if a change of section is made. The section adopted for A-13 will therefore be made continuous and used in both members.

Maximum load = 19·64 tons.
If two flats be used the minimum area required

$$= \frac{19{\cdot}64}{6} = 3{\cdot}27 \text{ sq. ins.}$$

Allowing one $\frac{15}{16}$ in. diameter rivet hole through the double section, and making the flat thickness $\frac{1}{2}$ in. the area of the rivet holes

$$= 2\left(\tfrac{15}{16} \times \tfrac{1}{2}\right) = 0{\cdot}94 \text{ sq. in.}$$

Gross area of tie $= 3{\cdot}27 + 0{\cdot}94 = 4{\cdot}21$ sq. ins.

$$= \frac{4{\cdot}21}{2 \times \frac{1}{2}} = 4{\cdot}21 \text{ ins.}$$

The nearest rolled width is $4\frac{1}{4}$ ins., and this will be adopted.

If angles be used in place of flats, the rivets being eccentrically placed with regard to the centre of gravity of the section will cause bending stresses in addition to the direct tension. To minimise the bending the rivets should be placed as near the neutral axis of the section as the forming of the rivet heads will allow.

Try two $5'' \times 3'' \times 0{\cdot}3''$ angles and place the rivets $1\frac{3}{4}$ ins. from the bottom edge of the section.

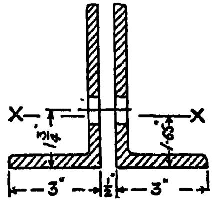


FIG. 249.

Then the eccentricity $= 0{\cdot}1$ in.

Bending moment $= 0{\cdot}1 \times 19{\cdot}64 = 1{\cdot}964$ in.-tons.

Modulus of section about X—X $= 1{\cdot}764 \times 2$

$$\text{Maximum bending stress} = \frac{1{\cdot}964}{1{\cdot}764 \times 2} = 0{\cdot}55 \text{ ton per sq. in.}$$

NOTE.—If the rivets had been pitched $2\frac{3}{4}$ ins. from the bottom edge the maximum tensile stress due to bending would

$$= \frac{2{\cdot}75 - 1{\cdot}65}{0{\cdot}1} \times 0{\cdot}55 = 6{\cdot}05 \text{ tons per sq. in.}$$

The net sectional area of the angles

$$= 2(2{\cdot}3 - \tfrac{15}{16} \times 0{\cdot}3) = 4{\cdot}04 \text{ sq. ins.}$$

$$\text{Direct stress} = \frac{19{\cdot}64}{4{\cdot}04} = 4{\cdot}8 \text{ tons per sq. in.}$$

Maximum combined stress $= 4{\cdot}8 + 0{\cdot}55 = 5{\cdot}35$ tons per sq. in.

Number of $\frac{7}{8}$ in. diameter rivets required at the connections

$$= \frac{19{\cdot}64}{3{\cdot}5} = 6$$

Tie A-7.—Maximum load $= 9{\cdot}45$ tons.

Using two $\frac{3}{8}$ in. flats and one $\frac{7}{8}$ in. diameter rivet in the section.

$$\text{Minimum sectional area required} = \frac{9{\cdot}45}{6} = 1{\cdot}57 \text{ sq. ins.}$$

Area of two $\frac{15}{16}$ in. dia. holes $= 2 \times \frac{15}{16}'' \times \frac{3}{8}'' = 0{\cdot}7$ „ „

Gross area $= 2{\cdot}27$ „ „

$$\text{Width of flats} = \frac{2{\cdot}27}{2 \times \frac{3}{8}} = 3 \text{ ins.}$$

Number of $\frac{7}{8}$ in. dia. rivets required in the connections $= \dfrac{9{\cdot}45}{3{\cdot}5} = 3.$

Very often a single angle is used for the middle tie, but if the effect of the eccentric loading be considered a most uneconomical section results. A $5'' \times 3'' \times 0{\cdot}3''$ angle has approximately the same sectional area as the two flats selected for this tie.

Assuming the rivets to be coincident with the axis X—X through

the centre of gravity of the section—their most favourable position—then—

$$\text{Eccentricity of force} = 0{\cdot}91 \text{ in.}$$
$$\text{Modulus of section about Y—Y} = 0{\cdot}69 \text{ in.}$$
$$\text{Bending moment} = 9{\cdot}45 \times 0{\cdot}91 = 8{\cdot}6 \text{ in.-tons.}$$

$$\text{Maximum stress due to bending} = \frac{8{\cdot}6}{0{\cdot}69} = 12{\cdot}4 \text{ tons per sq. in. compn.}$$

$$\text{Direct stress} = \frac{9{\cdot}45}{2{\cdot}3} = 3{\cdot}7 \quad \text{,,} \quad \text{,,} \quad \text{tension.}$$

$$\text{Max. combined stress} = 8{\cdot}7 \quad \text{,,} \quad \text{,,} \quad \text{compn.}$$

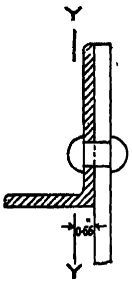


FIG. 249A.

This section therefore would be too highly stressed, and a single angle of the requisite strength would need to be much heavier and therefore less economical. It is desirable where angles are used as ties to employ only double-angle sections.

Ties 7–8 *and* 7–10. Again a continuous section for these members would be of practical advantage.

Maximum load = 10·36 tons.

Adopting two $\frac{3}{8}$ in. flats with one $\frac{7}{8}$ in. rivet in the section,

Minimum sectional area required

$$= \frac{10{\cdot}36}{6} = 1{\cdot}73 \text{ sq. ins.}$$

$$\text{Area of two } \tfrac{15}{16} \text{ in. dia. holes} = 0{\cdot}7 \quad \text{,,} \quad \text{,,}$$

$$\text{Gross area} = 2{\cdot}43 \quad \text{,,} \quad \text{,,}$$

$$\text{Width of flats} = \frac{2{\cdot}43}{2 \times \frac{3}{8}} = 3{\cdot}24 \text{ ins.}$$

Say two $3\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. flats.

Number of $\frac{7}{8}$ in. dia. rivets required in the connections

$$= \frac{10{\cdot}36}{3{\cdot}5} = 3$$

An alternative section of two angles can be found by following the procedure of the design of ties A-13 and A-11.

Ties 2–3 *and* 4–5. Maximum load = 3·2 tons. For direct stress only, the minimum sectional area required $= \frac{3{\cdot}2}{6} = 0{\cdot}53$ sq. in.

If a single flat be used there will be, in addition to the direct stress, bending stresses set up by the bending moment = $3{\cdot}2 \times l$ in.-tons, which will cause the total stress to be very high owing to the small resistance to bending of the flat. It is, therefore, desirable and economical to employ a double flat section to avoid the bending action.

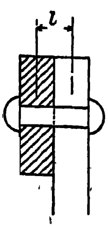


FIG. 249B.

If $\frac{3}{4}$ in. diameter rivets be used in the connections, the least width of flat should be 2 ins. to provide ample metal to the sides of the rivet holes, and a minimum thickness of metal of $\frac{3}{8}$ in. is desirable.

Two 2 in. $\times$ $\frac{3}{8}$ in. flats have a net sectional area

$$= 2(2 - \tfrac{13}{16})\tfrac{3}{8} \text{ in.} = 0{\cdot}89 \text{ sq. in.}$$

Number of $\frac{3}{4}$ in. diameter rivets required in the connections = 2.

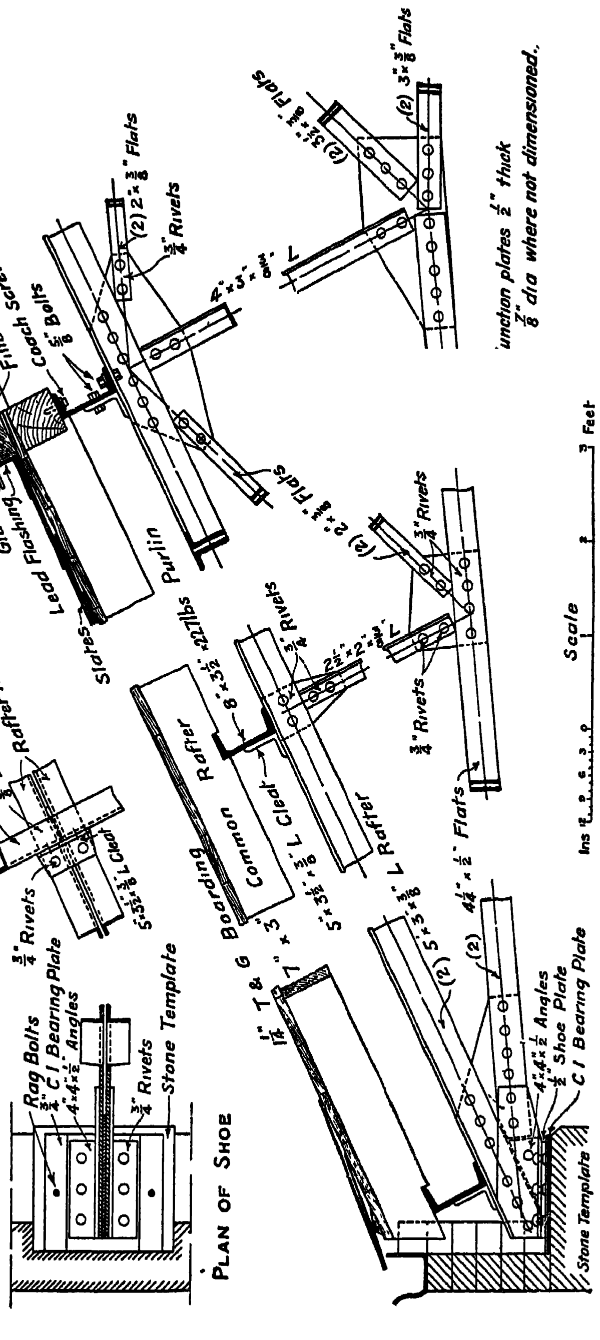

Glazing Bar
Fillers
Screw
Coach Bolts
5/8" Bolts
Glass
Lead Flashing
Slates
Purlin
(2) 2" × 3/8" Flats
3/4" Rivets
4" × 3" × 5/16" L
2" × 5/16" Flats
(2) 3 1/2" × 5/16" Flats
(2) 3" × 3/8" Flats
Junction plates 1/2" thick
7/8" dia where not dimensioned.
Purlin 5/8" Bolts
Rafter Angles
3/4" Rivets
5" × 3 1/2" × 3/8" L Cleat
Rafter
8" × 3 1/2" × 22.7 lbs
Boarding
Common
5" × 3 1/2" × 3/8" L Cleat
Rafter
3/4" Rivets
2 1/2" × 2" × 5/16" L
(2)
3/4" Rivets
3/4" Rivets
1 1/4" T & G
7" × 3"
(2) 5" × 3" × 3/8" L
(2) 4 1/4" × 1/2" Flats
Scale
Ins 12 9 6 3 0 1 2 3 Feet
Rag Bolts
3/4" C I Bearing Plate
4" × 4" × 1/2" Angles
3/4" Rivets
Stone Template
Plan of Shoe
4" × 4" × 1/2" Angles
1/2" Shoe Plate
C I Bearing Plate
Stone Template

FIG. 250.

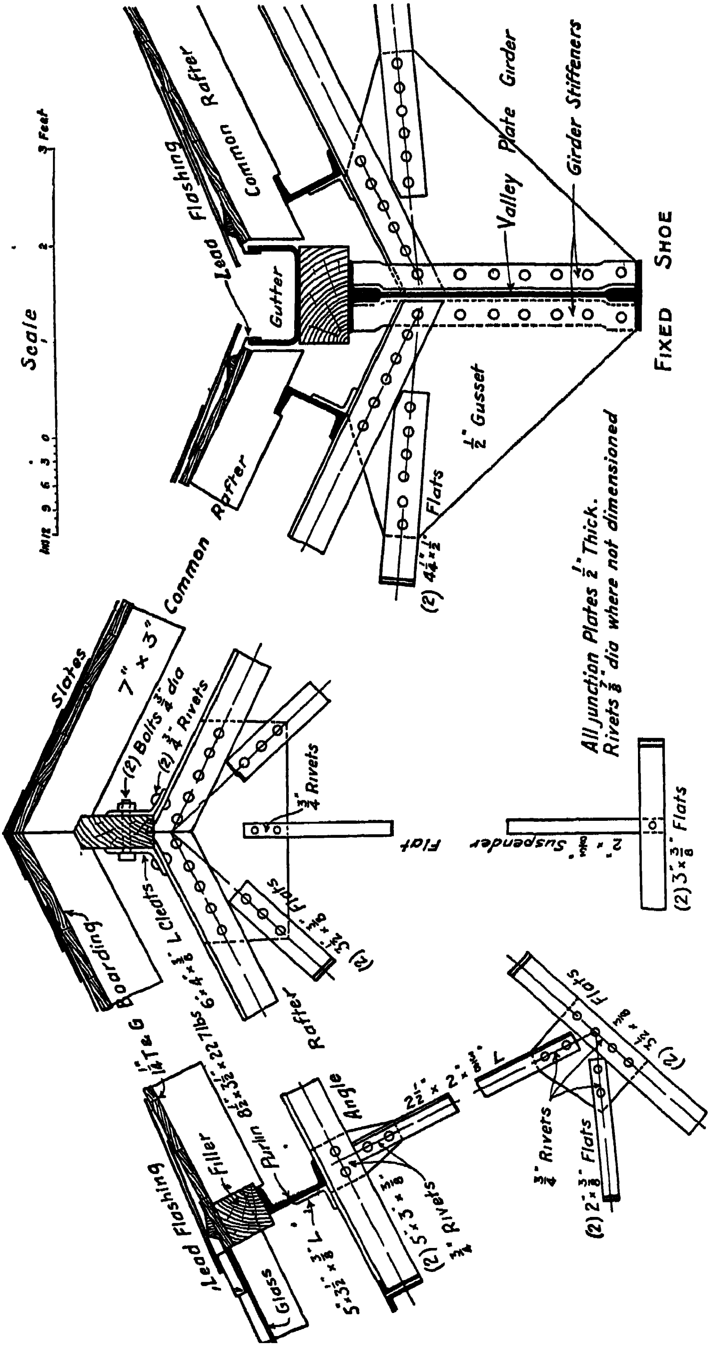

Scale
3 Feet
Lead
Flashing
Common Rafter
Gutter
Valley Plate Girder
Girder Stiffeners
Fixed Shoe
1/2" Gusset
Flats
(2) 4 1/4" x 1/2"
All junction Plates 1/2" Thick.
Rivets 7/8" dia where not dimensioned
Slates
7" x 3"
Common Rafter
(2) Bolts 3/4" dia
(2) 3/4" Rivets
Boarding
L Cleats
Rafter
3/4" Rivets
Flat
Suspender
(2) 3" x 3/8" Flats
1 1/4" T & G
Purlin
Filler
Lead Flashing
Glass
Angle
3/4" Rivets
(2) 2" x 3/8" Flats
3/4" Rivets

FIG. 251.

A vertical suspender has been inserted from the ridge to the middle tie to prevent excessive sag in the tie. This suspender does not take any primary stress.

The detailed design is shown in Figs. 250 and 251. The sliding shoe rests on a cast-iron bearing plate secured by rag bolts to a stone bearing block built into a wall. The wall is pocketed at the principal shoe. This shoe is formed by angles riveted to the frame members and to a shoe plate, the rivets on the underside of the plate being countersunk. The fixed shoe is connected to a plate girder valley beam by a junction plate riveted to the girder stiffener. The valley gutter rests on timber packings fixed to the valley girder by coach screws. Glazing is shown in the roof slope. The glazing bars are made of timber, notched on to and secured by wood screws to timber runners on the tops of the purlins. Patent metal glazing bars could, as an alternative, be fixed by screws in a similar manner.

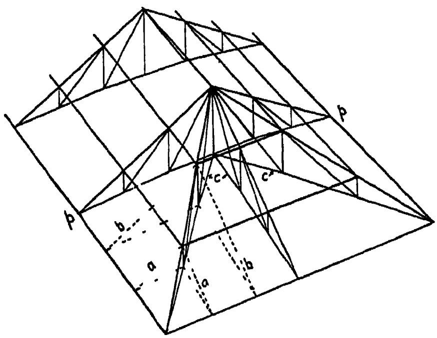


FIG. 252

Hipped ends of roofs are supported by one-sided principals, as shown in the diagrammatic sketch, Fig. 252. The full lines show three such hip principals converging on to the ridge of the first full principal. In large spans small principals shown dotted at *a* and *b* may be introduced to reduce the span of the purlins. Each of these hip principals is in the nature of a trussed rafter forming a lean-to supported at the shoe by the wall and at the ridge by the first full principal. The ties marked *c* are introduced for lateral stability only and take no primary stress. The purlins transmit dead and wind loads to the hip rafters, the panel loading on which is equal to the purlin reactions. The reactions at the shoe and ridge bearings of the hip rafters can be found in a similar manner as for the reactions of fixed ended principals, the elevation of the ridge bearing in no wise affecting these reactions. The vertical components of the ridge reactions form an additional load on the first full principal, whilst the horizontal components will be distributed throughout the length of the roof by the ridge bar and resisted by the roof structure as a whole. The stresses in the members of the hip rafters can be found by the usual stress diagram method.

The connections of the hip rafters to the full principal are formed by bent plates.

Cantilever Roofs.—Figs. 253 and 254 show the frame and stress diagrams for two single cantilever principals. The dead and wind loads can be estimated in the usual manner, and the stress diagrams drawn previous to calculating the reactions.

In Fig. 253 the reaction R_1 must be equal to and opposite in direction to the force in A5. Knowing, then, the resultant load P and the reaction R_1, R_2 must pass through x. In the stress diagrams R_1 is represented by $a5$ and P by ae, therefore $e5$ must represent R_2. The direction of R_2 is of course parallel to the line joining x, the intersection point of P and R_1 on the frame diagram, to the apex E.

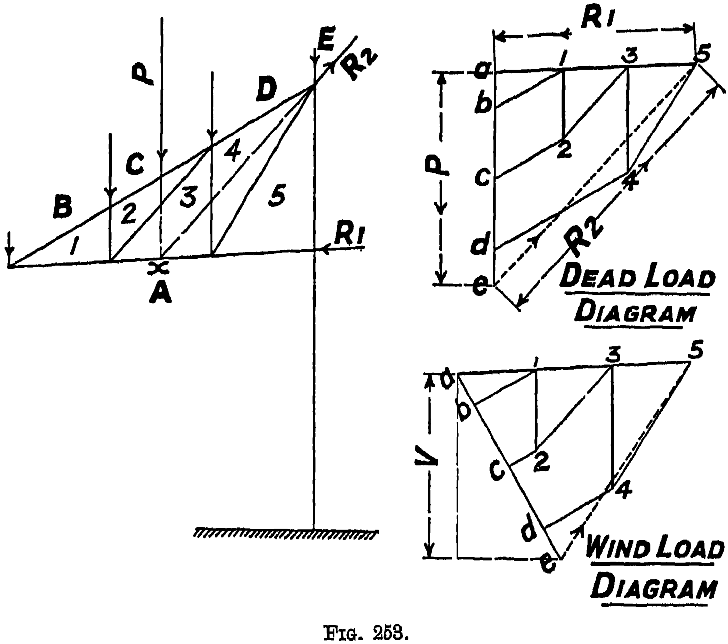


FIG. 253.

In Fig. 254, R_1 will be equal to and opposite in direction to the resultant of the forces in the members A4 and 4–5, *i.e.* $a5$ in the stress diagrams. Again, from the triangle of forces $e5$ represents the reaction R_2.

The signs of the stresses in the rafters and lower members correspond to those in the chords of any cantilever, being tensile in the rafters and compressive in the members A1, A3, etc. It will be noticed that the direction of slope of the web members affects the individual magnitudes of the vertical components of the reactions, but the horizontal components remain constant. The horizontal components produce an overturning moment on the roof support, which must be resisted by either the moment of resistance to bending of the column where such

is used, or by the stability of the wall to which the principals are fixed. Where the support consists of a column at each principal, the forces acting on any section X—X, Fig. 255, situated between R_1 and R_2, are :—

(*a*) The combined vertical downward pressures of the dead and wind load forces in the members at the apex $= V_1$.

(*b*) The combined horizontal dead and wind load forces at the apex $= H_1$.

The former is a direct load on the column and the latter produces a bending moment on the section $= H_1 \times Z_1$. The column must be

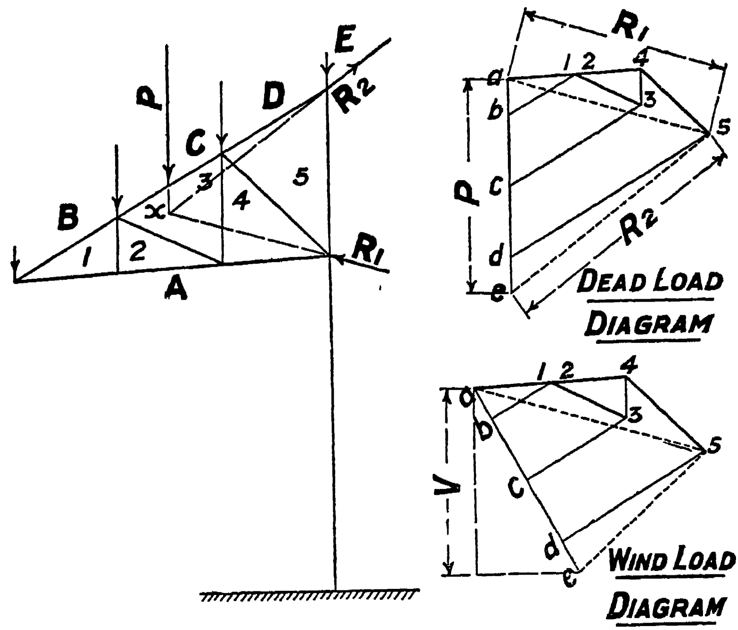


FIG. 254.

designed to resist the combination of direct load and bending moment.

If A = area of column section,
M = modulus of section,

then the maximum stress on the section

$$= \frac{V_1}{A} + \frac{H_1 \times Z_1}{M}$$

It is obvious that this stress increases from a minimum at the apex of the roof to a maximum at the lower principal connection to the column.

At any section Y—Y of the column below R_1 the direct load will

be the algebraic sum of the vertical components of the stresses in the principal members at the connections to the column. These equal the total dead load plus the vertical component of the wind load, *i.e.* P + V as shown on the stress diagrams. The horizontal components of the forces at the upper and lower connections act in opposite directions and the bending moment on the section = $H_1 \times Z_3 - H_2 \times Z_2$, where H_1 and H_2 are the horizontal components due to the combined dead and wind loads. The maximum stress on the section

$$= \frac{P + V}{A} + \frac{H_1 \times Z_3 - H_2 \times Z_2}{M}$$

The combined resultant of the dead and wind loads on the roof = R, Fig. 255, whose direction is inclined downwards towards the column. The bending moment on the column will therefore decrease towards

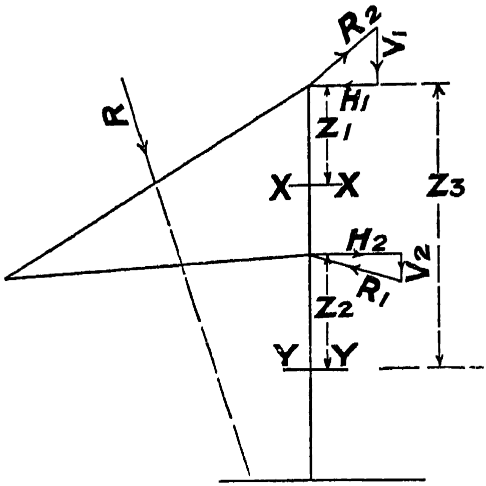


FIG. 255.

the bottom of the column, and hence the maximum stress will occur at the level of the connection of the lower principal member.

Where the roof principals are fixed to a wall, the rafters and lower struts should be connected to distributing beams B and B_1, Fig. 256, to prevent local failure of the brickwork and to bring the whole length of wall between the principals into action in resisting the overturning moment produced by the loads. The resistance to overturning will then be supplied by the weight of a length of wall equal to the distance between the principals and any loading D there may be from floors, etc., resting on the wall.

Let R = the resultant of the combined dead and live loads.

W = weight of wall above any section Z—Z under consideration.

D = weight of floors, etc.

Taking moments about the edge m of the wall :—

$$\text{Overturning moment} = R \times Z_4$$

$$\text{Moment of stability} = (W + D)\frac{t}{2}$$

For the wall to remain stable, the moment of stability must be at least equal to the overturning moment. To prevent tension on the joints in the right-hand face of the wall, the resultant pressure must fall within the middle third of the thickness t, see page 390.

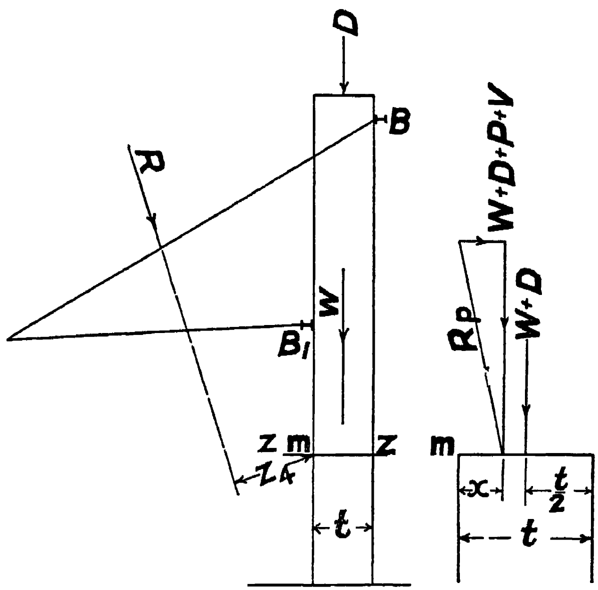


FIG. 256.

The vertical component of the resultant pressure R_p

$$= D + W + \text{vertical component of } R$$
$$= D + W + P + V$$

Then the moment of this force about m must be equal to the sum of the moments of the individual forces about the same point.

$$\text{That is, } R \times Z_4 - (W + D)\frac{t}{2} = (W + D + P + V)x$$

To avoid tension in the brickwork x must be at least equal to $\frac{t}{3}$ and must not exceed $\frac{2}{3}t$.

The horizontal component of R_p is the horizontal shear on the wall section at Z—Z.

If the wall projects considerably above the apex of the roof, the wind pressure on such area will have a stabilising moment on the wall, and the critical conditions may then be when the wind is not acting. Under such conditions $R = P$ and $V = 0$, and the foregoing equation reduces to

$$P \times Z'_4 - (W + D)\frac{t}{2} = (W + D + P)x$$

The wall may fail by shearing at B due to the pull in the rafter or by overturning or crushing between B and B_1. Below B_1 the overturning moment is constant when the wind is not blowing or a decreasing quantity under wind action, so then the critical section occurs at B_1.

Double Cantilever Roof.—Island platforms are often covered by a double cantilever roof supported by a central row of columns. The dead load is balanced about the column in which it produces no bending stresses. The wind acting on one slope produces a similar bending moment on the column to that produced by the wind acting on the single cantilevers of Figs. 253 to 256. The members of the leeward cantilever are not stressed by the wind pressure. The stresses in the windward cantilever and the bending moment on the column are found by the methods employed for single cantilevers.

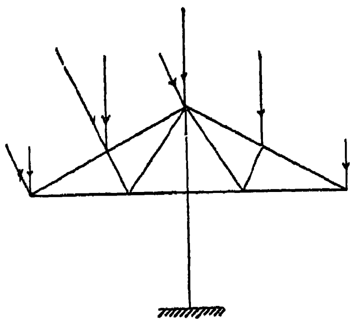


FIG. 257.

Principals with Curved Rafters.—The normal wind pressure on principals supporting curved roofs, varies in intensity from a maximum at the shoe to a minimum at the ridge, owing to the slope presenting a decreasing resistance from the shoe upwards, to the horizontal wind force. Each panel load will be the product of the area of slope supported and the intensity of wind pressure, as given in Fig 239, for a slope tangential to the curve at the panel point, and its direction of action will be normal to the tangent. Thus the panel point loads will be unequal and non-parallel for equal spacing of the panel points. The resultant of the panel loads can be found as in Fig. 258, where the loads to scale have been arranged end to end, the resultant being obviously equal to bg in magnitude and direction, since the sum of the vertical components of the loads $= bx$ and the sum of the horizontal components $= xg$. The position of the resultant on the frame diagram can be obtained as follows: Select on Fig. 258 any point o about opposite the midpoint of the load line and join o to $b, c \ldots g$. From any point k on the line of action of the load BC, draw kl parallel to oc and terminating on the line of action of the load CD. From l draw lm parallel to od and terminating on the line of action of DE. Continue this process to p. Through k and p draw lines kr and pr parallel to ob and og respectively. These lines intersect at r, which is a point on the line of action of the

resultant load P. Drawing P, through r, parallel to bg, establishes the position of the resultant.

If both shoes of the principal be fixed, the reactions will be parallel to the resultant load, and their magnitudes can be determined by taking moments about either shoe:

$$P \times s - R_2 \times t = 0$$

$$R_2 = \frac{P \times s}{t}$$

$$R_1 = P - R_2$$

If one shoe only be fixed the reactions can be found by the method of Fig. 244. The dead load presents no difficulties, the panel point

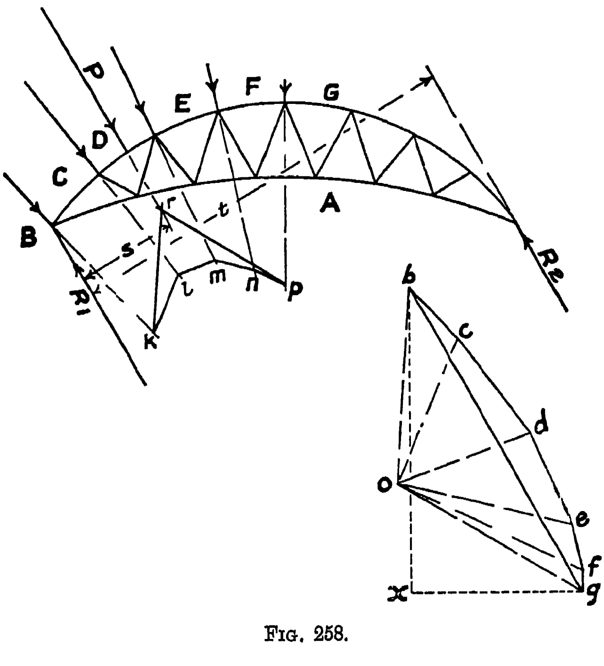


Fig. 258.

loads and reactions being determined as for V principals. If the upper member be actually curved and not straight between panel points, there will be a varying amount of eccentric loading on that member, and the bending moment thus produced must be taken into consideration when designing the section of the member (see remarks, page 254, on curved upper chord of bowstring girder).

Calculation of Stresses by the Method of Sections.—This method of calculating the stresses in any frame is based upon the following principle, proved in mechanics: if a structure be in equilibrium, the algebraic sum of the moments of all the forces acting in one plane, and to either side of any section, about any point in that plane, must be zero.

Suppose the roof principal in Fig. 259 be subject to the vertical loads indicated in the figure. To find the stress in any member, say AB, draw a section X–X cutting such member. The forces acting to the left of the section X–X are: the reaction at A and the stresses in the members AB and AD, cut by the section. For these forces to be in equilibrium the sum of their moments about any point in the plane of the principal must be zero. By taking moments about any point *b* in AD, Fig. 259, *c*, the moment of the stress in AD will be zero, and there remains then only the unknown moment of the force in AB. The portions of the figure to the left of the sections XX, YY, and ZZ are reproduced to a larger scale in Figs. *c*, *d*, and *e*.

Let S_{AB}, S_{BC}, S_{AD}, etc. = the stresses in the respective members.

Denoting all clockwise moments as positive and anticlockwise moments as negative, then for the stress in AB, taking moments about the point *b*, Fig. 259, *c*.

$$21 \times 6' + S_{AB} \times 2' \, 8'' = 0$$

$$\therefore S_{AB} = -\frac{21 \times 6}{2 \cdot 67}$$

$$= -47 \cdot 2 \text{ cwts.}$$

The above value of S_{AB} being negative indicates that its moment about *b* is anticlockwise, and therefore the force in AB acts *towards* the joint at A. Hence the stress in the member AB is compressive.

To calculate the stress in the member AD take moments about the point *a*, Fig. 215, *c*.

$$21 \times 4 \cdot 5 + S_{AD} \times 2 \cdot 33 + S_{AB} \times 0 = 0$$

$$\therefore S_{AD} = -40 \, 6 \text{ cwts.}$$

The negative sign again indicates an anticlockwise moment. The force in AD will therefore act *from* the joint at A and be of a tensile character.

To find the stress in the member BC take a section YY, cutting that member and the least number of other bars. The forces to the left of the section YY are: the reaction at A, the load at B, and the stresses in the members BC, BD, and AD, cut by the section. The forces in the members BD and AD acting through the point D, their moments about D are zero. The only unknown moment is therefore that due to the stress in BC. Taking moments about D, Fig. 259, *d*,

$$21 \times 15 - 14 \times 7\tfrac{1}{2} + S_{BC} \times 6 \cdot 75 = 0$$

$$S_{BC} = -31 \cdot 1 \text{ cwts.}$$

The value of S_{BC} being negative, the force in BC must act towards the joint at B, and therefore BC is a compressive member.

To find the stress in BD. Take moments about A, Fig. 259, *d*.

$$21 \times 0 + 14 \times 7 \cdot 5 + S_{BD} \times 7 \cdot 125 + S_{BC} \times 0 + S_{AD} \times 0 = 0$$

$$\therefore S_{BD} = -14 \cdot 7 \text{ cwts.}$$

The moment of S_{BD} about A being anticlockwise, the member BD will be in compression.

To find the stress in CD. Take the section ZZ cutting the members BC, CD, DE, and DF. The forces on the left of the section are: the reaction at A, the load at B, and the stresses in the members BC, CD, DE, and DF. As the principal is loaded symmetrically, the stresses in DE and DF are equal to the stresses in BD and AD

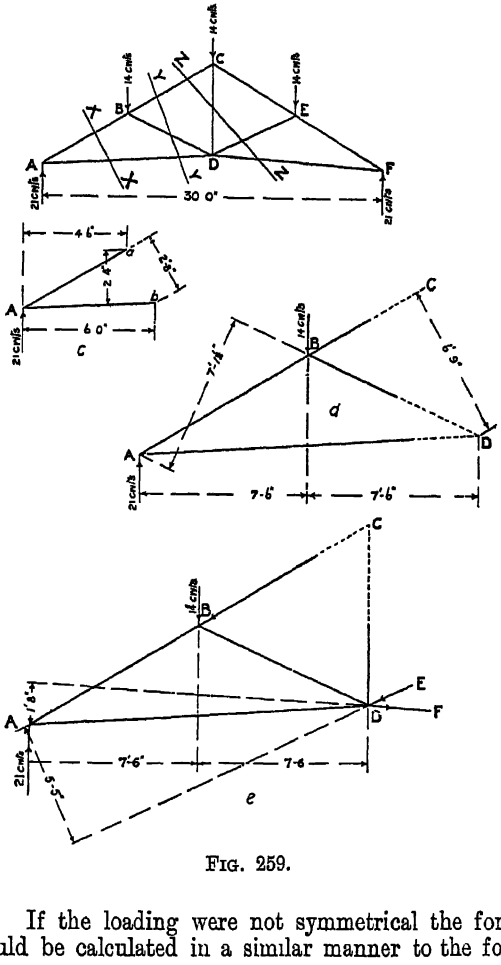


Fig. 259.

respectively. If the loading were not symmetrical the forces in DE and DF would be calculated in a similar manner to the forces in BD and AD.

Taking moments about A, Fig. 259, *e*,

$$21 \times 0 + 14 \times 7{\cdot}5 + S_{BC} \times 0 + S_{DE} \times 5{\cdot}41 + S_{DF} \times 1{\cdot}67 + S_{CD} \times 15 = 0$$

$$14 \times 7{\cdot}5 + 14{\cdot}7 \times 5{\cdot}41 + 40{\cdot}6 \times 1{\cdot}67 + S_{CD} \times 15 = 0$$

$$\therefore S_{CD} = -16{\cdot}8 \text{ cwts.}$$

The moment of S_{CD} about A being anticlockwise, the member CD is in tension.

Wind load stresses for all the members may be obtained in a similar manner.

It was shown in the wind diagram of Fig. 245 that no wind stress occurred in the member 3–4 when the wind acted on the left-hand slope of the roof. This is clearly demonstrated by the above method of calculation. Take a section XX, Fig. 260, and moments about the

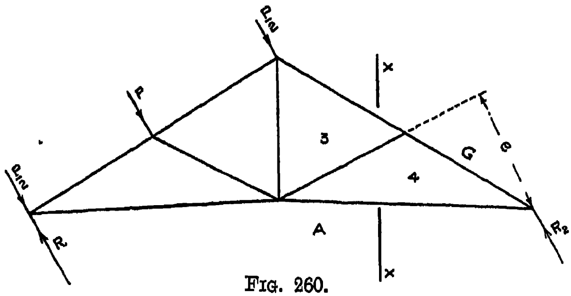


FIG. 260.

right-hand shoe, of all the forces acting to the right of the section—

$$(R_2 + S_{A4} + S_{G5}) \times 0 + S_{3-4} \times e = 0$$

$$\therefore S_{3-4} \times e = 0$$

that is, there is no stress in the member 3–4.

Fig. 261 is another example of the graphical method of determining

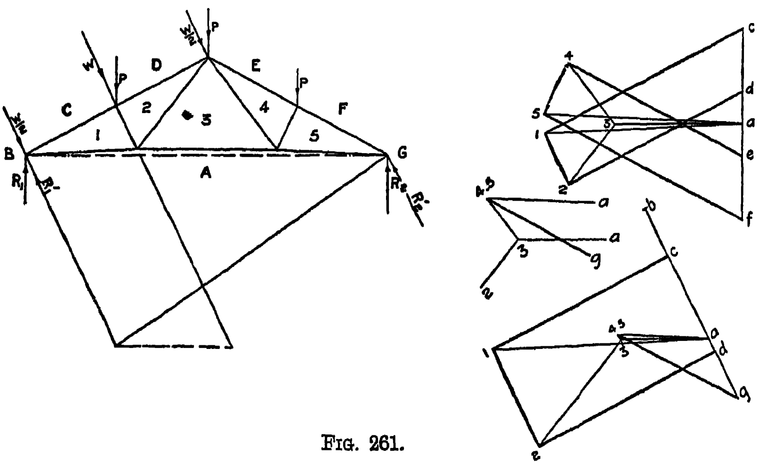


FIG. 261.

the dead load and wind load stresses in a roof principal. The compressive stresses are indicated by the heavier lines.

In Fig. 262 are drawn the stress diagrams for a double cantilever roof. The dead load diagram presents no difficulties, but the bending action in the member 3–4 makes it impossible to obtain a closed wind load diagram of the direct forces in all the members. The magnitude of the bending moment at the lower end of the member 3—4 equals R × *e*. By calculating the horizontal force at the apex necessary to

balance this bending moment and plotting it at $d4'$ on the wind diagram, a closing point $4'$ is obtained. The stresses produced by the wind in the member 3–4 are, the direct compressive stress 3–$4'$ and the stresses due to the bending moment $d4' \times f$. All other stresses in the diagram are the direct stresses in the respective members.

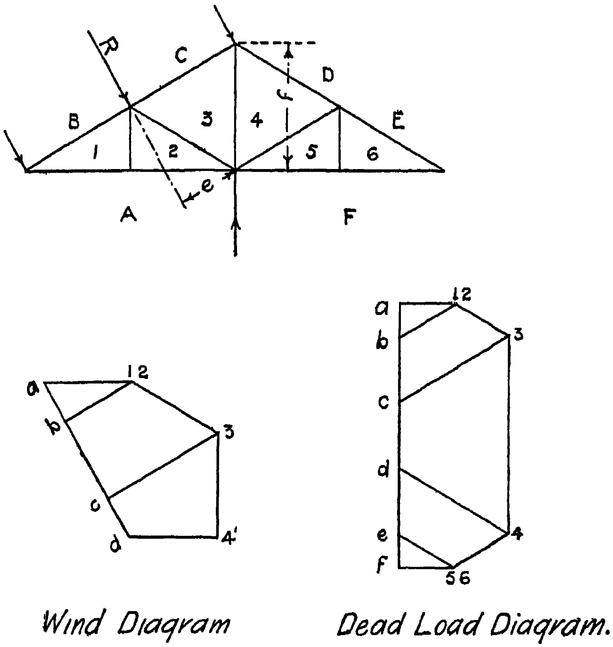


FIG. 262.

Three-hinged Arch Roofs.—Roofs of this description are frequently used for exhibition and other buildings requiring very large span roofs. The pins or hinged joints allow the apex of the roof to rise or fall when, owing to changes in temperature the expansion or contraction affects the length of the arch. The increase in stress in the members of rigid and two-hinged arches due to changes in temperature is considerable and very tedious to estimate, but the use of the third hinge at the crown allows of changes in the length of the arch without affecting the stresses in the frame.

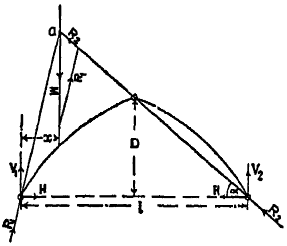


FIG. 263.

Suppose a vertical load W be supported by a three-hinged arch and the reactions be R_1 and R_2, Fig. 263

Let V_1 and V_2 be the vertical components of the reactions.

Then the vertical forces being in equilibrium

$$W = V_1 + V_2$$

Taking moments around the base hinges

$$Wx = V_2 l \quad \text{or} \quad V_2 = \frac{Wx}{l}$$

and $$W(l - x) = V_1 l \quad \text{or} \quad V_1 = \frac{W(l - x)}{l}$$

Taking moments about the crown, of the forces to the right

$$V_2 \frac{l}{2} = H \times D \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where H = the horizontal component of either reaction.

Substituting for V_2

$$\left(W\frac{x}{l}\right)\frac{l}{2} = H \times D$$

$$\therefore H = \frac{Wx}{2D}$$

To produce equilibrium the horizontal components, H, of the reactions R_1 and R_2 must be equal. Let α = the angle of inclination of the reaction R_2. Then $\tan \alpha = \frac{V_2}{H}$

From the equation (1) above $\frac{V_2}{H} = \frac{2D}{l}$

$$\therefore \tan \alpha = \frac{D}{\frac{l}{2}}$$

The reaction R_2 must therefore pass through the crown hinge. Knowing the lines of action of two of the forces acting on the arch, the direction of the third, R_1, may be obtained, since all three forces must pass through the same point. Producing R_2 through the crown hinge it cuts the load line at a. Joining the left-hand hinge to a gives the direction of R_1. The magnitude of the reactions may be obtained graphically by a triangle of forces.

The same construction holds good if the load be inclined to the vertical. Taking moments of the forces to the right, about the crown, Fig. 264,

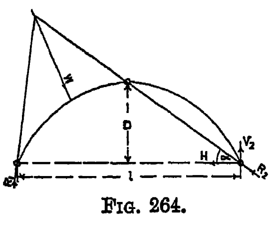


Fig. 264.

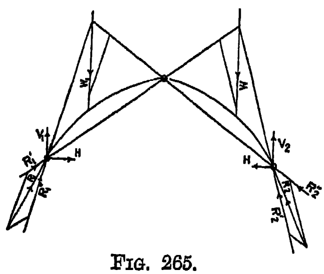


Fig. 265.

$$V_2 \frac{l}{2} = H \times D$$

$$\tan \alpha = \frac{V_2}{H} = \frac{D}{\frac{l}{2}}$$

R_2 must therefore pass through the crown.

If each half of the arch be loaded, the reactions may be found graphically by treating each load separately and combining the reactions so found, Fig. 265.

The reaction at the left support

$$\text{due to } W_1 = R_1''$$
$$\text{,, \quad ,, } W = R_1'$$
$$\text{The total reaction} = R_1$$

The reaction R_2 is obtained in a similar manner.

If the loads be equal and similarly disposed

$$\text{then } V_1 = V_2 = W_1 = W$$

and the vertical shear at the crown

$$= V_1 - W_1 = 0$$

The force on the hinge at the crown must therefore act horizontally, and the reactions and thrust at the crown may be found graphically as follows, Fig. 266. Through the crown draw a horizontal line to cut the load line in *a*. Join *a* to the base hinge. Then *ab* is the line of action of the reaction. The magnitudes of R_1 and H are again obtained by means of a triangle of forces.

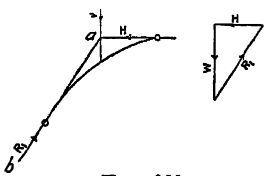


FIG. 266.

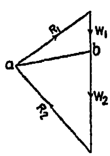


FIG 267.

If the two halves of the arch be unequally loaded the thrust at the crown will be inclined, Fig. 267. Its direction and magnitude may be found by the following construction. Obtain the reactions R_1 and R_2 by the above methods and draw the force diagram W_1, W_2, R_1, and R_2. The thrust on the crown is then equal to *ab*, the horizontal component of which equals the horizontal component of either reaction and the vertical component equals $W_2 - V_2$, or $V_1 - W_1$.

In Fig. 268 are drawn the dead load stress diagram for the left-hand half of a three-hinged arch and the wind stress diagram for the whole arch, the wind assumed acting on the right-hand slope. The positions and magnitudes of the resultants of the dead and wind loads are found by means of polar and funicular diagrams. The dead load on the two portions of the arch being equal, the thrust on the centre pin due to that load will be horizontal. The magnitudes of the reaction *ab* and the central thrust *am* are obtained on the dead load diagram. The dead load stresses in the members of the right-hand portion will be similar to the stresses in the corresponding members of the left-hand portion. Since the wind exerts no pressure on the left-hand half of the truss, the reaction R_1, due to the wind, passes through the centre pin. R_2 passes through the right-hand pin and the intersection of R_1 and the resultant wind pressure. The reactions on the wind stress diagram are obtained by drawing parallels to R_1 and R_2 from *l* and W, the ends

of the resultant, respectively. These lines intersecting very obliquely at a, the magnitudes of the reactions should be checked by calculation,

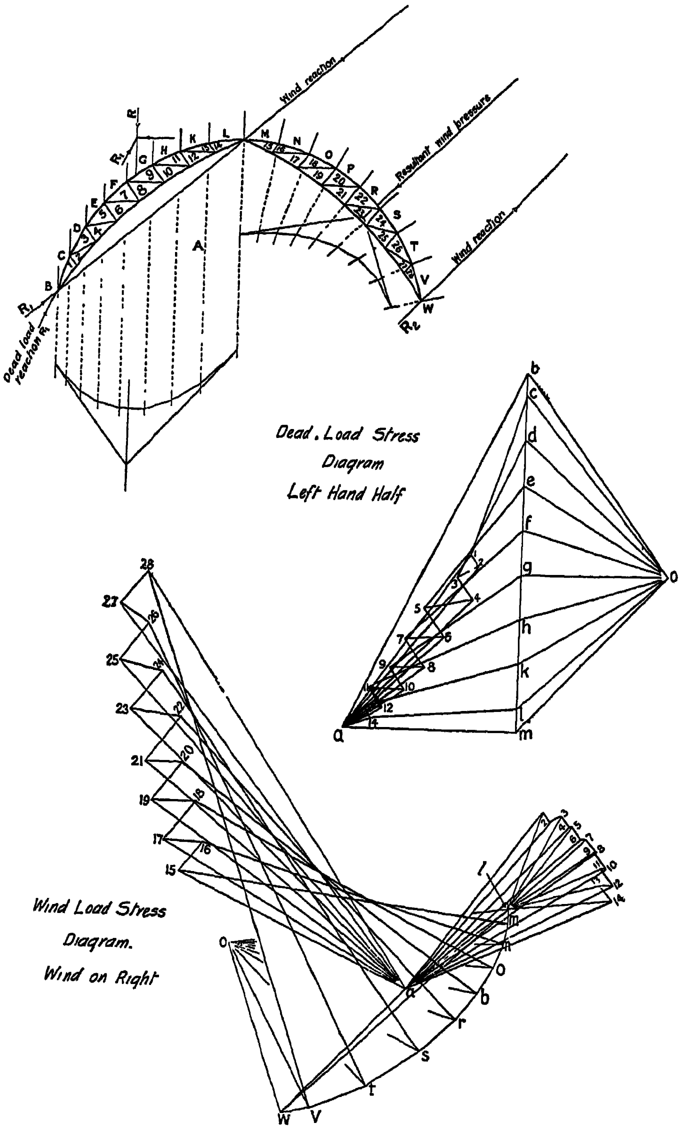


FIG. 268.

as a small error in the position of a will render a closed stress diagram impossible.

The character of the stress in some of the members will vary

according to the direction of the wind. For example, the stresses in the member 10–A are—

Due to dead load . . . . compressive;
„ wind on the right . . . . . . compressive;
„ „ left . . . . tensile.

The maximum compression in this member will therefore occur when the direction of the wind is right to left, and will be the sum of the dead and wind load stresses. The minimum compression or the maximum tension will be the difference of the stresses due to the dead load and the wind acting on the left-hand slope. The nature of the stresses in the other members of the same panel are given in the following table.

Member.	Stress due to			Maximum compression = stresses in columns	Maximum tension = stresses in columns.
	Dead load	Wind on right	Wind on left		
	1	2	3		
11–H	compression	tension	compression	1+3	2−1
10–11	tension	tension	compression	3−1	1+2
11–12	compression	compression	tension	1+2	3−1

Roof Details.—In Fig. 269 are shown details of a number of methods of attaching various coverings to the structural members of roofs. Glazing bars are usually fixed to timber purlin and ridge bars by means of copper screws, details A and B, and to steel purlins by copper bolts as indicated in detail C. Lead flashings preserve water-tightness at the junctions, and where the form of glazing bar necessitates a space between purlin and glass, wind bars of lead or other non-corrosive metal are employed to fill up such space. The glass is prevented from slipping down the slope by copper clips bolted to the glazing bars and bent under the glass. Details E and F are examples of the connection of skylight bars occurring in the middle of a roof slope covered with corrugated sheeting supported on steel purlins. The sheeting in each case is fixed to the purlins by hook bolts, as shown in detail D. The same detail also shows the ridge of such a roof with a pressed galvanized cover. Detail G is a suggestion for the apex of a northern light roof covered with glazing on the steeper slope, and slates on boarding on the opposite slope. Detail H is a common method of fixing weather boards to the eaves of a cantilever roof suitable for a station or loading platform.

Fig. 269A shows the general arrangement of a Warren girder roof The area is spanned by parallel Warren girders having bays of 12′ 6″ to 16′ 6″ length and spaced 20′ to 30′ apart. The web members of the girders act as principal rafters, and intermediate principals are carried by the longitudinal beams B framed in between the main girders. These also carry the valley gutters V. The roof slopes

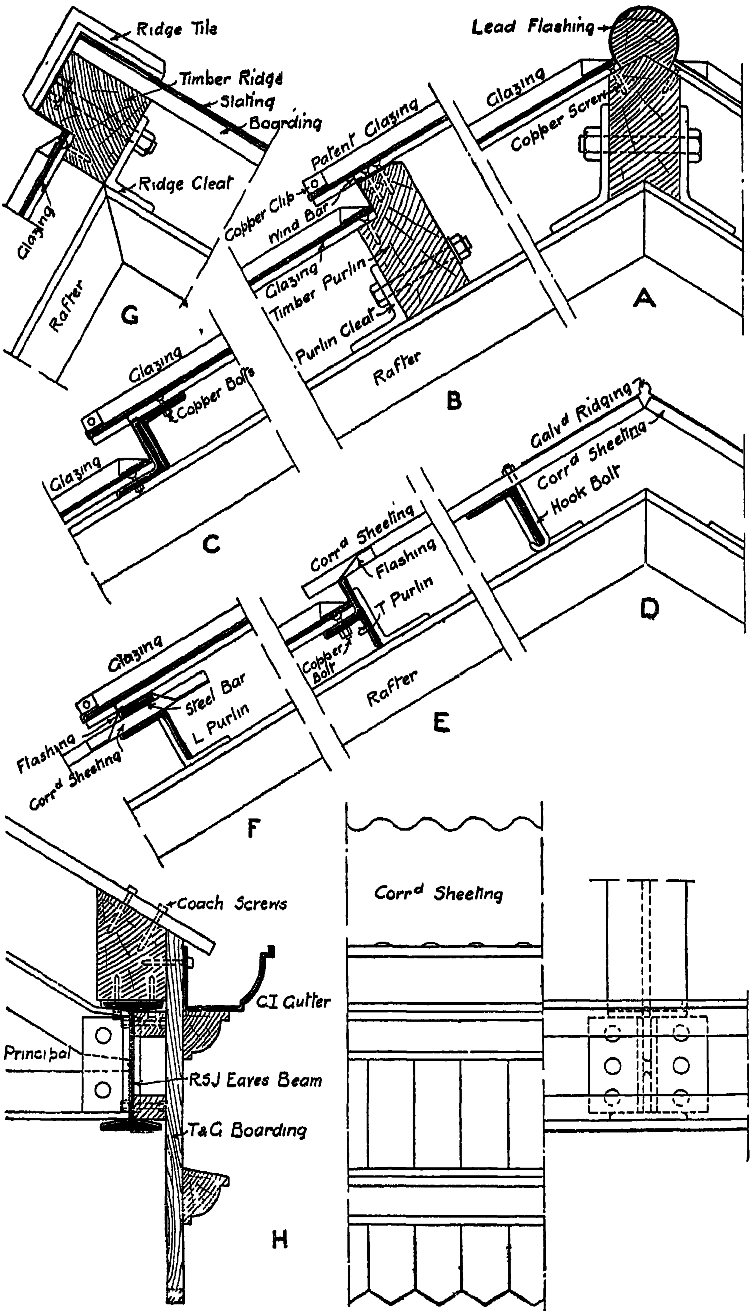

Ridge Tile
Timber Ridge
Slating
Boarding
Ridge Cleat
Glazing
Rafter
G
Lead Flashing
Glazing
Copper Screw
A
Patent Glazing
Copper Clip
Wind Bar
Glazing
Timber Purlin
Purlin Cleat
Rafter
B
Glazing
Copper Bolts
Glazing
C
Galvd Ridging
Corrd Sheeting
Hook Bolt
D
Corrd Sheeting
Flashing
T Purlin
Copper Bolt
Rafter
E
Glazing
Steel Bar
L Purlin
Flashing
Corrd Sheeting
F
Coach Screws
CI Gutter
Principal
RSJ Eaves Beam
T&G Boarding
H
Corrd Sheeting

FIG. 269.

intersect the upper chord C of the girders, the roof covering being suitably trimmed and flashed around the chord. This type of roof finds much favour for machine shops where absence of columns and good lighting are specially desired.

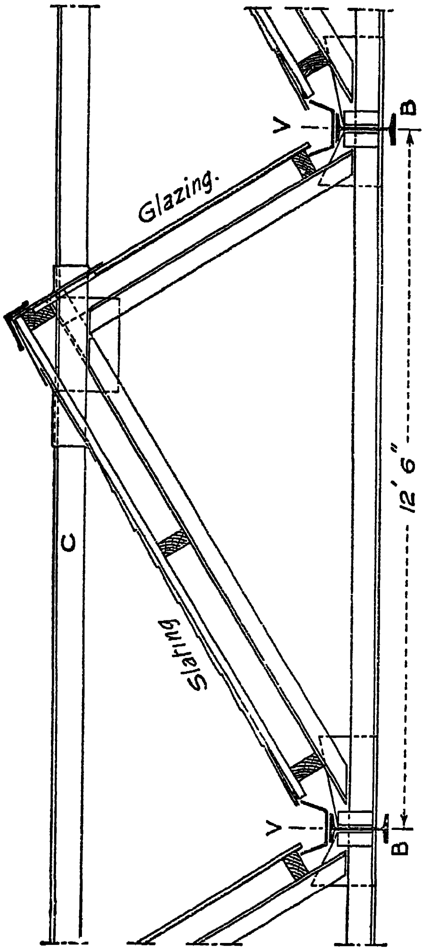


Fig. 269A.

CHAPTER XI.

MISCELLANEOUS APPLICATIONS AND TALL BUILDINGS.

Design for Lattice Roof Girder.—*Span* 36 *feet. Depth* 4 *ft.* 6 *in. To carry the feet of principals of two adjacent roof spans of* 50 *feet. Roof principals* 12 *feet apart.* The general arrangement is shown in Fig. 271, A The roof pitch is 1 to 2, and the loading is assumed as follows :—

Covering and snow at 20 lbs. per square foot of area covered $= 12 \times 50 \times 20 = 12{,}000$ lbs. for one principal.

Weight of principal $= \frac{7}{8}\mathrm{DL}\left(1 + \frac{\mathrm{L}}{12}\right) = \frac{7}{8} \times 12 \times 50(1 + \frac{50}{12})$ $= 2712$ lbs.

Normal wind pressure on one roof slope per 12 feet length, at 25 lbs. per square foot $= 28$ feet (length of slope) $\times 12 \times 25 = 8400$ lbs.

Reactions due to wind pressure

$$\text{at } a = 8400 \text{ lbs.} \times \tfrac{31}{45} = 5787 \text{ lbs.}$$
$$\text{and at } b = 8400 - 5797 = 2613 \text{ lbs.}$$

The central girder b provides the smaller reaction for the right-hand roof span and the larger reaction for the left-hand span when the wind blows from the right. The total inclined wind load on girder b at the points of support of the principals is therefore the total normal wind pressure of 8400 lbs. On the outer girders at a and e the inclined wind load will be equal to the larger reaction of 5787, say 5800 lbs.

The vertical component of the inclined pressure of 8400 lbs. $= 8400 \times \frac{bf}{bc} = 8400 \times \cos 26\frac{1}{2}° = 8400 \times 0{\cdot}894$

$$= 7509, \text{ say } 7500 \text{ lbs.}$$

The total *vertical* loading concentrated at points h and k Fig. 271, B, is therefore for the central girder,

due to covering and snow	. . . .	12,000 lbs.
„ two half principals	. . . .	2,712 „
„ vertical wind load	. . . .	7,500 „
		22,212 lbs., say 10 tons.

The principals at m and n being supported directly by the columns, do not affect the stresses in the girder.

The weight of the girder will be assumed as 3 tons, distributed

equally amongst the upper joints. The vertical loading and stresses are then as indicated in Fig. 270.

The horizontal component of the normal wind pressure causes lateral bending of the girder, and although some portion of the lateral bending moment will be resisted by the longitudinal roof members and transmitted

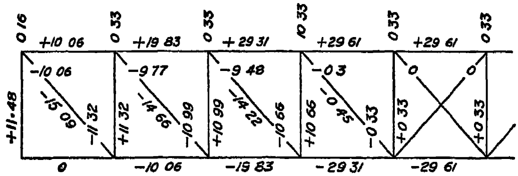


FIG. 270.

by diagonal wind bracing (where provided) to the columns, the amount of the horizontal wind load so resisted is very uncertain, and it is only reasonable to make the girders strong enough to resist the whole.

The horizontal component of wind pressure at h and k, Fig. 271, B, $= 8400 \times \sin 26\frac{1}{2}° = 8400 \times 0{\cdot}446 = 3746$, say 3750 lbs. $= 1{\cdot}67$ tons. Hence lateral bending moment between h and k

$$= \frac{3750 \times 12' \times 12}{2240} = 241 \text{ inch-tons.}$$

Flanges.—The flanges over the five central panels consist of two $4\frac{1}{2}'' \times 3'' \times \frac{1}{2}''$ angles and two $12'' \times \frac{3}{8}''$ plates. The outer plate is suppressed over the two end panels.

For the moment of inertia of the flanges about YY, Fig. 271, C,

$$\begin{aligned} I_y \text{ for one angle } &= 2{\cdot}55, \text{ from section book;} \\ \therefore I_Y \;,, \quad\; ,, \quad &= 2{\cdot}55 + (3{\cdot}5 \times 3{\cdot}75^2) = 51{\cdot}5 \\ I_Y \;,, \text{ four angles} &= 51{\cdot}5 \times 4 = 206 \text{ inch units.} \\ I_Y \;,, \text{ four } 12'' \times \tfrac{3}{8}'' \text{ plates} &= \tfrac{1}{12} \times \tfrac{3}{8} \times 12^3 \times 4 = 216 \\ \text{Total } I_Y \text{ for both flanges} &= 206 + 216 = 422 \end{aligned}$$

$$\text{Stress due to lateral bending} = \frac{241 \times 6''}{422} = 3{\cdot}43 \text{ tons per square inch.}$$

Sectional area of top flange = 16 square inches. Maximum compression in central bay = 29 61 tons, and compression per square inch due to vertical loading $= \frac{29{\cdot}61}{16} = 1{\cdot}85$ tons. Hence maximum intensity of compression in top flange due to vertical and lateral loading $= 3{\cdot}43 + 1{\cdot}85 = 5{\cdot}28$ tons per square inch.

The net section of the lower flange after deducting four $\frac{3}{4}$ in. rivet holes $= 14\frac{1}{8}$ square inches, and maximum tension due to vertical loading $= \frac{29{\cdot}61 \text{ tons}}{14{\cdot}125} = 2{\cdot}1$ tons per square inch. Adding the stress due to lateral bending, maximum intensity of tension in lower flange $= 2{\cdot}1 + 3{\cdot}43 = 5{\cdot}53$ tons per square inch. The actual value is a little higher than this, since no rivet holes were deducted in calculating I_Y

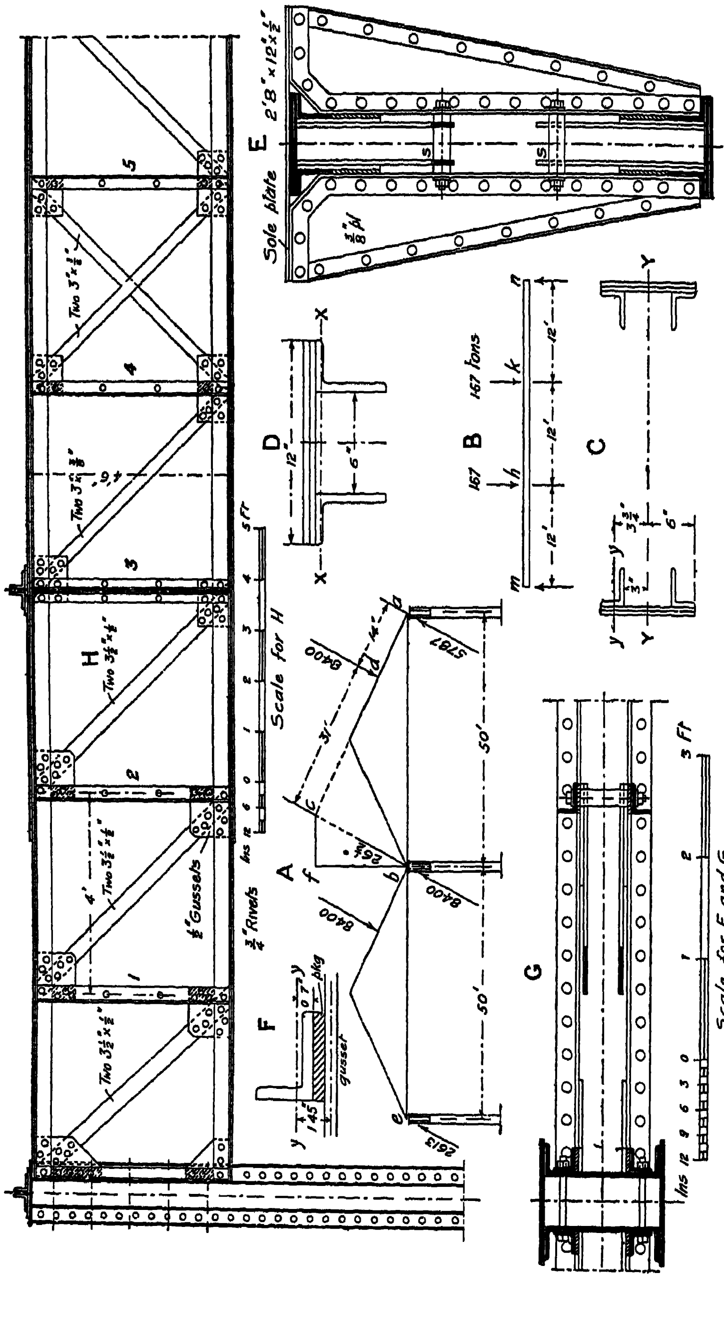

H
Scale for H
5 Ft
Two $3\frac{1}{2}" \times \frac{1}{2}"$
Two $3" \times \frac{1}{2}"$
$\frac{1}{2}"$ Gussets
$\frac{3}{4}"$ Rivets
E
Sole plate
$2'8" \times 12" \times \frac{1}{2}"$
$\frac{3}{8}"$ pl
D
B
167 tons
C
A
50'
31'
8400
5787
2613
F
gusset
pkg
G
Scale for E and G
3 Ft

FIG. 271

for the lateral bending. These working stresses are sufficiently low to allow a fair margin for the impact effect of the wind pressure. It is, however, very improbable that the maximum wind pressure would ever be applied instantaneously. It will be noticed further that the maximum wind load has been assumed on the left-hand roof span, whereas this would be sheltered to some extent by the right-hand roof.

The moment of inertia of the upper flange section about XX, Fig. 271, D, works out at 29 units, and least radius of gyration $= \sqrt{\frac{29}{16}}$ $= 1{\cdot}35$ in. The panel length is 48 in., whence $\frac{l}{r} = \frac{48}{1{\cdot}35} = 36$, and *average* safe load on top flange supposing the ends rounded = 13,700 lbs. or 6·1 tons per square inch. The *average* working stress is much below this, so that the flange is amply safe against buckling.

Struts.—Struts 1 and 2, Fig. 271, H, consist of two angles $3\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{1}{2}''$ tied together by bolts and tube separators *s*, *s*, Fig. 271, E. The maximum compression = 11·32 tons. Assuming one-half resisted by each angle, compression in one angle = 5·66 tons. This is eccentrically applied since the angle is riveted to the gusset plate by one leg only. Fig. 271, F, shows the dimensions concerned. The eccentricity is 1·45 in., y–y being the axis through the c.g. of the angle section. The sectional area = 2·75 square inches, and direct compression $= \frac{5{\cdot}66}{2{\cdot}75}$ = 2·06 tons per square inch. B.M. due to eccentricity of load = 5·66 × 1·45 = 8·2 inch-tons, I_y for the section = 1·43, hence compressive stress due to bending

$$= \frac{8{\cdot}2 \times 0{\cdot}7}{1{\cdot}43} = 4{\cdot}0 \text{ tons per square inch}$$

and maximum intensity of compression = 2·06 + 4·0 = 6·06 tons per square inch. As the struts are not long relatively to the radius of gyration, this intensity is not excessive, and they are further stiffened by the separators. Strut 3, beneath the shoes of the principals, is combined with gusset stiffeners to give lateral rigidity, and to assist in transferring the lateral wind load to both flanges of the girder. It consists of four $2\frac{1}{2}'' \times 2\frac{1}{2}'' \times \frac{3}{8}''$ angles, with $\frac{3}{8}$ in. gusset plates. Struts 4 and 5 have very little compression to resist, but cannot well be made of smaller sections than $3'' \times 2\frac{1}{2}'' \times \frac{3}{8}''$ for practical convenience in riveting. They may be subject to somewhat higher stresses under the action of a rolling gust of wind.

Ties.—These have been proportioned for a working stress of 6 tons per square inch on the net section. Those in the first three panels from the column, having practically the same stress, will be designed for 15·09 tons. Using two flats, stress in one bar = 7·55 tons. Net sectional area $= \frac{7{\cdot}55}{6} = 1{\cdot}26$ sq. in. Adding for one rivet-hole $\frac{3}{4}'' \times \frac{1}{2}'' = 0{\cdot}375$, gross section = 1·26 + 0·375 = 1·635 sq. in. A $3\frac{1}{2}'' \times \frac{1}{2}''$ flat gives 1·75 sq. in. The ties in the three central panels have very little stress. Two flats $3'' \times \frac{3}{8}''$ have been employed. The central panel is counterbraced. Fig. 271, H, shows the general elevation of half the girder, and G a part sectional plan with detail of

connection to a box section column. Sole plates 2′ 8″ × 12″ × ½″ are placed 12 ft. from each column, to which the feet of the adjacent roof principals are bolted. The number of rivets are arranged on a basis of 4 tons shearing stress and 8 tons bearing stress per square inch respectively. The connection to the column is made through two vertical angle cleats 3″ × 3″ × ½″ bolted to the column by ten ¾ in. through bolts. All gusset plates are ½ in. thick.

Design for Crane Jib.—*Length,* 45 *ft. Load,* 3 *tons. To lift at* 2½ *ft. per second and slew at* 6 *ft. per second, with* 1 *ft. per second acceleration for both lifting and slewing* (*Fig.* 272). *Lowest position of jib inclined* 30° *with the horizontal. Inclination of backstays* 15°, *and of rope* 20°.

The nominal load lifted = 3 tons. Allowing 0·25 ton for the weight of chain, bob and hook, total weight lifted = 3¼ tons. This will be doubled to provide against shock due to possible slipping of tackle, giving an equivalent load lifted = 6½ tons.

$$\text{Accelerating force in lifting} = \frac{W \times f}{g} = \frac{3\frac{1}{4} \times 1}{32} = 0{\cdot}1 \text{ ton.}$$

Maximum tension in chain = 6·5 + 0·1 = 6·6 tons.

In Fig. 272, A, AB inclined 30° represents the jib, and AT_1 and AT the chain. Setting off AT, AT_1, each = 6·6 tons, the resultant AR is obtained from the parallelogram $ATRT_1$. Draw RB parallel to the backstay AS, and AB scaling 25¼ tons gives the direct compression in the jib due to the tension in the two portions of the chain.

Approximate Weight of Jib.—Adopting 2½″ × 2½″ × ⅜″ angles and 2¼″ × ⅜″ flats for the bracing on upper and lower faces, and 2¼″ × 2¼″ × 0·3″ angles and 2¼″ × ⅜″ flats for the side bracing, an approximate estimate of the weight of the jib runs out at 1·75 tons.

Stress due to Weight of Jib.—With the jib in an inclined position, its own weight will give rise to both direct compression and bending stress in the main angles forming the section of the jib. In Fig. 272, A, set off AE = 45 ft, and draw the vertical CD through M the middle point of AE. Join EC. AC and EC give the directions of the reactions at the pulley and hinge ends of the jib respectively, due to its own weight. Note that the c.g. of the jib has been assumed at M. Although the lower half is wider and consequently somewhat heavier than the upper half, the additional weight concentrated around the pulley head will practically neutralize the effect of the greater weight of the lower half on the position of the c.g.

Make CD = 1·75 tons and draw DF parallel to the backstay AS. CF = the reaction at the hinge E, and DF scaling 2·96 tons = pull in backstay due to weight of jib. DF is transferred to AG, and resolved along and perpendicularly to the jib, giving AH = 2·85 tons and HK = 0·77 ton respectively. The component AH acting *along* the jib represents the direct compression applied by the backstay due to weight of jib, whilst the component AK acting at right angles to the jib causes bending moment at any section.

Considering the central section at M. The weight of the jib being distributed and not actually concentrated at M, the compression due to

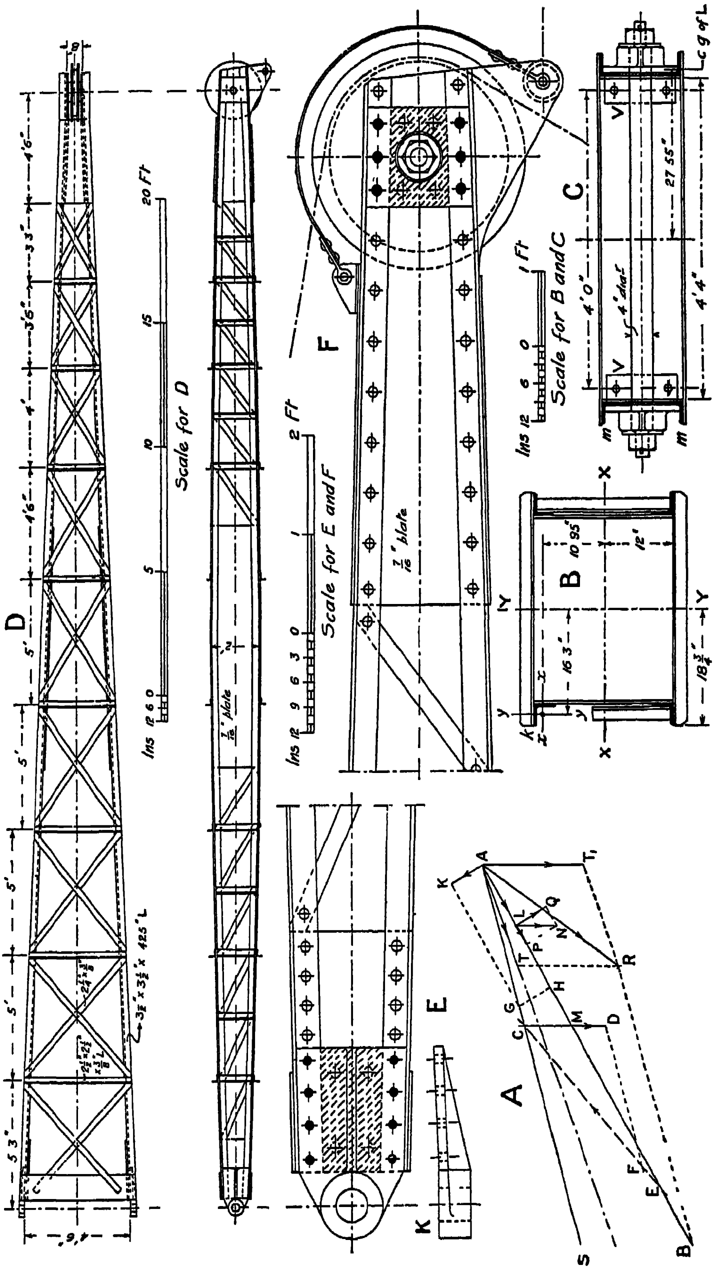

D
Scale for D
Scale for E and F
Scale for B and C
F
E
A
B
C
K
4'0"
4'4"
27 55"
c g of L

FIG 272.

its own weight will be cumulative from A towards E, consequently the weight of the upper half AM must be taken into account in calculating the compressive and bending stresses on section M. LN = $\frac{1}{2}$ weight of jib = 0·875 ton is resolved at LP and LQ. LP = 0·44 ton = the additional direct compression accumulated from A to M over and above that applied by the backstay. LQ = 0·76 ton = the component of the weight of AM causing bending moment at section M in the opposite sense to that caused by AK. Then at section M, direct compression due to weight of jib = AH + LP = 2·85 + 0·44 = 3·29 tons, and bending moment due to weight of jib = $0{\cdot}77 \times 22{\cdot}5' - 0{\cdot}76 \times 11{\cdot}25' = 8{\cdot}77$ foot-tons = 105·24 inch-tons.

The section adopted at M is shown at Fig. 272, B, consisting of four steel angles $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times 0{\cdot}425''$. From the section book the sectional area of one angle = 2·8 sq. in. and $I_x = 3{\cdot}22$.

Hence for the four angles $I_x = (3{\cdot}22 + 2{\cdot}8 \times 10{\cdot}95^2) \times 4 = 1356$.

$$\text{Stress due to bending} = \frac{105{\cdot}24 \times 12''}{1356} = \pm\ 0{\cdot}93 \text{ ton per square inch.}$$

Direct compression = 26·25 tons due to chain tensions + 3·29 tons due to weight of jib = 29·54 tons, or $\frac{29{\cdot}54}{11{\cdot}2} = 2{\cdot}64$ tons per square inch. The direct compression is slightly greater than this on account of the slight batter of the main angles. As the difference between slope length and axial length of jib is very small, this has been neglected.

Hence maximum compression due to load lifted, weight of jib, accelerating force in lifting and bending of jib in vertical plane = 0·93 + 2·64 = 3·57 tons per square inch.

Slewing.—The force necessary for accelerating the load during slewing acts at the pulley end of the jib horizontally, and gives rise to lateral bending moment.

$$\text{Accelerating force for load} = \frac{3{\cdot}25 \times 1}{32} = 0{\cdot}1 \text{ ton.}$$

Considering the central section of the jib, the force necessary for accelerating the upper half of the jib also acts horizontally and creates additional lateral bending moment on the central section. The centre of inertia of the upper half of the jib is situated at 0·77 × 45 ft. = 34·6 ft. from the slewing axis, here taken as the foot of the jib. The acceleration at outer end of jib = 1 ft. per second, therefore at 0·77 of the jib length, the acceleration = 0·77 ft. per second, and accelerating force for upper half of jib = $0{\cdot}875 \times 0{\cdot}77 \times \frac{1}{32} = 0{\cdot}021$ ton, acting at 34·6 − 22·5 = 12·1 ft. from central section. Hence, bending moment at central section due to accelerating the load and weight of upper half of jib = $0{\cdot}1 \times 22{\cdot}5' + 0{\cdot}021 \times 12{\cdot}1' = 2{\cdot}5$ ft.-tons = 30 in.-tons. To this must be added the bending moment caused by wind pressure on the side of the jib and on the load lifted.

Bending Moment due to Wind Pressure.—A safe outside allowance for wind pressure will be made by assuming 40 lbs. per square foot acting on the total projected area of one side of the jib, taken as a continuous surface. Only the ends and central portion are actually plated, and the

above allowance will cover the wind pressure acting on the latticing of the leeward side.

Mean breadth of upper half of jib = 19 in., and c.g. is 10·5 ft. from centre of jib.

Therefore wind pressure on upper half of jib $= 22\frac{1}{2} \times \frac{19}{12} \times 40 = 1425$ lbs., acting at a leverage of 10·5 ft., and bending moment at central section $= 1425 \times 10{\cdot}5 = 14{,}962$ ft.-lbs.

The wind pressure on the load lifted, assuming 30 sq. ft. of effective surface in the case of bulky loads, $= 30 \times 40 = 1200$ lbs., acting at a leverage of 22·5 ft. from central section, and bending moment due to this pressure $= 1200 \times 22{\cdot}5 = 27{,}000$ ft.-lbs.

Hence total bending moment due to lateral wind pressure $= 14{,}962 + 27{,}000 = 41{,}962$ ft.-lbs. $= 225$ in.-tons.

The lateral deflection caused by the wind pressure and accelerating forces still further increases the lateral bending moment, since the direct compression along the jib then acts eccentrically. This increase is, however, small—about 20 to 25 in.-tons, and is neglected, since ample allowance has been made for wind.

Total lateral bending moment = 225 in.-tons due to wind + 30 in.-tons due to accelerating forces = 255 in.-tons.

In Fig. 272, B, the moment of inertia of one angle about $yy = 3{\cdot}22$, hence I_Y for four angles $= (3{\cdot}22 + 2{\cdot}8 \times 16{\cdot}3^2) \times 4 = 2988$, and stress due to bending

$$= \frac{255 \times 18{\cdot}75}{2988} = \pm\, 1{\cdot}60 \text{ tons per square inch.}$$

Hence, maximum intensity of compression at central section of jib from all causes $= 1{\cdot}60 + 3{\cdot}57 = 5{\cdot}17$ tons per square inch. This intensity occurs in the upper leeward angle at k.

The stresses on the lower end of the jib are next considered. These are caused by direct compression, due to load and weight of jib, and B.M. due to wind pressure and accelerating forces.

Direct Compression.—This has been already determined, and = due to load 26·25 tons, and due to weight of jib = 2·85 (axial component AH applied by backstay) + 2PL (axial component of total jib weight) $= 2{\cdot}85 + 0{\cdot}88 = 3{\cdot}73$ tons.

Total direct compression $= 26{\cdot}25 + 3{\cdot}73 = 29{\cdot}98$ tons.

Bending Moment due to Wind Pressure.—Projected area of jib $= 45 \times \frac{19}{12}$ in., and total presssure $= 45 \times \frac{19}{12} \times 40 = 2850$ lbs.

B.M. at foot of jib $= 2850 \times 22{\cdot}5' = 64{,}125$ ft.-lbs.

B.M. due to wind on surface of load $= 1200 \times 45' = 54{,}000$ ft.-lbs.

Total B.M. $= 64{,}125 + 54{,}000 = 118{,}125$ ft.-lbs. $= 632$ in.-tons.

Bending Moment due to Accelerating Forces.—For the load $= 0{\cdot}1$ ton $\times\, 45' = 4{\cdot}5$ ft.-tons.

For whole weight of jib, centre of inertia is $\frac{45}{\sqrt{3}}$ ft. from foot, and accelerating force $= 1{\cdot}75 \times \frac{1}{\sqrt{3}} \times \frac{1}{32} = 0{\cdot}032$ ton.

Hence B.M. $= 0{\cdot}032 \times \frac{45}{\sqrt{3}} = 0{\cdot}83$ ft.-tons.

Total B.M. = 4·5 + 0·83 = 5·33 ft.-tons = 64 in.-tons.

Total B.M due to wind pressure and accelerating forces = 632 + 64 = 696 in.-tons.

The cross-section at foot of jib is shown at Fig. 272, C.

$$I_Y = (3{\cdot}22 + 2{\cdot}8 \times (27{\cdot}55)^2) \times 4 = 8514$$

$$\text{Stress due to bending} = \frac{696 \times 30''}{8514} = \pm\ 2{\cdot}46 \text{ tons per square inch.}$$

Direct compression $= \frac{29{\cdot}98}{11{\cdot}8} = 2{\cdot}54$ tons per square inch, and maximum intensity of compression at foot of jib = 2·46 + 2·54 = 5·00 tons per square inch. This intensity occurs at the outer edges *m*, *m*, of the leeward angles.

The general design of the jib is shown in Fig. 272, D. The middle portion of the sides is plated for a length of about 10 ft. to give greater stiffness against buckling. The details at hinge and pulley ends is shown at E and F. At E cast-steel bracketed eyes K are bolted through the angles, and side plates with packing plates inserted as indicated by the shading. These hinges bear on a 4-in. pin passing through a pair of bearings V, V, Fig. 272, C, bolted to the crane carriage or framework.

Shearing and Bearing Stresses on Pin.—Direct compression down each side of jib $= \frac{29{\cdot}98}{2} = 14{\cdot}99$, say 15 tons. Distance centre to centre of bearings V, V, = 4 ft. Total B.M. on side of jib = 696 in.-tons. Hence pressure on leeward bearing, or uplift on windward bearing, due to B.M $= \frac{696}{48''} = 14{\cdot}5$ tons. Adding the direct pressure of 15 tons, total pressure on leeward bearing = 29 5 tons. Bearing area = 4″ × 4″ = 16 sq. in., and bearing pressure per square inch $= \frac{29{\cdot}5}{16} = 1{\cdot}85$ tons. Distance between outer faces of bearing = 4 ft. 4 in. Shear force on pin due to lateral bending $= \frac{696}{52''} = 13{\cdot}4$ tons Adding the direct pressure, total shear force at leeward section of pin = 13·4 + 15·0 = 28·4 tons. Sectional area of pin = 12·57 sq. in., and mean shear on pin $= \frac{28{\cdot}4}{12{\cdot}57} = 2{\cdot}26$ tons per square inch. The bearing and shear stresses should be low, since the pin undergoes considerable wear and tear.

The pulley end carries a 30 in. diameter pulley and chain guard, the pin diameter being 2½ in., and the pulley eye bushed with bronze. The sizes of lacing bars and angles indicated will be found ample for resisting the stresses caused by the lateral loading of the jib, treated as a cantilever. It is unnecessary to calculate these in the case of a light jib of this type.

Design for Riveted Steel Tank.—*Capacity,* 20,000 *gallons. The tank to be carried on girders and columns, so that when filled the water-level shall be* 25 *ft. above ground-level.*

Such conditions are representative of those required for locomotive feed tanks, etc. Fig. 273 shows the general arrangement and details. Side and end elevations are shown at A, and an enlarged plan at B, on which

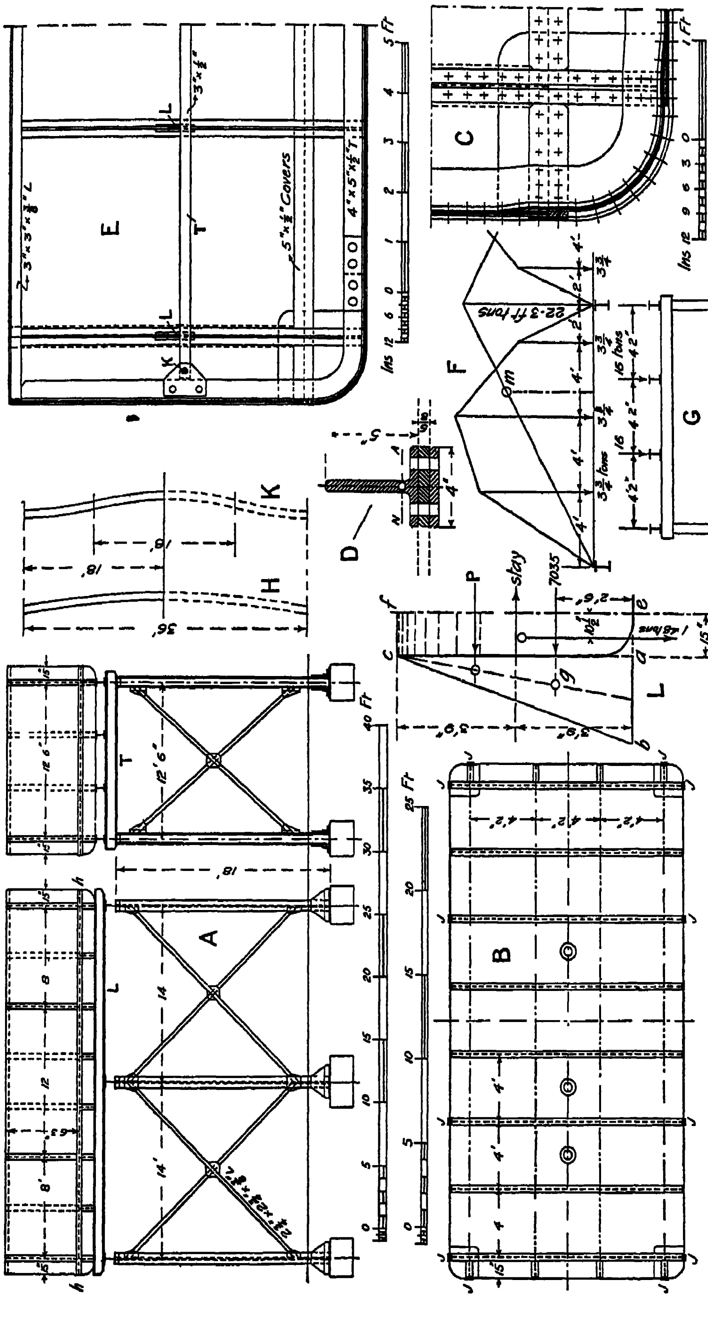

E
C
F
G
D
H
K
A
B
L
T
2-3"×3"×3/8" L
3"×1/2"
5"×1/2" Covers
4"×5"×1/2" T
22·3 ft tons
stay
7035
Ins 12 6 0 1 2 3 4 5 Ft
Ins 12 9 6 3 0 1 Ft
0 5 10 15 20 25 30 35 40 Ft
0 5 10 15 20 25 Ft

FIG. 273.

the arrangement of covers and joints is indicated. The sides consist of three plates, 8′, 12′, and 8′ long × 6′ 3″ high; the ends of one plate 12′ 6″ × 6′ 3″, and the bottom of seven plates 4 ft. wide, running transversely and turned up to form a butt joint at *h* with the side plates. The joints in the bottom are covered by tee bars inside, turned up to form vertical stiffeners for the sides. Vertical joints in the sides occur at J, J, in the plan. All outer joints and horizontal inner joints are covered by flat straps. The end and bottom corners are curved, and forged bosses are placed at the four bottom corners, the detail of which is shown at C. The tank is supported on four longitudinal rolled steel beams L, 4 ft. 2 in. centre to centre, resting on three transverse beams T, carried by six columns 18 ft. high. Stays are inserted as indicated subsequently.

The principal features of the design are as follows.

Thickness of Plates.—Span 4 ft. Head of water 7 ft. 6 in. Load per square foot on bottom due to water pressure $= 62{\cdot}5 \times 7{\cdot}5 = 469$ lbs. or $\frac{469}{12} = 39$ lbs. per inch width of plates.

Max B.M., assuming the plates simply supported at the joints,

$$= \frac{wl^2}{8} = \frac{39 \times 4 \times 4 \times 12}{8 \times 2240} = 0{\cdot}418 \text{ in.-ton.}$$

Assuming a working stress of 9 tons per square inch.

$$\text{Moment of resistance} = \tfrac{1}{6} \times 1 \times t^2 \times 9 = 0{\cdot}418$$

$$\text{whence thickness } t = 0{\cdot}528 \text{ in., say } \tfrac{5}{8} \text{ in.}$$

It may be noticed that a somewhat thinner plate would result if the plate be supposed to act as a fixed instead of a simply supported beam. The actual strength of flat plates employed under such conditions lies between the two above-mentioned limits, so that $\frac{wl^2}{8}$ is an outside estimate of the maximum bending moment on the plate. The side plates being subject to a maximum head of 6 ft. 3 in. would require a theoretical thickness of $\frac{1}{2}$ in., but for practical reasons would be made the same thickness as the bottom plates. A small additional stress will be caused by the weight of plate, which has been neglected since amply covered by the calculation.

Bottom Transverse Tee-covers.—These transfer the load on the plates to the longitudinal bearing girders. Their span is 4 ft. 2 in., and their actual resistance lies between that of a simply supported and a fixed beam. Each rib carries the weight of a volume of water 7 ft. 6 in. deep × 4′ × 4′ 2″

$$= \frac{4 \times 4\frac{1}{6} \times 7\frac{1}{2} \times 62\frac{1}{2}}{2240} = 3{\cdot}49 \text{ tons.}$$

$$\text{B.M. at centre} = \frac{3{\cdot}49 \times 4\frac{1}{6} \times 12}{8} = 21{\cdot}81 \text{ in.-tons.}$$

The effective beam section resisting this moment is shown at Fig. 273, D, and is assumed to consist of the inner tee stiffener, outer cover $\frac{1}{2}$ in. thick, and portion of $\frac{5}{8}$ in. plate between. The modulus of this section after deducting two $\frac{13}{16}$ in. rivet holes $= 4{\cdot}3$ ins.[3]

$$\text{Hence maximum bending stress} = \frac{21{\cdot}81}{4{\cdot}3} = 5{\cdot}07 \text{ tons per square inch.}$$

Stays.—Transverse and longitudinal stays are taken across the tank at 4 ft. and 4 ft. 2 in. intervals respectively, at the centre of height of the sides. Fig. 273, L, is a diagram of intensity of water pressure against the side per 4 ft. width of tank.

$$ab = 4 \times 62\tfrac{1}{2} \times 7\tfrac{1}{2} = 1876 \text{ lbs. per 4 ft. width,}$$

and the horizontal breadth of triangle *abc* gives the intensity of pressure at any depth. The total pressure against the side per 4 ft. width

$$= ab \times \frac{ac}{2} = 1876 \times 3\tfrac{3}{4}' = 7035 \text{ lbs.}$$

acting through the centre of pressure at level *g*.

Taking moments round *a*—

$$\text{tension in stay} \times 3'\,9'' = 7035 \times 2'\,6''$$

whence tension in stay due to horizontal water pressure = 4690 lbs. = 2·1 tons.

This tension will be augmented due to the overhang at the sides. The increase of tension from this cause will be calculated by taking moments about *e*.

Weight of volume of water *cfea* per 4 ft. width

$$= \frac{4 \times 1\tfrac{1}{4} \times 7\tfrac{1}{2} \times 62\tfrac{1}{2}}{2240} = 1{\cdot}05 \text{ tons.}$$

Adding for weight of side of tank per 4 ft. width, 950 lbs. or 0·43 ton, total overhanging weight per 4 ft. width = 1·05 + 0·43 = 1·48 tons. The common centre of gravity is $10\frac{1}{2}$ in. to the left of *e*.

Taking moments about *e*—

$$\text{tension in stay due to overhang} \times 3\tfrac{3}{4}' = 1{\cdot}48 \times \tfrac{7}{8}'$$
$$\text{whence additional tension in stay} = 0\,35 \text{ ton.}$$
$$\text{Total tension in stay} = 2{\cdot}1 + 0{\cdot}35 = 2\,45 \text{ tons.}$$

In Fig. 273, E, which shows an enlarged cross-section of one half of the tank, flat stays L and T are employed. Round stays better resist corrosion, but sag more severely under their own weight. The longitudinal stays L, being 30 ft. 6 in. long, are arranged to rest on the transverse stays T in order to relieve them of part of the bending stress due to their own weight. Adopting flat bars $3'' \times \frac{1}{2}''$, the bending stress due to the weight of the transverse stay T, and the 4 feet lengths of the longitudinal stays L, L, resting on it, works out to 0·93 ton per square inch.

$$\text{Direct tension} = \frac{2\,45 \text{ tons}}{3 \times \frac{1}{2}} = 1{\cdot}63 \text{ tons per square inch}$$

and maximum tension = 1 63 + 0·93 = 2·56 tons per square inch.

Smaller stays might be used, but this size will provide a margin for corrosion ; $\frac{3}{4}$ in. bolts at K will be suitable.

The upper half of the sides above the stays, Fig. 273, L, resists the horizontal water pressure P by cantilever action, when not stayed across the top of the tank.

Pressure P per 4 ft. width of side = $\frac{1}{4}$ the pressure for the 7 ft.

6 in. depth $= \frac{1}{4} \times 7035 = 1759$ lbs. applied at 1 ft. 3 in. above the level of attachment of stays.

$$\text{Hence B.M. on side} = \frac{1759 \times 15}{2240} = 11{\cdot}8 \text{ in.-tons.}$$

This is resisted by the section Fig. 273, D, the modulus of which is 4·3,

$$\therefore \text{ bending stress} = \frac{11{\cdot}8}{4{\cdot}3} = 2{\cdot}8 \text{ tons per square inch.}$$

The stress due to this bending action will be somewhat higher on the ribs intermediate between the vertical joints, since the effective beam section is there reduced by the absence of the outside $\frac{1}{2}$ in. plate.

There will be direct tension across the tank bottom, due to the reaction required to balance the horizontal forces acting on the sides, which = total horizontal pressure on side − horizontal pull in stay due to this pressure = 7035 − 4690 = 2345 lbs. per 4 ft. width of tank. This creates a negligibly small tension in the plates, which would be entirely annulled if the stays were attached at *g*.

A close pitch of riveting, say $2\frac{1}{2}$ in., will be required to ensure water-tightness, and all joints will be caulked. Rivets $\frac{3}{4}$ in. diameter will be suitable.

The weight of the tank as designed runs out at 16·8 tons.

Longitudinal Girders.—The load applied at each bearing-point of the tank on the two central longitudinal girders = $3\frac{3}{4}$ tons. These girders are continuous, and one-half the B.M. diagram is drawn at Fig. 273, F, and the characteristic point *m* found by the method of Example 9, Fig. 47, Chapter III. The maximum moment occurs at the centre, and = 22·3 ft.-tons = 267·6 inch-tons.

Using $9\frac{1}{2}'' \times 9\frac{1}{2}'' \times 51$ lbs. B.F. beams, the modulus = 52·2, and working stress $= \dfrac{267{\cdot}6}{52{\cdot}2} = 5{\cdot}12$ tons per square inch. The B.M. on the two outer girders is less than on the central ones, but all four are necessarily of the same depth.

Transverse Girders.—The central reaction of the longitudinal girder, calculated from the bending moment diagram F, by the method of Example 9, is 16 tons. Each of the two central longitudinal girders therefore applies a load of 16 tons to the central transverse girder. This loading is shown at Fig. 273, G. The two outer loads do not affect the bending moment, being directly over the column.

$$\begin{aligned}\text{Maximum B.M. on transverse girder} &= 16 \times 4\tfrac{1}{6} \times 12 \\ &= 800 \text{ inch-tons.}\end{aligned}$$

Using a $12\frac{1}{2}'' \times 12'' \times 85$ lbs. B.F. beam, the modulus = 115, and working stress $= \dfrac{800}{115} = 6{\cdot}95$ tons per square inch.

Columns.—The maximum load comes on the two central columns, and equals the sum of the central reactions from one central and one outside longitudinal girder, plus a portion of the weight of the beams. If the B.M. diagram for the outside longitudinal girder be drawn in a

similar manner to that at F for the inner girder, the central reaction will be found to be 12·3 tons.

Hence load on one central column

= from inner longitudinal girder . . .	16·00	tons
„ outer „ „ . . .	12·30	„
$\frac{5}{16}$ of weight of longitudinal girder . .	0·85	„
$\frac{1}{2}$ weight of transverse girder	0·27	„
Vertical component of stress in wind tie (determined below)	2·50	„
Total . .	31·92	„

say 32 tons.

The column, if free to bend as at Fig. 273, H, will act as a round-ended column 36 ft. long. The stiffness of the connection with the central transverse girder and the action of the diagonal wind braces, will tend to cause bending as at K, in which case the equivalent round-ended column would have a length of 18 feet. The real strength will lie between these two extremes, but it will be advisable to design the column as at H, since the connections are not likely to be very rigid, and the wind pressure acts on a relatively large surface.

Using an $11\frac{1}{2}'' \times 11\frac{1}{2}'' \times 75$ lbs. B.F. beam—

Sectional area = 21·9 sq. in., least radius = 2·66 in., and $\frac{l}{r} = \frac{36 \times 12}{2{\cdot}66} = 163$. The safe load per square inch for this ratio, from Fig. 109 = 3200 lbs.

$$\text{Total safe load} = \frac{3200 \times 21{\cdot}9}{2240} = 31{\cdot}3 \text{ tons.}$$

The maximum load to be carried is 31·92 tons.

Wind Pressure.—The wind pressure on side of tank at 35 lbs. per square foot

$$= \frac{30\frac{1}{2} \times 7\frac{1}{2} \times 35}{2240} = 3\,5 \text{ tons.}$$

Frictional coefficient with tank empty $= \frac{3\,5}{16\,8} = 0{\cdot}21$. Tanks presenting a large area to the wind pressure may require bolting to the pillars or girders by suitable brackets and bolts.

Length of wind ties = 24 feet.

Horizontal component of wind stress in central ties $= \frac{5}{8} \times 3{\cdot}5$ tons $= 2{\cdot}2$ tons.

Inclined stress $= 2{\cdot}2 \times \frac{24}{12{\cdot}5} = 4\,2$ tons, and vertical stress $= \frac{14}{24} \times 4{\cdot}2 = 2{\cdot}5$ tons, which gives the pressure to be added as above to the load on the leeward central pillar.

For the diagonal braces a small angle section, say $2\frac{3}{4}'' \times 2\frac{3}{4} \times \frac{3}{8}''$, flat, or round bars may be used.

Three holes are provided in the tank bottom for supply, draw-off, and overflow pipes.

Tall Buildings.

These comprise types of construction principally developed in the United States, in which steel framework is carried to much greater height than usual. Some of the most notable examples are the Singer building, New York, 612 feet high, the Woolworth tower, New York, 792 feet; the Penobscot building, Detroit, 551 feet, of forty-five storeys, the Chicago Tribune and the Chicago Board of Trade building of forty storeys. The Larkin Tower, it is stated, will be 1208 feet high and will contain one hundred and ten storeys. The distinguishing feature of these buildings is the lightness of the covering employed for the walls, which, together with that of the steel frame, enable such heights to be attained, without unduly loading the foundations, which in New York and Chicago are of generally poor bearing power. The foundations for the columns are consequently usually constructed of the grillage type already referred to.

In the older buildings the exterior walls are self-supporting, the floors being carried independently by the steel framework. In the more modern structures, both walls and floors are carried by girders and ultimately by the columns. This method enables any portion of the walls to be commenced independently as convenience allows, and renders it possible to provide more closely for uniform settlement of foundations. In the case of an existing party wall, W, Fig. 274, the footings

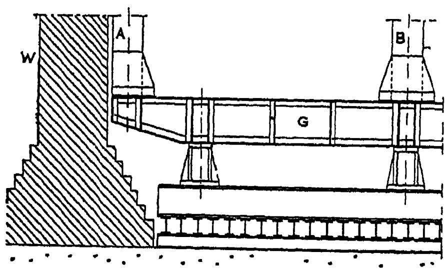


Fig. 274.

of which may not be further loaded or interfered with, exterior columns such as A are frequently carried on cantilever girders, G, balanced by the load on one or more neighbouring columns as B.

Live Loads on Floors.—The Building Code of New York City prescribes the following uniformly distributed live loads (1928):—

For residence purposes	40	lbs. per square foot.
„ places of assembly	100	„ „
„ class-rooms in schools	75	„ „
„ offices	60	„ „
„ business and other occupancies, other than those stated above .	120	„ „

The Code also provides that every steel floor beam in any building hereafter erected, to be used for business purposes, shall be capable of sustaining a live load concentrated at its centre of at least 4000 lbs.

Columns, posts, or vertical supports must be designed for the total live load and dead load to be supported, excepting that in buildings more than five storeys in height, the live load on the floor next below the top floor may be assumed at 95 per cent. of the allowable live load, and on the next lower floor 90 per cent., and on each succeeding lower floor a correspondingly decreasing percentage, provided that in no case shall less than 50 per cent. of the allowable live loads be assumed.

For side walks between the curb and building lines the live load shall be taken at 300 lbs. per square foot. For yards and courts inside the building line the live load shall be taken at not less than 120 lbs. per square foot.

The Building Ordinance Regulations in Chicago prescribe (1928):

Buildings for the sale, storing, and manufacturing of merchandise, and garages .	100 lbs. per square foot.
Offices, hospitals, hotels, and club rooms .	50 „ „
Private residences	40 „ „
Assembly halls, theatres, and churches . .	100 „ „
Apartment buildings	40 „ „
Schools	75 „ „
Department stores	100 „ „

In school buildings those parts exclusive of the floors in assembly halls, the corridors and stairs shall not be required to support a live load exceeding 40 lbs. per square foot. The stairways and their stair halls in buildings of all types shall be required to carry 100 lbs. per square foot. The roofs of all buildings shall bear, in addition to the weight of their structure and covering, 25 lbs. per square foot.

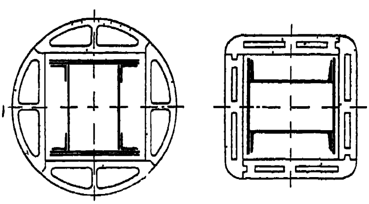


FIG. 275.

Details of Construction.—The steelwork of columns is covered by hollow fireproof terra-cotta tiles 2 in. to 3 in. thick, bonded together with an air space left between the tiles and column faces. Examples of such coverings are shown in Fig. 275.

Floors.—The floor girders are similarly protected by the systems of terrr-cotta floors in general use. Fig. 276 gives a typical example of

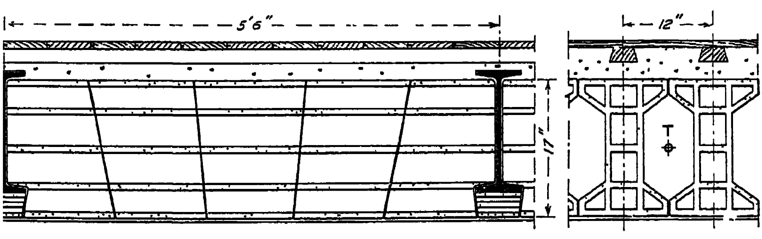


FIG. 276

these floors by the Pioneer Fireproof Construction Company. The floor consists of flat arches of tile voussoirs, abutting on the parallel

joists, forming the primary framing of the floor. Table 28 gives particulars and weights of such types of floors. The weight of concrete and timber covering forming the floor surface requires adding to the weights stated in the table, and a further 5 to 8 lbs. per square foot if the under surface be plastered. In the best systems the vertical webs of the hollow tiles run transversely to the carrying joists, these being known as "end-systems," whilst those having the webs parallel to the joists are called "side-system" arches, the former being lighter for equal strength.

TABLE 28.—WEIGHT OF HOLLOW TILE FLOOR ARCHES.

Span of arch	Depth.	Weight per sq ft
ft	in	lbs
5 to 6	8	27
6 „ 7	9	29
7 „ 8	10	33
8 „ 9	12	38

Tie-rods T, Fig. 276, are employed for resisting the end thrust of such floor arches. The spacing of tie-rods should not exceed twenty times the width of flange of floor beams. The approximate thrust in flat tile arches is given by—

$$T = \frac{3wL^2}{2d} \text{ lbs. per linear foot of arch.}$$

Where w = load per square foot on arch, L = span in feet, and d = depth in inches from top of arch to bottom of supporting beams. The spacing of the tie-rods being decided, the diameter may be calculated from the above thrust, 7 tons per square inch being allowed on the net section of bars.

Partitions are constructed of wire lathing or expanded metal tied to verticals of light channel section filled in with concrete and plastered on the face; or of slag wool and plaster slabs, or terra-cotta blocks 2 in. to 4 in. thick. The last named are the soundest and most fireproof. For thicknesses of 3, 4, 5, and 6 inches these weigh 16, 19, 22, and 23 lbs. per square foot respectively.

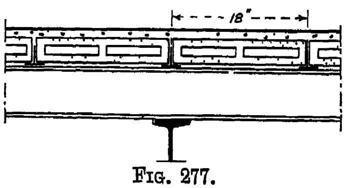


FIG. 277.

Roofs are covered usually with 3 in. book-tiles laid on T-bars as in Fig. 277, the T-bars being supported by the beams and girders forming the roof framing. An outside coating of cement, tar and gravel is applied over the tiles. The weights of the floors and roof in the Fisher building, Chicago, are as follows :—

		lbs per sq ft.
Floor—$\frac{7}{8}$ in. Maple floor	. . . .	4
Deadening, cinder concrete on top of floor arch	.	15
15 in. hollow tile floor arch	. . . .	41
Steel joists and girders	. . . .	10
Plaster on ceiling	. . . .	5
	Total	75

		lbs. per sq ft.
Roof—3 in. Book tiles	. . . .	22
6-ply tar and gravel roof	. . . .	6
T-bars	. . . .	4
Steel roof framing	. . . .	8
	Total	40

Chimney Stacks are usually constructed of a steel tube lined with firebrick to a height of 60 or 70 ft., and with hollow tiles thence to the top, a 2 in. air space being left between the steel and lining. Fig. 278 shows the sliding joint J provided between the top of the stack and the roof in the Fisher building, which allows of expansion and contraction without dislocating the roof tiles.

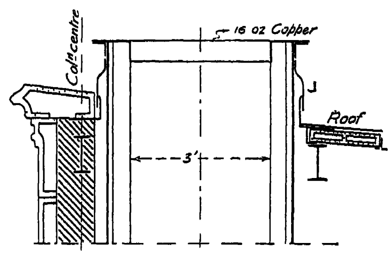


FIG. 278.

Exterior walls are of brick or terra-cotta, or brick with terra-cotta facing, and are most usually carried by the horizontal girders of the outer framing. The weight of the steel framing for buildings of 16 to 20 storeys varies from $1\frac{1}{8}$ to 2 lbs. per cubic foot of the building, and the cost from $2\frac{1}{2}$ to 3 pence per cubic foot, or £14 8*s*. to £17 5*s*. per ton of steel, representing from $\frac{1}{7}$ to $\frac{1}{5}$ the total cost of the building.

Wind Bracing.—In the case of tall buildings of light construction, the walls and partitions being thin and not bonded together, the steel framing must provide all resistance to the wind. In buildings of usual proportions of height to width of base, the dead weight is sufficient to resist bodily overturning, and the effect of the lateral wind pressure is to create bending moment on the columns and horizontal shearing stress on the connections, or to increase the compression in the leeward columns and relieve that in the windward columns if the building is braced diagonally from top to bottom.

In the case of exceptionally tall buildings which are practically narrow towers, the wind pressure may create tension in the windward columns exceeding the compression due to the dead weight of the building, and so produce an actual uplift on the foundations of the windward columns, necessitating their being securely anchored down.

The usual methods of bracing against the action of wind pressure are illustrated below. In Fig. 279 diagonal ties are introduced in a

suitable number of vertical panels between the columns. This is probably the most efficient method. The framing is thereby converted into a series of cantilevers fixed at foundation level and subject to horizontal loads p, p. The principal objection to this system is its interference with window and door openings, for which reason diagonal bracing is more frequently placed in some of the internal partition walls than in the outer walls. This difficulty is avoided in the 612 ft. tower of the Singer building by arranging the bracing as in Fig. 280.

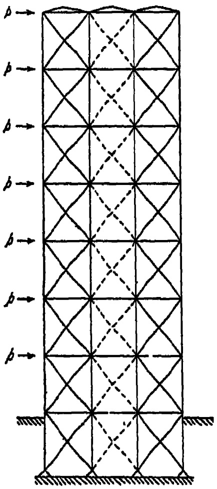


FIG. 279.

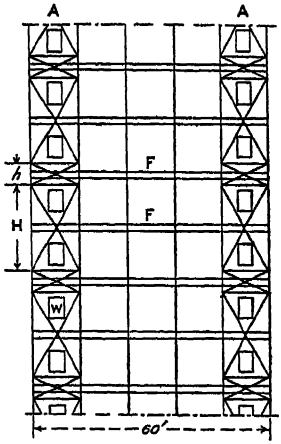


FIG. 280.

The corner bays A, A, are braced for the whole height in alternate long and short panels H and h, the longer panels extending through two storeys. The bracing bars thus intersect near the floor levels F, F, leaving uninterrupted openings for windows W in all storeys.

Fig. 281 shows the more usual methods adopted by employing deep and stiff girders G between the columns at the floor levels, or by introducing stiff brackets B in the case of shallower girders. The effect of this type of bracing is to create local bending stresses in the columns at their junctions with the horizontal girders, and the general effect on the framing is shown in Fig. 282, the distortion being greatly exaggerated. The rigidity of the connections is here relied on to distribute the bending action of the wind pressure equally amongst those columns which are efficiently braced by deep girders, and this assumption is the only one on which any approximate calculation of the wind stresses can be based. Points of contra-flexure p, p, occur at or near

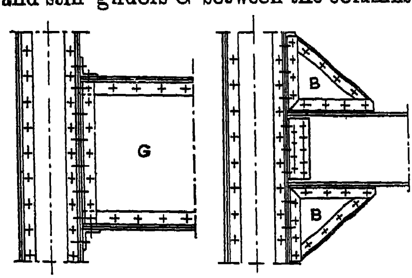


FIG 281.

the centre of each storey length l of columns, where the bending moment due to wind pressure will be zero. Assuming wind pressures P, P, to act horizontally at the centre of each storey, and the building to be three columns deep as in Fig. 282, the bending moment on each column at the horizontal section A will be $\frac{P}{3} \times \frac{l}{2} = \frac{Pl}{6}$. At section B, one-third of the total wind pressure above section B may be supposed acting at each of the points p_1, and the bending moment on each column at section B $= \frac{2P}{3} \times \frac{l}{2} = \frac{Pl}{3}$. Similarly at C, the bending moment on each column section $= \frac{3P}{3} \times \frac{l}{2} = \frac{Pl}{2}$, and at D, $\frac{4P}{3} \times \frac{l}{2} = \frac{2Pl}{3}$. This method of transferring the wind load to the foundations is often referred to as the "table-leg" principle.

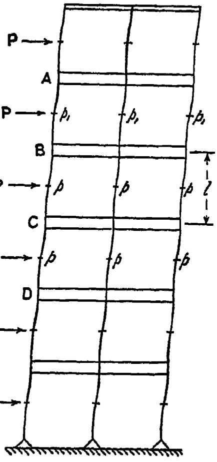


FIG. 282.

The column section is augmented from roof to basement in order to meet the increasing direct compression and bending moment, and the ultimate effect of this type of bracing is to increase the intensity of compression alternately on windward and leeward faces of *every* column so braced.

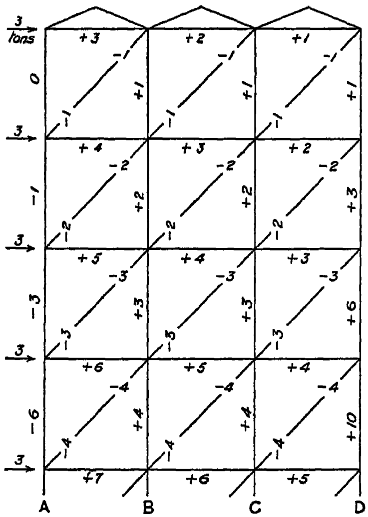


FIG. 283.

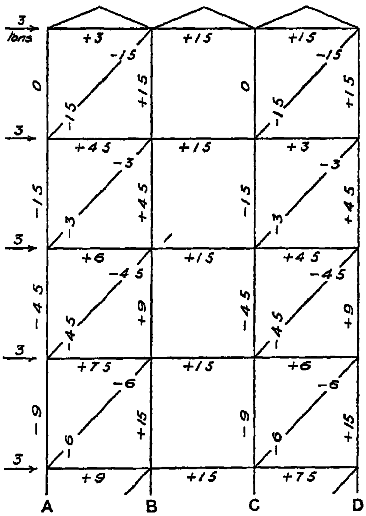


FIG. 284.

To be efficient, the rigidity of the horizontal girders should be large relatively to that of the columns, and this result is obtained in modern practice by giving such girders a liberal depth.

When diagonal bracing is employed, the skeleton of the building

acts wholly or partially as a braced frame or girder according as the bracing extends throughout the whole system of columns, or is only employed in certain bays. Fig. 283 indicates the stresses in one frame of the four upper storeys of a building four columns deep from back to front with diagonal bracing throughout, assuming the wind stress to be equally shared by the diagonal braces in each storey; and Fig. 284 the stresses in the same building when the central bay is unbraced. The diagonals are assumed inclined at 45°. In Fig. 283 it will be seen that the vertical tension in the diagonals of the windward bay AB is augmenting at the rate of 1 ton per storey, so that the uplift at the foundation of column A, if carried down through twelve storeys, would amount to $(1 + 2 + 3 + 4 \ldots + 12) = 78$ tons. The foundation of column D would be superloaded to the same extent under the maximum wind pressure. The compression in columns B and C is augmented at the rate of 1 ton per storey, but the ultimate load on their foundations is not affected, the compression in each storey length of columns B and C being balanced by the vertical tension in the wind tie attached at its lower end. In Fig. 284 the bays AB and CD constitute separate cantilevers, CD receiving its wind load through the horizontal girders traversing the bay BC. The uplift on columns A and C at a depth of 12 storeys $= (1\frac{1}{2} + 3 + 4\frac{1}{2} + 6 \ldots + 18) = 117$ tons, whilst an increased pressure of 117 tons comes on the foundations of columns B and D at that depth.

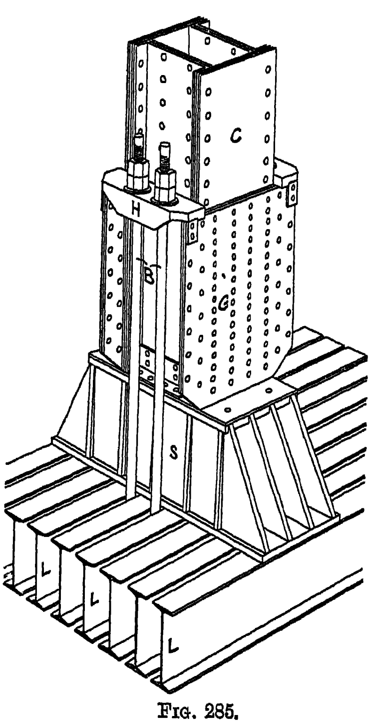


Fig. 285.

In buildings of exceptional height, the uplift on foundations of windward columns may exceed the pressure due to the dead and live loads on the structure. In such cases the columns require efficient anchorage. As an example, ten of the main columns of the Singer building are anchored down in the manner shown in Fig. 285 The maximum uplift in this instance is 413 tons. Four steel bolts B, 4 in. in diameter, pass through steel cross-heads H bearing on heavy gussets G riveted to the base of the column C. The lower edges of the gussets and column plates are planed to bear on the cast-steel shoe S. The bolts pass between the beams L, L, of the grillage, below which they are attached to a second steel casting embedded in the concrete 4 ft. below the grillage. From this casting a series of steel eye bars are

embedded for a further depth of about 40 ft. in the concrete filling of the foundation shaft. These anchor bars are diminished in number towards the lower end, an adhesive force of 50 lbs. per square inch between their faces and the concrete being allowed in designing their sectional area.

CHAPTER XII.

MASONRY AND MASONRY STRUCTURES.

Masonry.—The following classes of masonry are principally employed in masonry structures.

1. *Unsquared or Random Rubble* for unimportant and temporary walling.

2. *Squared Rubble*, built either in courses or uncoursed. Built in courses, this class of masonry is perhaps most widely used for general work. For small walls, the individual stones are relatively small and the courses of moderate height, whilst for retaining walls, heavy piers and abutments of bridges, the scale of the construction may be magnified to any desired degree. Squared uncoursed rubble is probably superior to coursed rubble for work demanding great strength, since vertical bonding is obtained in addition to horizontal bonding. It is, however, more difficult to ensure really first-class work in this than in coursed rubble masonry, and largely for this reason uncoursed rubble is not so extensively used. Probably no class of masonry is so liable to be scamped as rubble masonry, since in the absence of very thorough and constant inspection, it is an easy matter to give the face work an excellent appearance whilst the backing of heavy walls may be practically devoid of bond and simply consist of a mass of small and relatively useless material.

3. *Rubble Concrete*—This is principally employed for the bulk of the heaviest engineering structures, such as masonry dams. It may be compared to rough uncoursed rubble on the largest scale, in which the individual stones consist of masses of rock ranging from a few hundredweights to 7 or 8 tons in weight, whilst they are set in concrete instead of in ordinary mortar. Such work requires the external faces covered with large squared rubble facing set in cement mortar.

4. *Ashlar Masonry* is built of blocks of large size very carefully worked, in courses usually of equal height and laid with joints seldom exceeding one-eighth of an inch in thickness. It is the most expensive class of masonry, and its place in engineering work is limited to the facing of works to which it may be desired to give a highly finished appearance, and to those portions of a structure which must of necessity be given very accurate shapes and faces. Such, for example, are internal facing of docks, locks, gate sills, quoins, copings, overflows for dams, arches and the water face of quay walls. Piers are frequently given ashlar quoins, whilst occasionally ashlar lacing courses are laid at intervals of 20 ft. to 30 ft., in rubble-stone piers, to bring up the work to a dead

level in order to more uniformly distribute the pressure before commencing a further rise. Fig. 303 illustrates such an example. The blocks in ashlar facing are usually worked smooth, but may be draughted with rock face or other treatment as desired, depending largely on the degree of ornamental appearance or otherwise required.

Although the above names are ordinarily accepted as implying a certain class of masonry in general building construction, it is unwise for an engineer to undertake to designate a particular class of masonry by a special name without at the same time stating in the specification a careful description of the kind of masonry intended to be built. Class names for masonry vary considerably in different parts of the country, and in different countries. The names applied to various classes of masonry in America, for instance, would be almost unrecognizable by an Englishman. Different qualities of coursed rubble are variously referred to as "ranged rockwork," "random range work," and ashlar is commonly known as "dimension stone masonry." "Rip-rap" is a purely American term applied to rough irregularly shaped stones used for pitching the slopes of earthen dams, as distinct from "paving," which is usually understood to imply roughly rectangular hammer-dressed stone laid dry by hand in regular courses.

Piers, abutments and retaining walls are frequently built either entirely in blue brick or with blue brick facing and stock brick hearting, or with masonry facing and brick hearting. In laying masonry and in writing specifications for masonry three essential aims should be borne in mind. 1. Uniformity of bearing on each course or horizontal section. 2. Absolute solidity with absence of voids. 3. The most perfect bonding attainable so that the mass may be, as nearly as possible, monolithic in character. Whilst it is impossible in a work of this scope to give complete specifications at length, some of the essential points which should be included in specifications for masonry will now be noticed. In the following notes on specifications, illustrations are inserted for the purpose of rendering more clearly the intended arrangement and bonding of the work, but it will be understood that such drawings do not usually accompany specifications.

Outline Specifications for Masonry.—Stone.—The stone employed for general rubble masonry should be of an approved kind, sound and durable and free from all flaws, seams, cracks and discolorations. The size of stones employed will depend on the magnitude of the work and the sizes conveniently obtainable from the quarries. In general such stones will be from 6 in. to 14 in. thick, 2 feet to 5 feet long, 10 in. to 36 in. wide, the larger blocks being used for the heavier classes of work. Strongly acute angles on stones should be entirely prohibited, and any angles less than 80° are undesirable. Blocks of the softer kinds of sandstone and limestone should be thicker in proportion to length and breadth than those of harder varieties, to mimimize danger of cracking under heavy pressure. All stones in squared rubble masonry should have the top and bottom beds parallel to the natural quarry bed and be approximately rectangular in shape.

Face Dressing.—Fig. 286 shows various kinds of face dressing on stones. A and B are examples of ashlar faced masonry, with rebated joints, the rebate in A being formed entirely on one block, and in B

half on each adjacent block. The faces are finished smooth by rubbing after the final tooling or sawing. Deep rebated joints are employed where heavy characteristic lines are desired, the deep shadows imparting

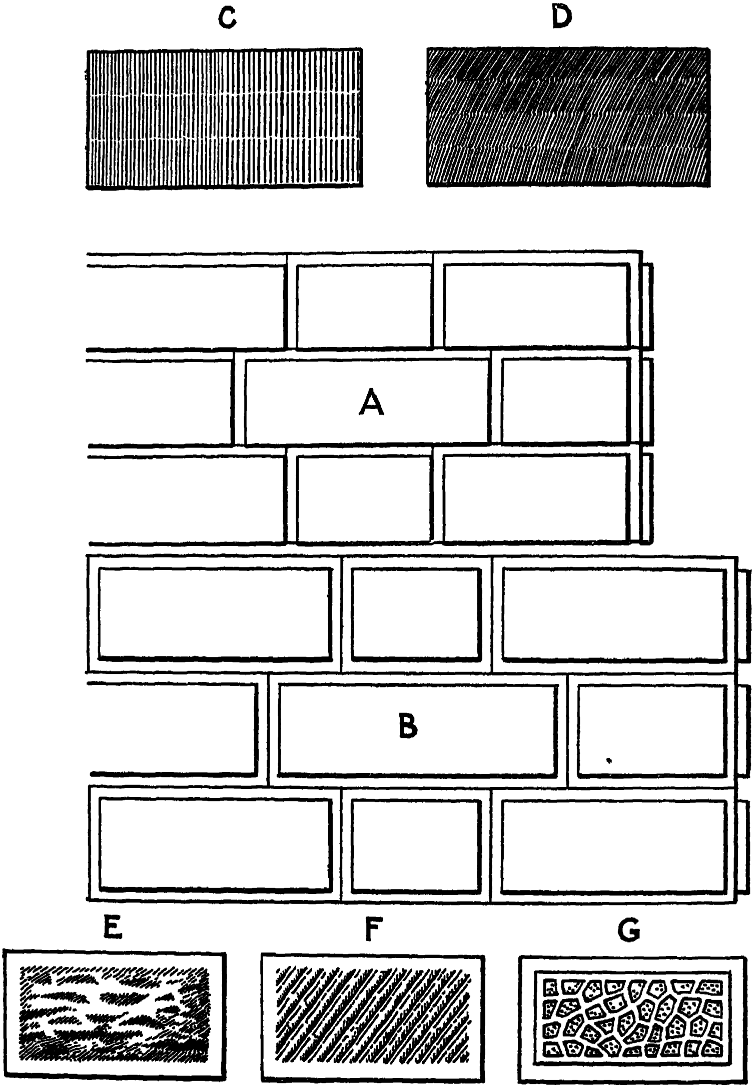


FIG. 286.

boldness of appearance to the work. Fig. 286, C, shows a block finished with fine and coarse axed face, on left- and right-hand sides respectively. At D, the face is "droved" or "tooled," being worked with a broad flat chisel. E is a rock-faced block with draughted margin. This type of

dressing without the draught is always employed for squared rubble masonry, the draughts being occasionally cut on the quoins. The face F is "broached" and draughted, whilst G is a rebated and vermiculated block. The more ornamental face dressings are seldom employed on engineering work.

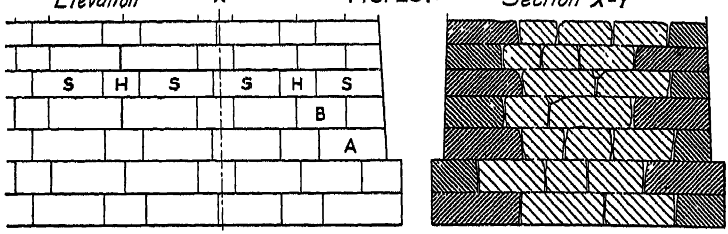
General clauses regarding measurement for payment by the cube yard (or cube foot in case of ashlar), conditions of inspection and execution according to drawings, usually precede a detailed specification for masonry.

Heavy Squared Coursed Rubble (occasionally referred to as Bridge Masonry, Coursed Blockstone, and Block in Course).—Foundation courses which are laid immediately on a concrete foundation slab, to be of selected stones not less than 18 in. thick and to have a bed area of not less than 15 square feet. No course to be less than 14 in. nor more than 24 in. in height, and each course to be continuous both through and around the wall. If courses of unequal height are permitted, the deeper ones to be at the bottom diminishing regularly towards the top. Face stones to have rock-faced surfaces, the edges being dressed to straight lines conforming to size of courses, and no part of rock face to project more than 3 in. in heavy courses or 2 in. in medium courses.

In Figs. 287 the elevation and plans of two consecutive courses of part of a rubble masonry pier are shown, together with a vertical section on XY. The beds and vertical joints of face stones to be dressed back at least 12 in. from face of wall and the under beds to extend to the extreme back of stone as at B, Fig. 288. Overhanging stones as at C leave spaces A which are liable to be indifferently packed. Stretchers to have a length not less than $2\frac{1}{2}$ times their height, and no stone to have a less width than $1\frac{1}{2}$ times its height. The face bond to show not less than 12 in. lap. The size and disposal of headers and bond stones will depend on the thickness of wall. Headers should preferably be not less than 3 feet to 4 feet long. For walls less than 4 feet thick the headers should extend from back to front, giving the arrangement shown in plan in Fig. 289. For walls exceeding about 4 feet in thickness, there should be an equal number of headers built into both back and face, so arranged that a face header shall be roughly midway between two headers in the rear as shown in plan in Figs 287, 290, and 293. In the case of retaining and wing walls which are backed by earth, the back face may of course be left much rougher than the exposed front face, but this should not be allowed to excuse defective bonding at the back of the wall. There should be one header to every two stretchers, and they should sensibly retain their full face size for the whole length of stone beyond the dressed surfaces, as A in Fig. 291. At B is shown an inferior header, the dressed joints (shaded) not extending far enough into the wall, whilst the tail end tapers off, requiring packing up with rubbish or leaving voids beneath the stone. This is the commonest defect in rubble masonry, and cannot be detected from the face appearance once the course is covered up. In walls of 6 feet to 7 feet thickness the inner ends of headers should overlap laterally.

The hearting or packing stones to be of large well-shaped stones of the same general thickness as the face stones. No voids exceeding 6 in. width to be left between these stones, and all voids to be thoroughly filled

FIG. 287.



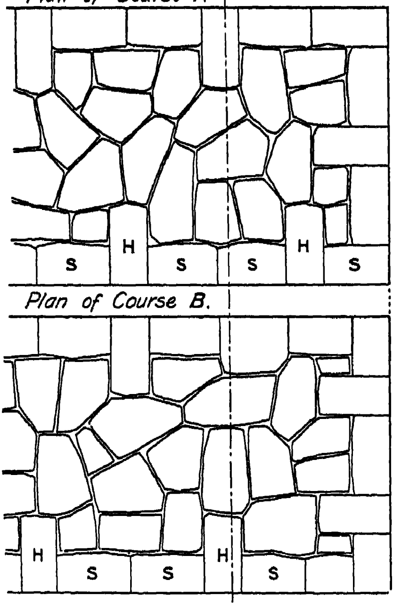


FIG 288.

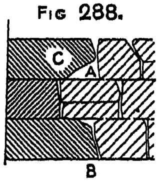


FIG 291.

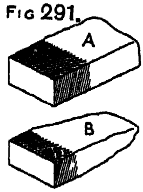


FIG 289.

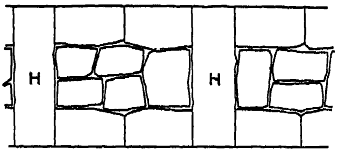


FIG 290.

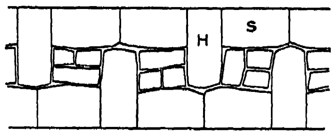


FIG 292.

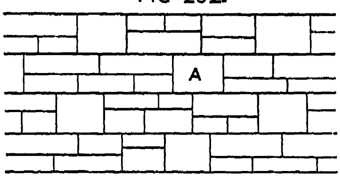


FIG 294

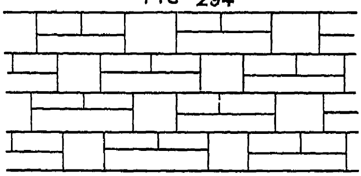


FIG 295.

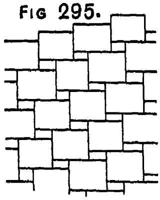
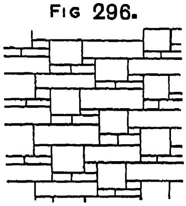


FIG 296.



Plan of Course A

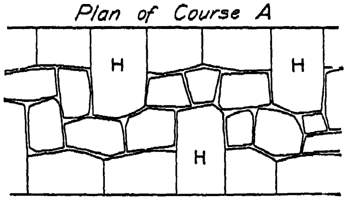


FIG 293.

with spalls bedded in cement mortar. The superposition of the larger hearting stones in each two consecutive courses should have regard to the preservation of the best possible bond and minimum number of coincident vertical joints. (See plans of courses in Figs. 287.) Where provision is to be made for leading off drainage water through masonry, the positions of weep holes or inlaid pipes will be either shown on drawings or personally indicated by the engineer in charge.

Other points to notice are that heavy hammering or dressing of stone on the wall itself should be prohibited; stones accidentally broken or moved after setting, to be removed and properly replaced; each stone to be laid in a full bed of mortar, and joints not to exceed $\frac{1}{4}$ in. to $\frac{3}{8}$ in. when laid. Stones and masonry to be kept free from dirt, and all stones and adjoining masonry to be wetted with clean water just before laying, especially in hot weather. If the masonry is to be extended by building future work, the old masonry should be stepped back uniformly to ensure a break in vertical bond between old and new work of at least 12 in. Quoins in massive work are usually draughted 2 in. to 3 in. wide, the width of draught being proportioned to the scale of the work, and they may be further finished with rebated joints, heavy rock face, or by rusticating, according to the degree of finished appearance required.

The above notes relate more particularly to the massive types of masonry structures. For work intermediate between these and ordinary walls occurring in general building construction, the following class of squared rubble is employed.

Ordinary Squared Coursed Rubble.—Fig. 292 shows the general face appearance of such work, and Fig. 293 a plan of a typical course. The main points of difference are the generally smaller size of stone, and less regularity of arrangement. The usual height of each course is 12 in. to 14 in. The stones in each course are not all the full height of the course, but all headers and bond stones should be so. The number of headers may be specified as 1 to 3, 1 to 4 stretchers, etc., or as so many per square yard of surface, or per 6, 10, or 12 ft. run of the course. The smaller face stones should not have a less thickness than one-third the course height, and preferably not more than two stones should be allowed to the course. In highly finished work of this class, greater regularity of appearance on the face is obtained, as in Fig. 294. Specially skilled masons are required for executing really good work of this character. Exceptionally good examples may be seen in the masonry of the overflows and training walls at the Langsett reservoir near Sheffield, executed by Mr. William Watts, M.Inst.C.E. The general remarks on backing for heavy masonry apply equally with regard to the backing in this case. Figs. 295 and 296 are other examples of regularly faced rubble, although the bond in Fig. 295 is necessarily inferior.

Rubble Concrete.—This class of masonry is practically confined to the hearting of reservoir dams, breakwaters, etc. Fig. 297 indicates the general disposition of such work when laid. Large irregular blocks called *plums* or *displacers*, with fairly flat but rough beds, and weighing from 1 ton to 8 tons, constitute the bulk of the work. They are bedded on a thick layer of cement concrete, or very coarse mortar,

consisting of 2 parts sand, 4 crushed rock spoil, small enough to pass a $\frac{3}{16}$ in. mesh, and 1 part cement. The blocks are usually slung on to the work from an overhead cableway, and are arranged to break joint both horizontally and vertically, no two stones being nearer than about 6 in., to allow of perfect ramming of the joints with concrete. The stones are settled on their beds by slowly working them backwards and forwards by crowbars, and by heavy hammering with wooden malls. It is important in such work to use as heavy stone as is procurable, since the stability depends on the weight of the mass, whilst the closer together the stones can be laid, the greater the ultimate mean density of the mass, and the less the amount of cement required. The stones should not be of too irregular a shape, or very large voids require to be filled with concrete. Smaller stones are used for partially filling the joints. A fair proportion of stones, as indicated in section in Fig. 297, should bond transversely, as well as longitudinally. All such overhanging parts as A in Fig. 298 should be removed, otherwise the c.g. of the block is thrown appreciably away from the centre of the bed, causing unequal settlement and tilting on the soft bed of concrete. Work of this character over large areas is necessarily interrupted, and where new masonry is joined to old, the surface of the concrete is picked over, carefully brushed and washed and covered with cement mortar before commencing the new work. In this connexion, the late Mr. G. F. Deacon considered that a hydraulic lime was probably superior to cement, since the much slower setting enabled new work to be bonded to work several days old whilst still in a plastic state.[1] The face of such work is finished in ashlar, or heavy, square, rock-faced rubble. Fig. 299 shows the typical appearance of the outer face work of the Vyrnwy dam.

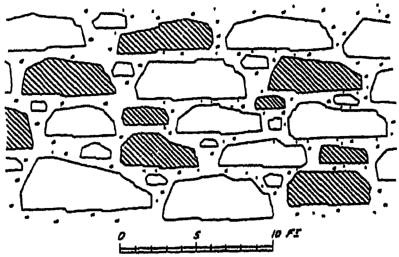


FIG. 297.

FIG. 298.

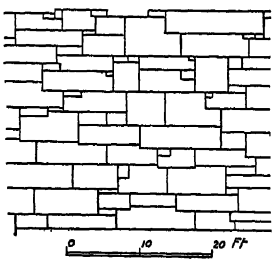


FIG. 299.

Ashlar.—The stones are cut to exact dimensions, with surfaces of beds and joints dressed back to the full depth of block, and joints should not exceed $\frac{1}{4}$ in. Where a considerable area is faced with ashlar, the kind of bond is usually specified, and the stones laid and bonded with the regularity of brickwork. Courses may be of equal or unequal height, preferably the former. In the highest class work the beds and joints are required to be plane throughout. As this entails heavy waste in dressing, other specifications allow depressions below the level of the

[1] *Mins. Proceedings Inst. C.E.*, vol. cxlvi. p. 27.

beds, provided they do not exceed, say, 6 in. in width, are not nearer than 4 in. to an edge, and do not aggregate more than one-quarter the whole area of bed. To produce ashlar blocks quite free from such depressions or *plug-holes* considerably increases the expense of labour in cutting, and reduces the general bulk of the finished blocks.

Arched Masonry Blocks require cutting to accurate radial lines, and if large, are usually worked to drawings supplied. The plane of the natural quarry bed should be perpendicular to the direction of pressure acting through the arch. In arches with ashlar ring-stones, and brick or rubble sheeting, the bond with the sheeting should not be less than from 6 in. to 12 in., depending on size of work. The ring-stones, or face voussoirs, V, Fig. 300, should preferably be cut with square heads, in order to obtain a satisfactory bond with the stones of the head wall, H.

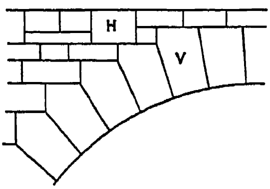


FIG. 300

Bonding of Brick and Masonry Arches.—Fig. 301 illustrates various methods of arranging and bonding the bricks and masonry in arches. A shows a $13\frac{1}{2}$ in. brick arch with bricks laid alternately header and stretcher. B shows the same bond applied to an 18 in. arch. Although the bond is satisfactory throughout the depth of the arch, the disadvantage of laying the bricks in this way is that the joints become excessively wide at the extrados unless tapered or radius bricks are used. For this reason, arches exceeding 18 in. are generally built in duplicate rings, each 9 in. thick, as at C, or in separate $4\frac{1}{2}$ in. rings, as at D. With either of these methods, the bond is deficient, and unless very carefully built, unequal settlement on striking the centering, is liable to cause one or more rings to fall away from the others, as shown at F, when the load comes on a much reduced thickness of arch. In order to secure some degree of through bonding, the arrangements at G and E are often adopted. That at G shows the bond inserted in the $22\frac{1}{2}$ in. portions of the lining of the Totley tunnel. The inner and outer 9 in. are built in English bond as at B and C, whilst the central $4\frac{1}{2}$ in. constitutes an independent ring, which is bonded alternately with the inner and outer rings at sections where the joints JJ are flush throughout the whole depth. At E bond stones S or specially constructed brick voussoirs V are inserted at flush joints. In stone arches any desired arrangement of bond may be adopted, since the stones are cut to taper shapes. The expense is, however, much greater than in the case of brick arches, and excepting where architectural effect is to be secured, arches built of stone throughout are seldom employed in ordinary structural work. The most usual construction for arches of from four to eight half bricks in thickness, which comprise the majority of engineering arches and tunnel linings, is to build the exposed soffit ring in blue brick, and the backing rings in good stock brick. In the best work, the outside faces are of masonry bonded alternately header and stretcher with the brickwork. At D a label course, L, of stone is shown. This is sometimes added in large brick arches to secure a more finished appearance, and is usually from 4 in. to 6 in. thick. At

H is a masonry arch regularly bonded header and stretcher, the face stones being rebated and rock-faced. This is a suitable treatment for large span arches, where appearance is of importance. K illustrates a

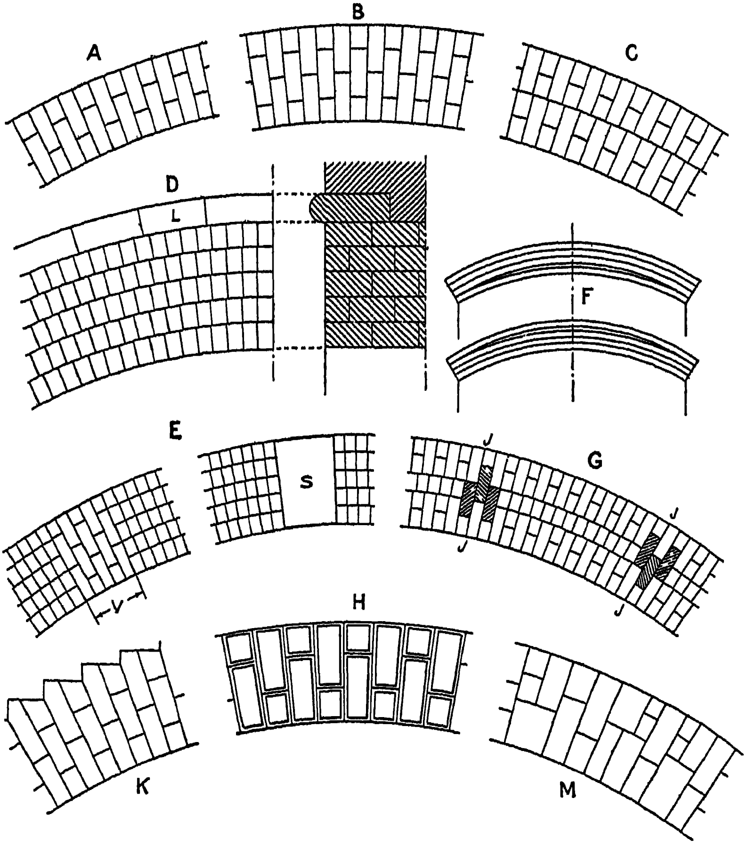


FIG. 301.

heavy masonry arch ring having every second voussoir square-shouldered to bond with the head wall. The rough square rubble arch at M is occasionally adopted for rough or temporary work.

Piers.—Masonry piers are usually of rock-faced square rubble masonry of the kind already described. Where, however, the work is imposing by reason of exceptional magnitude or where considerable architectural effect is desirable, all external faces are built in ashlar with varying degrees of enrichment in the matter of rebates, mouldings, etc. Fig. 302 shows one of the piers for the aqueduct over the river Loire at Briare in France, and is an excellent example of highly finished and well-treated engineering architecture. This structure carries the Loire canal over the river Loire in fifteen spans of 131·25 ft.,

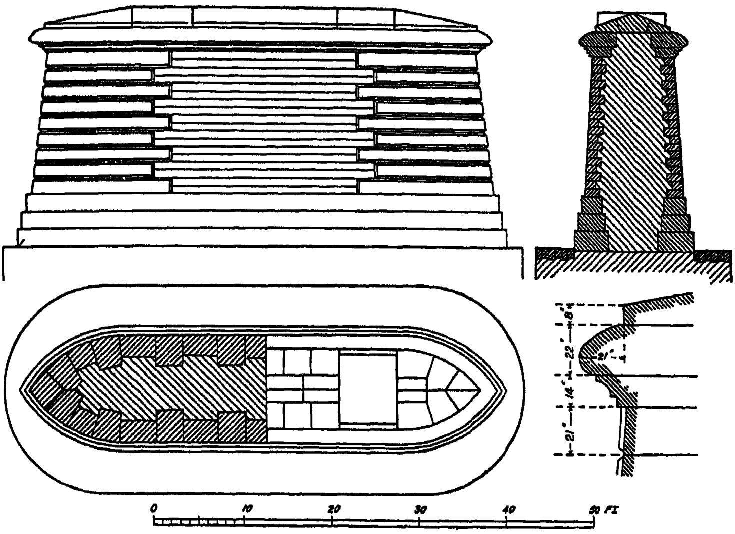


FIG. 302

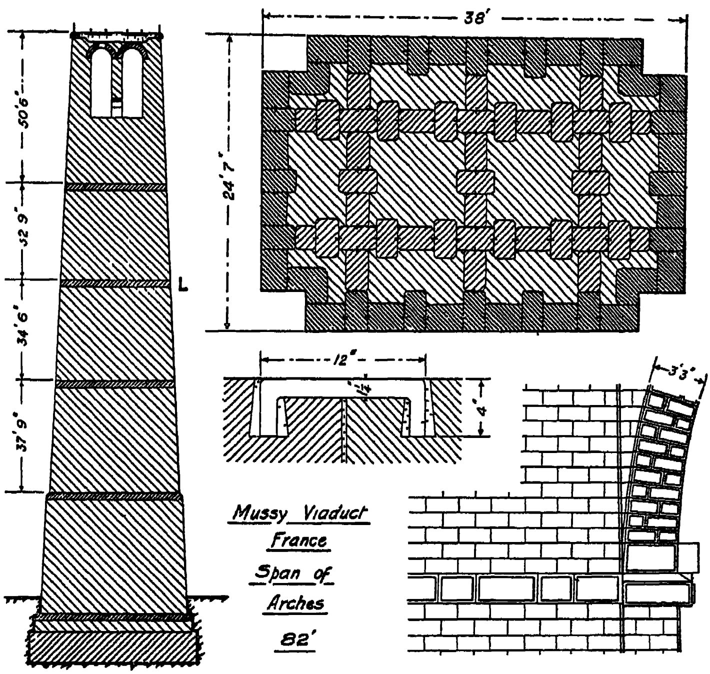


FIG. 303.

and owing to its prominent situation and imposing appearance, has been given a high degree of architectural finish.[1]

Fig. 303 illustrates the construction of one pier of the Mussy viaduct, also in France. This, one of the largest masonry viaducts erected, comprises 18 arched spans of 82 feet each, the maximum height from foundation to rail level being 180·25 ft. The piers are of rubble masonry, with ashlar lacing courses at intervals of about 35 ft. A plan of one of these courses L is shown in the figure, all the main blocks being tied together by iron cramps, the detail of which is also shown. The lower figure indicates the face treatment, and the structure constitutes a fine example of massive bridge masonry on the largest scale.[2]

Masonry-Faced Concrete Blockwork.—Fig. 304 shows the type of construction followed in the re-constructed north Tyne breakwater.

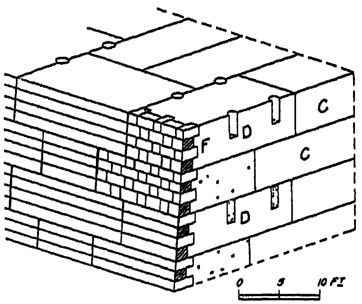


FIG 304.

The blocks C are of concrete alone, and vary from $5\frac{1}{3}$ to 19 cube yards content. The facing blocks are covered with an outer face F of Aberdeen granite, this masonry facing being built inside, and forming one end of the timber box in which the blocks were moulded. The blocks are regularly bonded alternately header and stretcher, and are further dowelled by filling the vertical semi-cylindrical grooves D with 4 to 1 cement concrete, the blocks consisting of 6 to 1 concrete. An extremely durable outer face having the appearance of a masonry structure is thus secured.[3]

Footing Courses.—The lowest portions of masonry structures usually rest on footing courses, which project some distance beyond the faces of the superstructure in order to distribute the pressure due to the weight of the structure over a larger area. These footing courses again rest on a mass of concrete laid in the bottom of the excavation for the foundation. The concrete serves the following purposes. By being made of still greater area than the footing courses it reduces the intensity of pressure to the safe bearing power of the soil; and, secondly, it fills up all irregularities of the foundation and provides a smooth and level surface on which to commence building the masonry. The thickness given to the concrete bed depends largely on a variety of practical considerations, although, as will be seen later, it must have not less than a certain minimum thickness fixed by the weight and disposition of load on the superstructure. It is, for example, often cheaper or more convenient to put in a considerable depth of concrete instead of building additional masonry on a thinner layer, whilst in other cases the lower portion of the structure may require to be formed by depositing concrete below water-level until a suitable height is reached on which to

[1] *Annales des Ponts et Chaussées*, 1898, 2nd Trimestre.
[2] *Ibid.*, 1901, 1st Trimestre
[3] *Mins. Proceedings Inst. C.E.*, vol. clxxx., p. 133.

commence the masonry. Footing courses of masonry should consist of specially selected large and well-shaped stones. Hardness is essential, since they are subject to the heaviest pressure. The successive offsets or projections should be made very gradually. If the footing courses project too far beyond the superstructure or beyond each other, they are liable to be cracked when bearing on concrete or to tilt up as in Fig. 305, if bearing directly on the foundation earth. In the latter case the advantage of the footing courses is quite lost, since the effective bearing area is reduced by the breadth AB.

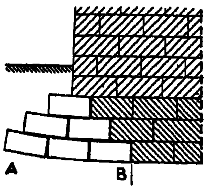


Fig. 305.

No stone in a footing course should project more than $\frac{1}{4}$ to $\frac{1}{5}$ of its length, and such stones should further have a liberal depth. (See preceding specification.) Piers, abutments, wing-walls and retaining walls are frequently built of brickwork, and for these structures each footing course should not be less than four bricks in thickness, and more for heavy works. The projection of each course should not exceed $2\frac{1}{4}$ in., and the upper course of each ledge or offset should be all headers as at H, H, Fig. 306. The bonding is shown in the figure for a wall executed in English bond, three-quarter bricks being inserted at A, A, and whole bricks at B, B, in order to properly break joint.

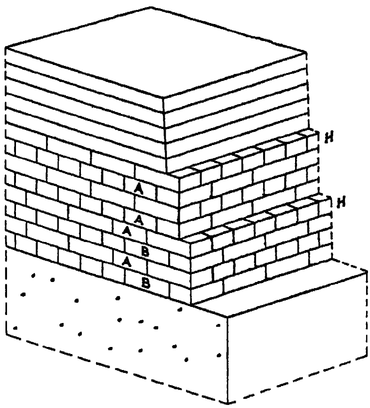


Fig. 306

Pressure on Foundations.—The bearing power of soils naturally varies widely, and any stated values can only be regarded as general standards Before commencing any important structure, and especially where the bearing capacity of the foundation is an unknown or doubtful quantity, careful examination and testing are desirable. If the foundation be near the surface, the bearing power may be conveniently tested by erecting a strong timber platform carried on four legs of 12 in. square cross-section, and gradually and uniformly loading it until any desired maximum settlement takes place. The settlement of each leg is noted at intervals of one or two days by taking level readings. For foundations at considerable depths below the surface, a tank resting on a plate of 2 or 3 square feet area may be placed at the bottom of trial excavations and gradually filled with water. Whether a considerable amount of settlement may be allowed depends on the kind of structure to be erected. An appreciable amount of settlement is not objectionable, provided it takes place uniformly over the whole area of the foundation. Where a definite amount of settlement is anticipated, the original level of the foundation is adjusted so that the desired permanent level may be established when the total weight of

the completed structure comes upon the foundation. It is, however, desirable to minimize settlement in such structures as piers and abutments for bridges, chimneys, retaining walls, and dams. In the former, settlement entails subsequent trouble with girder bearings, cracking of arches, and frequent packing up of the track; whilst in retaining walls and dams, settlement, if of appreciable amount, is necessarily unequal, and results in vertical displacement, creation of unknown shearing stresses, or heeling over of the structure. In the case of tall framed buildings, some amount of settlement is usually unavoidable. The accuracy with which the bearing area of the foundations of the supporting columns may be proportioned to their loads, however, realizes a high degree of uniformity of settlement. In the tall buildings of Chicago a settlement of 6 in. to 12 in. is general, and cannot be avoided, owing to the compressibility of the clay in which they are founded.

TABLE 29.—PERMISSIBLE SAFE PRESSURES ON FOUNDATIONS AND MASONRY.

Material.	Tons per sq. ft.
Very hard rock	25 to 30
Ordinary rock	8 „ 16
Dry clay of considerable depth	4 „ 6
Moderately dry clay	2 „ 3
Soft clay	1 „ 2
Compact gravel and coarse sand	6 „ 8
Clean dry sand, laterally confined	2
Alluvial soils	½ to 1
Concrete in foundations	8
„ walls	7
Good red brickwork in cement mortar	10
Blue „ „ „	15
Squared rubble freestone masonry in cement mortar	8
Ashlar freestone masonry in cement mortar	15
Granite ashlar „ „ „	25
Freestone ashlar bearing blocks	12
Granite „ „	20
Granolithic „ „	12

Distribution of Pressure on Foundations.—Any masonry structure symmetrical about its vertical axis, such as the rectangular and pyramidal piers in Fig. 307, will have its centre of gravity G vertically over the centre of gravity M of the area of foundation *abcd*. The point P, where the vertical through G cuts the foundation level, is the *centre of pressure* on the foundation, and in these cases obviously coincides with M. Further, in such cases the pressure per square foot on the foundation area will be uniform, provided the upward resistance of the soil is also uniform. If W = total weight of structure in tons, and A = area of foundation in square feet, then pressure per square foot on foundation $= \frac{W}{A}$ tons. If, as usually happens, the area *abcd* be too small to suitably distribute the weight W, it must be increased by either footings, a projecting concrete base, or grillage beams until the

unit pressure $\frac{W}{A}$ does not exceed what may be safely imposed on the soil. Thus, if the pier A be 24 ft. high, 30 ft. wide, and 5 ft. thick, and built of masonry of 140 lbs. to the cube foot, its weight will be 225 tons. Suppose it to carry a further central load of 300 tons applied by girders resting on it, the total load W = 525 tons. The foundation area A, without footings = 30 × 5 = 150 sq. ft., and the pressure per square foot = $\frac{525}{150}$ = 3·5 tons. If it is undesirable to load the foundation soil beyond 2·5 tons per square foot, the necessary area *with* footings will be $\frac{525}{2 \cdot 5}$ = 210 sq. ft., which might be obtained by making the bearing area a rectangle 31′ 6″ × 6′ 8″. It is often convenient to represent the intensity of pressure on foundations diagrammatically. If *eh* be made = 2½ tons to any convenient scale, then the pressure per square foot being uniform, the ordinates of the rectangle *efgh* indicate the pressure per square foot at any point along the foundation from *e* to *f*, and *efgh* is a **pressure intensity diagram.**

In the case of an unsymmetrical structure, such as the reservoir dam in Fig. 308, the centre of gravity G is not vertically over the

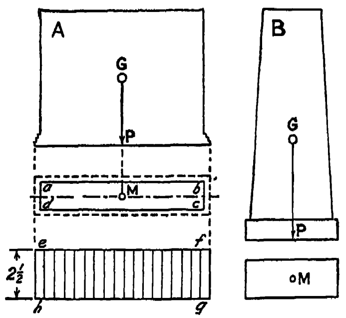


FIG 307.

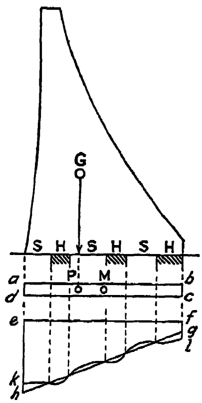


FIG. 308.

centre of area M of the foundation *abcd*, and the centre of pressure P vertically below G no longer coincides with M, but falls to one side of it. The weight of the structure is here concentrated more towards *ad* than *bc*, and the intensity of pressure on the foundation will be greater at the inner face *ad* than at the outer face *bc*. The pressure intensity diagram *efgh* will now be a trapezium, having *eh* considerably greater than *fg*. If the values of the intensities of pressure *eh* and *fg* be calculated, the straight line *hg* will cut off ordinates representing the pressure per square foot at other points along the base. It should be noticed that by drawing a *straight* line from *h* to *g*, it is assumed that the ground beneath the dam is of uniform character. If, for instance, in the neighbourhood of the points H, H, H, the foundation were appreciably harder than in other parts, the weight would bear more

intensely on these points, and the intensities of pressure on the base would then be represented by some undulating line as *kl*, cutting off deeper ordinates beneath H, H, H, and shallower ordinates beneath the intervals of softer ground S, S. The *mean* pressure would remain the same, that is, the area *efgh* would equal the area *eflk*. Local defective places such as S, S, are made good in all carefully prepared foundations, and the intensity of pressure may reasonably be expected to vary practically uniformly in well-executed works.

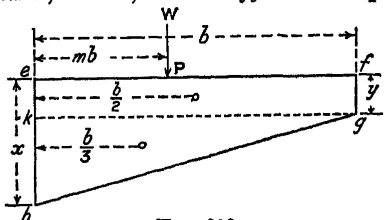


FIG. 309.

General Expression for Intensities of Pressure on Rectangular Foundations. — To ascertain the values of the pressure intensities *eh* and *fg* in Fig. 309, let b = breadth of foundation in feet, W = total weight or vertical pressure on foundation in tons per foot run acting at the centre of pressure P, distant mb feet from e. Let x and y tons per square foot be the intensities of pressure at e and f respectively.

Considering a one-foot length of the structure, the *mean* pressure on foundation $= \frac{x+y}{2}$ tons per square foot, and the total pressure per foot run

$$= \frac{x+y}{2} \times b, \text{ which must} = \text{W tons} \quad . \quad . \quad . \quad . \quad (1)$$

Also, taking moments about e, $\text{W} \times mb$ = moment of area *efgh* about e. Dividing the area *efgh* into a rectangle and triangle by the dotted line *gk*, its moment about e

$$= by \times \frac{b}{2} + \frac{b(x-y)}{2} \times \frac{b}{3}$$

$$= \frac{b^2y}{2} + \frac{b^2(x-y)}{6} = \frac{b^2x + 2b^2y}{6} = \text{W} \times mb . \quad (2)$$

But from (1), $\frac{x+y}{2} \times b = \text{W}$, whence $y = \frac{2\text{W}}{b} - x$.

Substituting in (2)

$$\frac{b^2x + 2b^2\left(\frac{2\text{W}}{b} - x\right)}{6} = \text{W} \times mb,$$

from which

$$x = \frac{2\text{W}}{b}(2 - 3m) \quad . \quad . \quad . \quad (3)$$

and by substituting in (1)

$$y = \frac{2\text{W}}{b}(3m - 1) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

These results are important, and are capable of general application in all cases of rectangular surfaces in contact under a known pressure,

acting through a known centre of pressure. The factor $m = \frac{eP}{ef}$, so that when the position of the centre of pressure P is known the value of m is also fixed. Applying this result to the dam in Fig. 308, suppose the base to be 60 ft. wide, the weight of the dam per foot run to be 200 tons, and the centre of pressure P to be 21 feet from the inner face. Then $m = \frac{21}{60}$, $b = 60$, and $W = 200$ tons. Therefore pressure per square foot

$$\text{at inner face} = x = \frac{2 \times 200}{60}(2 - 3 \times \tfrac{21}{60}) = 6\tfrac{1}{3} \text{ tons}$$

$$\text{and at outer face} = y = \frac{2 \times 200}{60}(3 \times \tfrac{21}{60} - 1) = \tfrac{1}{3} \text{ ton.}$$

These, of course, are the pressures at opposite edges of the foundation due simply to the dead weight of the dam, no account having been taken of any water pressure acting against it.

It will be seen that here the centre of pressure falls to one side of the centre of the foundation area, entirely on account of the *shape* of the section of the dam, the material being bulked up more to the left- than to the right-hand side. In many structures the centre of pressure is caused to fall nearer to one edge of the foundation by the action of external forces. In Fig. 310 the pier A, under the action of its own weight only, would have the centre of pressure on its foundation at P, the centre of base. When subject to a lateral wind pressure F, the resultant pressure on the base takes the direction OP_1, the centre of pressure being thereby displaced from P to P_1. In the retaining wall B the earth pressure E causes the centre of pressure to be displaced from P to P_1, and in the case of the dam C the centre of pressure falls at P_1 under the action of the horizontal water pressure W, instead of at P when the reservoir is empty.

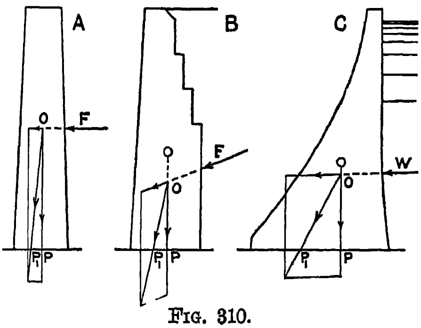


FIG. 310.

The intensities x and y have particular values, according as the fraction m is greater than, equal to, or less than $\frac{1}{3}$. Four distinctive cases occur, which are illustrated in Fig. 311. In each case a total vertical load of 16 tons per foot run acts on a foundation 8 ft. wide.

Case 1.—Centre of pressure at middle of foundation. $m = \frac{1}{2}$; $W = 16$ tons; $b = 8$ ft. Inserting these values in equations (3) and (4), $x = y = 2$ tons per square foot, or the intensity of pressure is uniform over the whole foundation area.

Case 2.—Centre of pressure 3 ft. from one edge of foundation. $m = \frac{3}{8}$, or is *greater* than $\frac{1}{3}$. Hence $x = +3\frac{1}{2}$ tons, and $y = +\frac{1}{2}$ ton per square foot.

Case 3.—Centre of pressure $2\frac{2}{3}$ ft. from one edge, or $\frac{1}{3}$ of 8 ft. $m = \frac{1}{3}$, whence $x = +4$ tons per square foot, and $y = 0$.

Case 4.—Centre of pressure 2 ft. from one edge, or *less* than $\frac{1}{3}$ of 8 ft. $m = \frac{1}{4}$, whence $x = +5$, and $y = -1$ ton per square foot.

In the last case the value of y, being negative, signifies the intensity y to be a *tensile* instead of a compressive stress, and its value is accordingly plotted on the opposite side of the horizontal line EF to that of x. As the centre of pressure moves from the centre towards the left-hand edge of the foundation area, the pressure per square foot on the left steadily increases, whilst that on the right decreases. In Case 3 the right-hand intensity diminished to zero, and in Case 4 it has passed beyond zero and become an uplift instead of a downward pressure. Assuming the foundation to be appreciably compressible—clay, for instance—in Case 1 uniform settlement will take place, and a wall standing on this 8 ft. base will settle from the level E′F to some level $e'f$, the vertical depth E′e' representing the settlement produced by a pressure of 2 tons per square foot on the subsoil in question. In

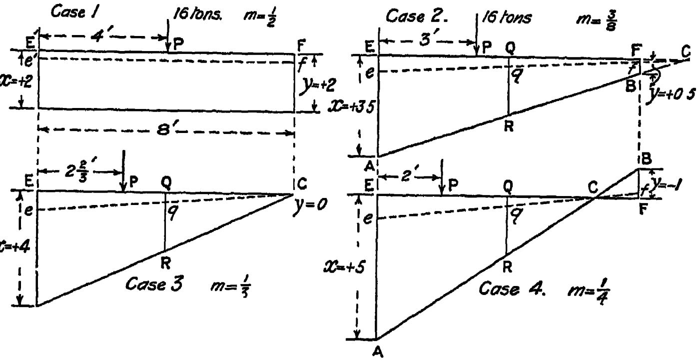


FIG. 311.

Case 2 an intensity of 2 tons per square foot exists at QR. $Qq =$ E′e' therefore represents the settlement beneath this point of the foundation. If AB be produced to C, then at this point the intensity of pressure on the foundation would be zero, and the settlement also zero. Joining Cq and producing to e, the depths Ee to Ff represent the unequal settlement taking place in this case, which is everywhere proportional to the intensity of pressure. Similarly in Case 3, QR = 2 tons per square foot. $Qq =$ E′e', and Ee represents the settlement at E, whilst that at C is nothing. In Case 4, QR = 2 tons per square foot. $Qq =$ E′e', and eCf determines the settlement Ee at E, whilst an uplift Ff occurs at F, the point C on the base of the wall neither depressing nor rising. The effect of the unequal settlement in Cases 2, 3, and 4 is to cause the structure to heel over towards the side where the greater intensity of pressure exists, and in each case the structure rotates slightly about the point C as centre. This action is especially apparent in the case of heavy masonry structures on yielding foundations.

Masonry, concrete, and mortar possessing but small tensile strength, it is generally admitted as undesirable to allow tensile stress to exist in such

materials. In Case 4 tension exists from O to F, and whether EF represent a horizontal joint in a masonry structure, a horizontal section of a monolithic concrete structure, or the actual foundation level, whilst it is undesirable to have tension existing from F to O, it does not necessarily follow that the structure is unsafe. Provided the intensity of compression EA at the opposite face does not exceed the safe compressive resistance of the mortar, concrete, or subsoil, the structure may be perfectly safe. If, however, the maximum intensity of tension FB exceed the tensile resistance of the mortar or concrete, then in a masonry structure the joint will open for some distance in from F; in a concrete structure cracks will be developed; and at a foundation level a certain amount of uplift will take place. The immediate effect of such actions is to reduce the effective breadth of joint, as in Fig. 312, from EF to EF′. Employing the same values as in the previous case, suppose the joint to open for a length of 1 ft. The reduced breadth EF′ = 7 ft., $m = \frac{2}{7}$, and $x = 5{\cdot}23$ tons per square foot compression, and $y = 0{\cdot}65$ ton per square foot tension. If this tension is incapable of causing further opening of the joint or crack at F′, and the compression of 5·23 tons does not exceed the safe resistance of the material, the structure will still be safe. If further opening occurs at F′, EF′ is still further reduced, the intensity x increased, and safety ultimately depends on the extent of this increase in the value of x. In the case of piers, retaining-walls, and arches the existence of tension at one face of the joints does not necessarily render the structure unsafe; but in masonry or concrete dams any opening of joints or cracks, however slight, on the water face is accompanied by entry of the water, and consequent application of an upward hydrostatic pressure H in Fig. 313, which creates a new resultant pressure P_1R_1 with centre of pressure at P_1, in place of the previous resultant PR with centre of pressure at P. This resultant, acting on the reduced bearing surface EF_1, gives rise to further tension at F_1 which extends the width of opening FF_1, introduces increased upward water pressure with further displacement of the resultant pressure P_1R_1 and still further reduction in the width of bearing surface. The intensity of pressure at E thus rapidly increases until failure takes place by crushing of the material near E, or bodily overturning of the portion CDEF. In the design of these structures, therefore, it is important to ensure the centre of pressure at any horizontal section falling within the middle one-third of the width of the section, and so to avoid the formation of cracks or open joints on the water face.

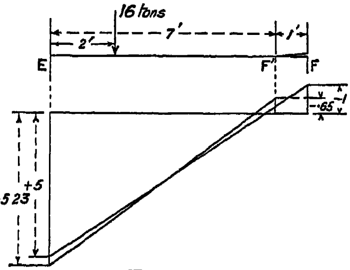


FIG. 312.

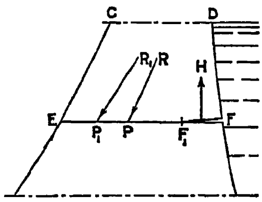


FIG. 313

Retaining Walls.

A face of earth exposed to the action of the weather eventually assumes a more or less uniform slope AB, Fig. 314, the inclination of which with the horizontal is called the **angle of repose or natural slope** of the particular soil in question. The natural slope of different kinds of earth varies very widely, and cannot be closely specified for any particular variety. In the case of the more homogeneous and uniform varieties, as fine dry sand and gravel free from large stones, the angle of repose is fairly constant. The presence of varying amounts of water in the same soil, however, greatly modifies the angle of slope, and the nature of soils in general is so varied that although many may be classed as *similar*, yet it is seldom any two of the same class exhibit the same properties. Any table of angles of repose and unit weights of stated varieties of earth can only be accepted as representing approximate average values, and if more accurate ones be required they should be obtained by actual measurement of a sample of the earth involved. The following are fairly representative values of these quantities :—

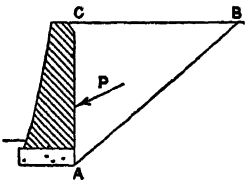


Fig. 314.

Table 30.—Natural Slope and Unit Weight of Earth, etc.

Material.	Natural slope	Weight
	deg.	lbs. per cub. ft.
Sand, dry	30 to 35	90
„ wet	26 „ 30	118
Vegetable earth, dry . . .	29	90 to 95
„ „ moist . . .	45 to 49	95 „ 110
„ „ wet	17	110 „ 120
Loamy soil, dry	40	80 „ 100
Clay, dry	29	120
„ damp	45	120 to 130
„ wet	16	135
Gravel, stone predominating .	45	90
Gravel and sand	26 to 30	100 to 110

These values being so variable, it is useless, in the theoretical design of retaining walls, to expect the same degree of accuracy which may be relied on in dealing with a constructive material like steel, whose properties are much more closely established. Moreover, the earth behind a high retaining wall is seldom uniform from top to bottom. Two or three distinct beds of different classes of earth may be present, which further increases the difficulty of applying mathematical procedure in estimating the probable earth pressure against the wall.

Earth Pressure.—In Fig. 314, in order to hold up the triangular mass of earth ACB, a wall is required of sufficient stability to safely resist the pressure P exerted on it by the earth. Many mathematical formulæ have been devised with a view to calculating the magnitude of

the earth pressure behind retaining walls, but they are necessarily based on the fundamental assumption that the earth is perfectly uniform throughout, and consequently do not provide for the variations constantly encountered in practice. Some rule, however, is necessary as a guide in the choice of a suitable section, and probably all formulæ in connexion with earth pressure at least err on the side of safety, since the varieties of earth met with in practice are generally less uniform and more cohesive than the ideal granular earth assumed in theory.

Rankine's Theory.—This theory has for many years been adopted principally in England. It is based on a mathematical analysis of the conditions of equilibrium of a particle in the interior of a mass of earth, assumed to be perfectly dry, uniform and granular. It makes no allowance for adhesion or for friction between the earth and back of the wall. Stated briefly, if A = angle of repose of earth, H = height of wall in feet, and w = weight of a cubic foot of earth in lbs., then for a wall retaining earth with a level upper surface, the horizontal earth pressure per foot run of the wall

$$= P = \tfrac{1}{2}wH^2\left(\frac{1 - \sin A}{1 + \sin A}\right) \text{ pounds.}$$

Modifications of the formula are employed for giving the pressure in cases of surcharged walls. It may be stated, however, that earth pressures calculated by this formula are 25 to 30 per cent. in excess of the actual pressures behind existing walls.

Rebhann's Method.—The following simple graphical method, due to Professors Rebhann and Haseler, for determining the magnitude of the earth pressure behind a wall has been very generally adopted on the continent of Europe for many years past. It has the advantage of being applicable alike to cases of horizontal or surcharged upper surface, and certainly gives results both economical and satisfactory in practice. The general construction is shown in Fig. 315. BC represents the back of the wall and CD the upper surface of the earth. Draw BD making an angle A with the horizontal, equal to the angle of repose of the earth. On BD describe a semicircle BGD. From C draw CE making an angle = 2A with the back of the wall. At E draw EG perpendicular to BD to cut the semicircle in G. With centre B and radius BG, cut BD in K. Draw KL parallel to CE, cutting the upper surface in L. Make KM = KL and join LM. The triangle KLM is called the earth pressure triangle. Suppose it to represent a triangular prism of earth one foot thick, as shown at T. Then the resultant pressure P per foot run behind the wall, including the effect of friction between the earth and wall, is given by the weight of this

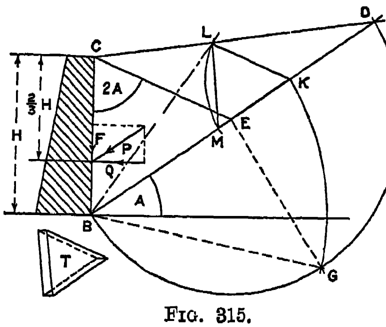
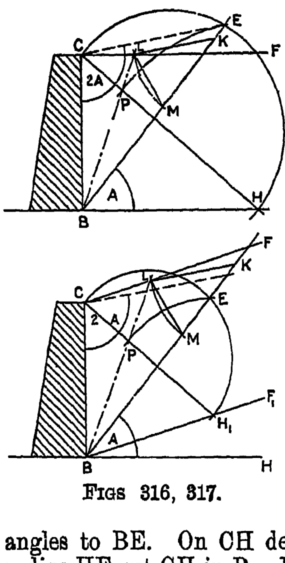


FIG. 315.

prism of earth, or P = area KLM in square feet × 1 foot × weight of earth per cubic foot.

It is unnecessary here to state the proof of this construction, for which the reader may be referred to a paper by Professor G. F. Charnock.[1] It may be stated briefly, however, that the method is based on Coulomb's theory, and that the object of the construction is to determine the position of point L, and consequently the location of the line BL. This line BL marks what is termed the plane of rupture for maximum earth pressure behind the wall. In other words, if the wall be supposed to yield, the triangular mass of earth BCL will tend to break away along the plane BL, and the wall must be sufficiently stable to resist the pressure caused by the tendency of the mass BCL to slide down the incline LB. The actual *earth* pressure acts perpendicularly to the back of the wall at two-thirds its depth H from the surface ; but the stability of the mass of earth BCL is obviously partially maintained by the frictional force F existing between the earth and back of the wall. The resulting pressure P on the wall, as given by the earth-pressure triangle, is therefore compounded of a certain *horizontal* pressure Q and a *vertical* force F, depending on the coefficient of friction between the earth and wall. It may reasonably be assumed that the coefficient of friction between the earth and back of wall will be at least equal to that between the particles of the earth itself, since the backs of most walls are usually very rough. The inclination of the resultant pressure P may therefore be safely taken as making an angle with the perpendicular to the back of the wall equal to the angle of repose of the earth. Possibly in the case of a concrete wall constructed between moulding boards back and front, the above amount of friction might not be realized, and a suitable reduction in the inclination of P might be made accordingly.



FIGS 316, 317.

The above construction is inapplicable if the angle of repose exceed 45°. The following modification may be adopted in such cases, although the angle of repose will seldom approach 45° in practice. In Fig. 316, CF is the horizontal surface of the earth. Draw BE making the angle of repose A, with BH and CH at right angles to BE. On CH describe a semicircle, and with centre H and radius HE cut CH in P. BP is the plane of rupture, and producing it to L, draw LK parallel to CE, which makes an angle of 2A with the back of the wall. Cut off KM = KL, and KLM is the earth-pressure triangle. (The coincident intersection at E of BE and the dotted line CE in Fig. 316 is accidental.) For a surcharged wall as in Fig. 317, first draw BF_1 parallel to the upper surface CF, and repeat the same

[1] *Stability of Retaining Walls*, by Prof. G. F. Charnock. Read before the Association of Yorkshire Students of the Inst. C. E., March 10, 1904.

construction excepting that CH_1, perpendicular to BE, is used as the diameter for the semicircle instead of carrying it through to intersect BH.

An application of the method to the design of an actual wall will now be given.

EXAMPLE 36.—*Design a suitable section for a retaining wall, 20 ft. high, of rubble masonry weighing 140 lbs. per cubic foot, the angle of repose being 27° and weight of earth 110 lbs. per cubic foot. The maximum pressure on foundation not to exceed $2\frac{1}{2}$ tons per square foot.*

In Fig. 318, for a clear height of 20 ft. the total height of wall from coping to foundation is taken as 24 ft., allowing for a concrete base

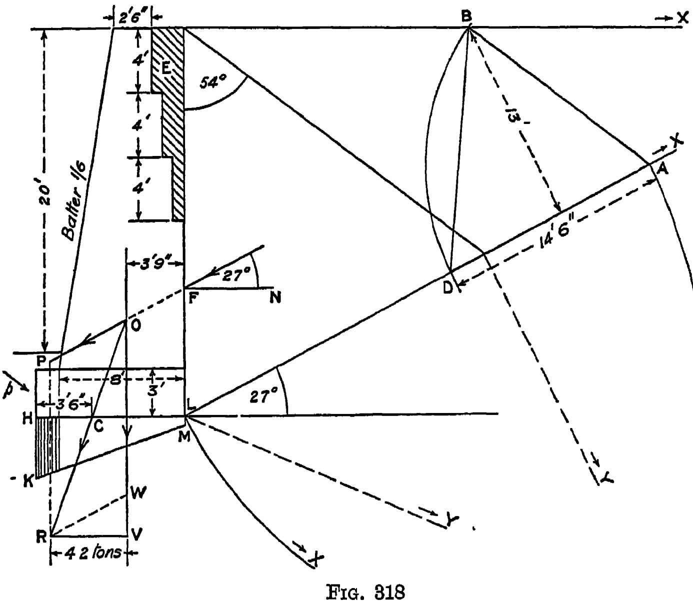


FIG. 318

3 ft. thick, with the level of foundation 4 ft. below the lower ground-level, and the section indicated is assumed. In all practical walls there may be a slight counter-pressure p against the front of the toe. It is, however, very small compared with the back pressure, and is frequently reduced to zero by shrinkage of the earth. Also in walls with stepped or battered backs, the shaded block of earth E, which apparently rests on the wall and adds to its stability, has often been found to be quite out of horizontal contact with the wall, owing to compacting under pressure and subsequent shrinkage, or settlement of the wall away from the earth, and for these reasons the effect of the pressure p and weight of block E will be neglected. Assuming the concrete base as of practically the same weight as the masonry, the total sectional area including the base $= 143\frac{3}{4}$ square feet, and weight of wall per foot run $= 143\frac{3}{4} \times 1 \times 140$ lbs. $= 9$ tons. Applying the construction for the earth pressure, the area of the resulting earth-pressure triangle ABD

$= \frac{1}{2} \times 14\frac{1}{2} \times 13$ square feet, and the resultant pressure against the wall $= \frac{1}{2} \times 14\frac{1}{2} \times 13 \times 110 = 10{,}367$ lbs. $= 4{\cdot}63$ tons per foot run of wall. The centre of gravity of the wall section (including the base) is situated on the vertical line OW, 3 ft. 9 in. from the back of the wall. This may be found by dividing the section into a convenient number of parts and taking moments about the back of the wall, or by suspending a cardboard template of the section. Mark the point F at $\frac{2}{3}$ of $24 = 16$ ft. from the surface, and draw FN perpendicular to back of wall. FO making 27° with FN gives the direction of the resultant pressure behind the wall. From O, where FO cuts the vertical through the centre of gravity of the wall section, set off OP = the pressure of 4·63 tons, and OW = weight of wall, 9 tons, to any convenient scale. OR to the same scale gives the resultant pressure on the foundation. The centre of pressure C, in which OR cuts the foundation level, is situated 3 ft. 6 in. from the outer edge of foundation, and the fraction m to be used in calculating the intensity of pressure on foundation $= \frac{3'\ 6''}{9'\ 6''} = \frac{7}{19}$. The resultant *vertical* pressure on foundation acting through C = OV, the vertical component of OR, which, it should be noted, consists of the wall weight OW + the frictional component WV of the inclined pressure WR. OV scales off 11·2 tons. The intensities of pressure on the foundation are then—

$$\text{at H} = \frac{2W}{b}(2 - 3m) = \frac{2 \times 11{\cdot}2}{9{\cdot}5}(2 - 3 \times \tfrac{7}{19}) = 2{\cdot}11 \text{ tons per sq. ft.}$$

$$\text{and at L} = \frac{2W}{b}(3m - 1) = \frac{2 \times 11{\cdot}2}{9{\cdot}5}(3 \times \tfrac{7}{19} - 1) = 0{\cdot}25 \text{ ton per sq. ft.}$$

Setting off HK = 2·11 and LM = 0·25, and joining KM, the area HKLM shows by its ordinates the varying intensity of pressure on the foundation. As the maximum intensity of 2·11 tons per square foot is within the specified limit of 2·5 tons, a slightly more economical section might be adopted. For further illustrating the application of principles, however, this section will be adhered to. The *horizontal* component RV of the resultant pressure OR scales off 4·2 tons, and tends to cause horizontal sliding of the wall on the foundation. This horizontal component should not exceed the frictional resistance of the wall to sliding. The following coefficients of friction may be applied in checking the liability to sliding :—

Masonry and brickwork joints, dry	0 6 to 0·7
,, ,, ,, wet mortar .	0·47
,, ,, on dry clay	0·50
,, ,, on wet ,, ...	0·33

In the case of retaining walls on fairly dry foundations there is little risk of sliding, since the foundation level is invariably sunk some feet below the surface, and the resistance to displacement of the earth in front of the wall assists the frictional resistance of the wall itself. Walls on wet clay foundations require careful examination in this respect.

The total vertical pressure = 11·2 tons. Assuming an average

foundation with coefficient of friction of 0·4, the frictional resistance to sliding $= 11 \cdot 2 \times 0 \cdot 4 = 4 \cdot 48$ tons, which exceeds the horizontal pressure of 4·2 tons, the resistance of the earth in front being unconsidered. It remains to ascertain if the thickness of the concrete base is sufficient to allow of a projection of 18 in. in front of the wall. This portion of the base is shown enlarged in Fig. 319. It is subject to an upward pressure from the foundation, represented by the portion HK*kh* of the area of the pressure intensity diagram. The intensity *hk* scales off 1·71 tons. HK = 2·11 tons. The mean intensity over $Hh = \frac{2 \cdot 11 + 1 \cdot 71}{2} = 1 \cdot 91$ tons per square foot. The area of foundation beneath H*h* = 1·5 sq. ft., and total upward pressure $= 1 \cdot 91 \times 1 \cdot 5 = 2 \cdot 865$ tons, acting through the c.g. G of area HK*kh*. G is 9·4 in. to the left of *hk*. The B.M. on the section *hf* of the concrete cantilever $HFfh = 2 \cdot 865 \times 9 \cdot 4 = 27$ inch-tons. The cross-section of this cantilever is shown at S. Its moment of resistance $= \frac{1}{6}bd^2f_b = \frac{1}{6} \times 12 \times 36 \times 36f_b =$ the B.M. of 27×2240 inch-lbs.,

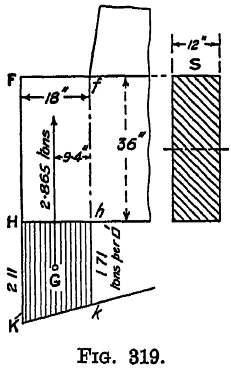


FIG. 319.

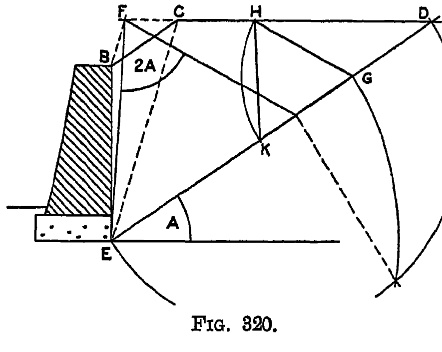


FIG. 320.

whence $f_b = 22 \cdot 3$ lbs. per square inch tension at *h* and compression at *f*. The safe transverse bending stress f_b for concrete should not exceed about 50 lbs. per square inch. In a deep foundation the B.M. on *fh* would be appreciably reduced by the action of the weight of the concrete block F*h* and the earth above it, which here has not been considered. The section *fh* is also subject to a vertical shear of 2·865 tons. The mean shear $= \frac{2 \cdot 865 \times 2240}{12 \times 36} = 14 \cdot 9$ lbs. per square inch, and maximum shear intensity $= 14 \cdot 9 \times 1 \cdot 5 = 22 \cdot 4$ lbs. per square inch. The safe shear on concrete should not exceed 60 lbs. per square inch.

Surcharged Walls.—The following construction may be applied in the case of walls retaining a bank of earth BCD, Fig. 320. Join CE and draw BF parallel to CE. Join FE and proceed as before to construct the earth-pressure triangle GHK, setting off the angle 2A at F instead of at B.

Many walls, as in Fig. 321, are subject to additional pressure due to weight of traffic, buildings, etc., in streets carried along the upper level. It is impossible to estimate correctly the effect of such loads on

walls. Such additional loading is usually reduced to an approximately equivalent layer of earth and the wall then treated as being surcharged to that extent. Thus, if the traffic along the street in Fig. 321 be taken as equivalent to a load of 150 lbs. per square foot, this may be increased to, say, 250 lbs. to allow for vibration, and if the earth weigh 100 lbs. per cubic foot an additional layer LL, 2 ft. 6 in. deep, being superimposed, the effective depth behind the wall will then be LH.

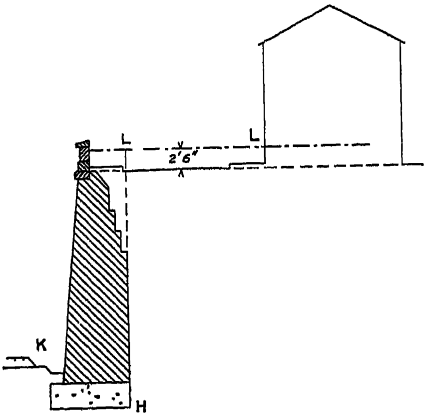


FIG. 321.

Needless to say, buildings should not be erected immediately in rear of existing retaining walls unless provision has been made at the time of building the wall. A more usual occurrence is when railway cuttings have to be made through towns subsequently to the erection of the buildings, in which cases suitable provision for the super-loading may be made as above. The passage of heavy railway traffic along a cutting as at K makes it desirable to ensure a good margin of stability in the supporting walls.

Types of Walls.—Retaining walls generally are of one or other of the types shown in Fig. 322. A is a wall with battered face and

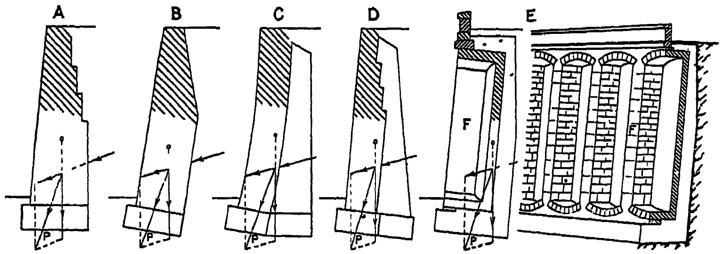


FIG. 322

stepped back, which is a satisfactory section on an unyielding foundation such as hard gravel or rock. C and D are sections of leaning walls with buttresses or counterforts at intervals behind the wall. Theoretically such counterforts should make for economy in material, but their practical value is doubtful, especially in masonry walls, where they are very liable to break away from the wall and so become useless Their employment is more satisfactory in concrete walls, where they are moulded *en bloc* with the body of the wall. B is a leaning wall of similar section to A. Curved walls as at C are not now very frequently employed. The approximate position of the c.g. is marked in each case, and it will be noticed that in type A the centre of pressure P

falls much nearer the outer edge of the base than the back, with a consequently large variation in pressure intensity from front to back of foundation. This is of little moment on hard foundations where the settlement is very slight. In types C, D, and E the centre of gravity of the wall is thrown well back, and the resultant pressure may be designed to intersect the base at or near its centre, giving nearly uniform pressure, and therefore uniform settlement on a yielding foundation. These types are more suitable for clay or other soft foundations where heavy pressure at the outer toe would result in heeling over of the wall towards the front. Panelled walls as at E are very generally used for the sides of railway cuttings through towns. The buttresses F, roof and back arching and inverts are built of rubble masonry or blue brick, and the backing either of rubble or concrete. This type of wall is most suitable under conditions of heavy backing and soft foundation. Whether the back of a wall is built in steps or uniformly battered matters little as regards the stability, steps however being usual in brickwork, and forming convenient ledges during building for the support of staging. Where sliding forward of the wall is apprehended, the base is often sloped upward towards the front.

Example of Panelled Retaining Wall.—In designing a panelled section such as E, Fig. 322, a length equal to the spacing of the panels must be considered instead of a one-foot run of the wall, since the section is not uniform throughout. The position of the c.g. of one bay may be found as follows.

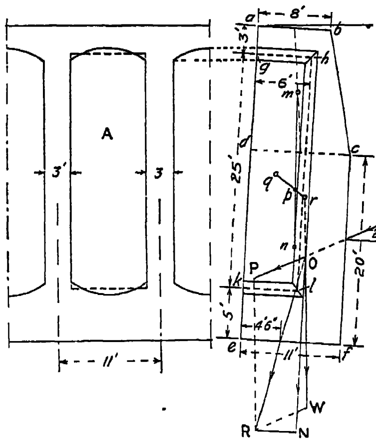


FIG. 323

Suppose Fig. 323 to represent a proposed section for a panelled wall. First equalize the curved surfaces of the recess A by the dotted rectangular boundary lines. Next locate the c.g. of the section *abcfe*, supposing the wall to be built solid throughout. Divide *abcfe* into two portions *abcd* and *cdef*. The c g. of *abcd* is at *m*, and of *cdef* at *n*. Area *abcd* $= 13 \times \frac{8+11}{2} = 123\frac{1}{2}$, and of *cdef* $= 11 \times 20 = 220$ sq. ft., and total area *abcfe* $= 343\frac{1}{2}$ sq ft. Divide *mn* in *p* so that $\frac{mp}{pn} = \frac{220}{123 \cdot 5}$. *p* is the c.g. of the whole section *abcfe*. The c.g. of the panel recess *ghlk* is at *q*. Next—

		cub. ft.
Cubic content of one bay of 11 ft. of the *solid* section	$= 343\frac{1}{2} \times 11$	$= 3778 \cdot 5$
„ „ panel recess	$= 25 \times 8 \times 6$	$= 1200 \cdot 0$
„ „ bay of the *hollow* wall	= difference	$= 2578 \cdot 5$

These volumes being proportional to the weights of the corresponding masses may be considered as forces acting respectively through centres of gravity p, q, and r, r being the c.g. of the hollow wall. Taking moments about the back of the wall and denoting by x the unknown distance of r from cf,

$$\begin{array}{llll} \text{Moment of solid section about } cf = 3778{\cdot}5 \times 5{\cdot}8 & = 21915 \\ \quad ,, \quad \text{panel recess} \quad ,, \quad cf = 1200 \times 8 & = 9600 \\ \quad ,, \quad \text{hollow wall} \quad ,, \quad cf = 2578{\cdot}5 \times x & = 2578{\cdot}5x \end{array}$$

But the moment of the solid wall = sum of moments of panel recess + hollow wall;

$$\therefore\ 2578{\cdot}5x + 9600 = 21{,}915, \text{ whence } x = 4{\cdot}77 \text{ ft.}$$

Join qp and mark the position of r on qp produced distant 4·77 ft. from cf. r is the required c.g. of the 11 ft. bay of the wall. This calculation assumes the wall of uniform material throughout. If the facing and backing be of materials of appreciably different unit weights, a suitable mean value may be assumed without sensibly affecting the exact result, which would otherwise be very tedious to work out. With the materials in ordinary use, the above determination is sufficiently close.

Continuing, the weight of one 11 ft. bay of the wall at 140 lbs. per cubic foot $= \dfrac{2578{\cdot}5 \times 140}{2240} = 161$ tons.

Assuming a face batter of 1 in 16 and earth backing at 100 lbs. per cubic foot having a natural slope of 25°, the area of the earth pressure triangle = 124 sq. ft. The resultant pressure against wall *per* 11 *foot run* $= \dfrac{124 \times 11 \times 100}{2240} = 61$ tons. Setting off OP = 61 tons and OW = 161 tons, the line of resultant pressure OR cuts the base 4 ft. 6 in. from e. Draw ON perpendicular to ef and RN parallel to ef. ON the *normal* pressure on foundation = 187 tons per 11 foot run of wall, or 17 tons per foot run. Hence for pressure intensities on foundation, $m = \dfrac{4{\cdot}5}{11} = \frac{9}{22}$, $b = 11$ ft., and W = 17 tons.

$$\text{Intensity at } e = \frac{2 \times 17}{11}(2 - 3 \times \tfrac{9}{22}) = 2{\cdot}4 \text{ tons per square foot.}$$

$$,, \qquad f = \frac{2 \times 17}{11}(3 \times \tfrac{9}{22} - 1) = 0\ 7 \qquad ,, \qquad ,,$$

These intensities may be more nearly equalized if desired by extending the base of the wall at the front. In these walls the back pressure is transmitted to the buttresses by the horizontal arching forming the back of the recesses, and from the buttresses to the foundation by the inverted arches at the base. It is assumed the pressure on the foundation per foot run of the wall is rendered uniform through the action of the inverts. This distributing effect of the inverts will be very closely realized in a well-executed wall. If, as is sometimes the case, the panels be not provided with inverts as in Fig. 324, the

pressure must be considerably more intense immediately beneath the buttresses, and the basal concrete slab S will yield as shown in an exaggerated form by the curved lines. In such a case the greater part of the pressure on the foundation per length L should be taken as coming on a width W very little greater than the width of footing of the buttress. A specially thick slab, or steel reinforcement should be used in such cases, but inverts are desirable.

Methods of relieving Pressure behind Walls.—Various means are employed for relieving the pressure on retaining walls, especially such as may exhibit signs of weakness after erection. In Fig. 325 tiers of

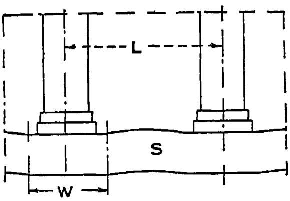


FIG. 324

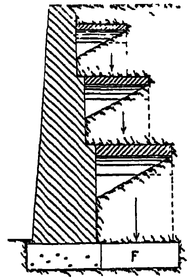


FIG. 325.

arches are built behind the wall, which reduces the depth of earth in contact with the wall, and throws a portion of the weight of the earth on to the footings F of the piers carrying the arches. Such arches may be part of the original design, or may be inserted subsequently to the building of the wall. This method is occasionally adopted where buildings have unavoidably to be erected immediately behind an existing wall.

In Fig. 326 tie-rods, chains, or cables are passed through the wall and taken back to anchor-plates A or raking piles P driven well in

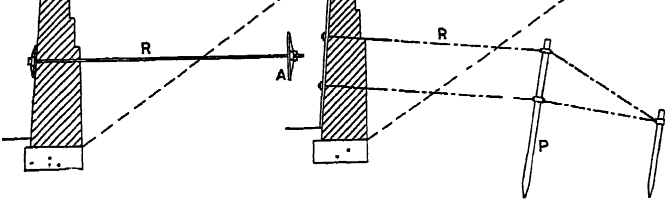


FIG. 326.

rear of the natural slope. Both are expedients involving no great difficulty of execution in applying to existing walls. An extension of the concrete base B in Fig. 328, or the subsequent insertion of a concrete block C, is occasionally effected in order to counteract forward sliding of a wall.

In Fig. 327 the heavy backing H is benched back, and the space

immediately in rear of the wall filled in with lighter material L. In all cases where the backing is filled in after the building of the wall,

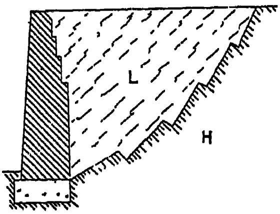


FIG. 327

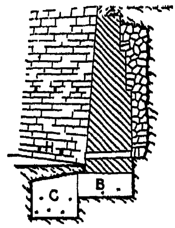


FIG. 328.

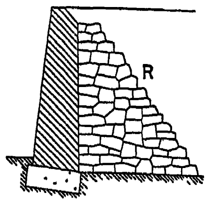


FIG. 329.

the material just behind the wall should be carefully deposited so as to exert the minimum pressure. Carefully stacked rubble R, as in Fig. 329, is largely self-supporting, and will considerably relieve the pressure on the wall, at the same time acting as a drain. It is advisable to pack the space immediately behind a wall for a width of 2 or 3 ft. with dry stone filling, as in Fig. 328, and to provide water outlets or weep-holes H at frequent intervals. In very wet soils a properly constructed drain should be laid behind the wall with efficient discharge pipes passing through or beneath the wall. The walls on either side of deep cuttings are frequently strutted by steel or arched masonry ribs R, as in Fig. 330. An invert V is required in cases where the subsoil is liable to flow under pressure.

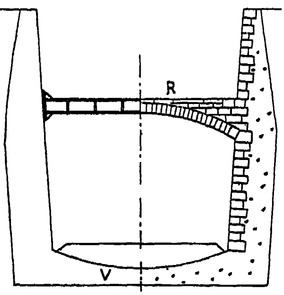


FIG. 330.

Walls of Variable Section.—Triangular wing-walls of bridge abutments retain a gradually diminishing height of the embankment behind

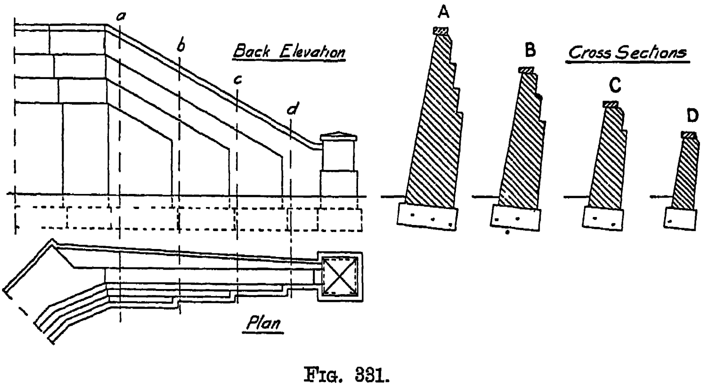


FIG. 331.

them, and are usually built as in Fig. 331, with battered face and inclined steps at the back, the steps ending vertically at regular

intervals. Successive vertical sections *a*, *b*, *c*, *d*, of the wall are shown at A, B, C, and D.

A similar case arises as in Fig. 332, where a long retaining wall W, is built in order to avoid a heavy cutting C in the side of a hill H. The

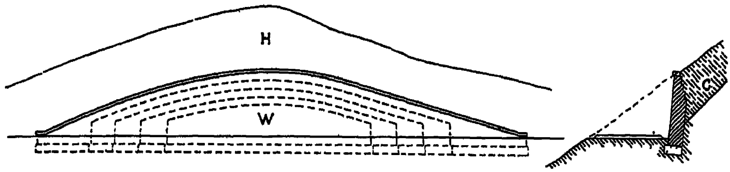


FIG. 332.

wall W being of variable height, the back is conveniently stepped as indicated by the dotted lines on the elevation.

MASONRY DAMS.

Reservoir dams of masonry or concrete are classed under the name of *masonry* dams. The modern practice in regard to these works is to construct them of rubble concrete with squared rubble masonry facing. Many existing high dams, however, are composed entirely of rubble masonry. Masonry dams are further classed as "gravity dams" and "arched dams." The former resist the water pressure by the action of dead weight only. The latter may only be employed on sites where a reliable rock abutment is provided by the sides of the valley across which the dam is built. They are curved in plan, and resist the pressure behind them by acting as horizontal arches, and are therefore of much lighter section than gravity dams. Arched dams are not often employed, and are subject to complicated stress conditions which cannot be accurately estimated, and the remarks here will be confined to the consideration of gravity dams. A dam is virtually a retaining wall for water, but since the magnitude of the water pressure may be accurately calculated, the design of a dam is capable of more rigid treatment than that of a retaining wall for earth. Further, most dams are works of much greater magnitude than retaining walls, and it is the more necessary to employ the minimum quantity of material consistent with safety, since a relatively small increase in sectional area involves heavy additional cost. Dams possessing the minimum section consistent with safety are said to be designed of *economical section.*

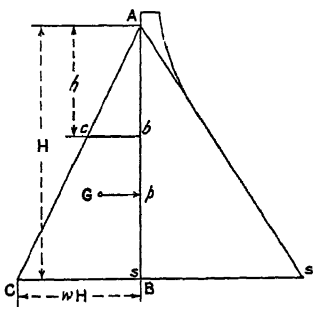


FIG. 333.

Pressure of Water on the Inner Face of a Dam.—In Fig. 333 let H feet be the depth of water above any horizontal section *ss* of a dam. The pressure per square foot at B = $w \times$ H lbs., where w = weight of

1 cubic foot of water. If BC be made $= w$H pounds to scale, since the pressure at the surface A is nothing, by joining AC the triangle ABC forms a diagram of intensity of pressure at any desired horizontal level. Thus, at the depth h feet, the pressure per square foot $= bc$ lbs., to the same scale as BC represents wH. The *resultant* pressure per foot run of the dam is represented by the area ABC, and

$$= \tfrac{1}{2}\text{BC} \times \text{AB} = \tfrac{1}{2}w\text{H} \times \text{H} = \tfrac{1}{2}w\text{H}^2 \text{ lbs.}$$

This resultant pressure acts through the c.g. G of the triangle ABC in a direction perpendicular to the surface AB Gp at right angles to AB therefore determines p, the centre of pressure on the inner face, which is obviously situated at two-thirds of the depth AB below the surface. The inner surface of a high dam cannot, for reasons to be presently explained, be made vertical for the whole depth, and is therefore given a slight straight or curved batter. As this batter is invariably small, the following construction is sufficiently accurate for all practical purposes. To obtain the resultant water pressure above any horizontal section ss in Fig. 334, join AB, and set off BC $= w$H at right angles to AB. Join AC, then resultant pressure = area ABC $= \tfrac{1}{2}w\text{H} \times \text{AB}$ lbs.

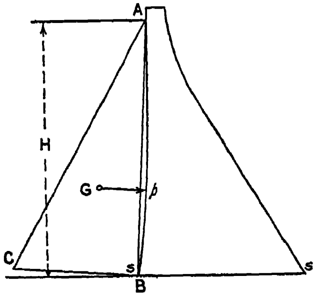


FIG. 334.

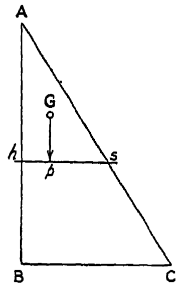


FIG 335.

The centre of pressure p is obtained by projecting the c.g. G of triangle ABC perpendicularly on to AB, and the direction of action of the resultant pressure is sensibly Gp. For the slight batters employed in practice, the line AB differs very slightly from the actual inner face of the dam, and any closer estimate is both tedious and unnecessary.

Section of Dam Wall. — The stability of any proposed section requires to be examined under two conditions · first, when the reservoir is empty, and the material of the dam subject to the action of its own weight only; secondly, when filled with water, and subject to the combined effect of the water pressure and weight of masonry.

1. **Reservoir Empty.** — The ideal theoretical section for a dam would be a right-angled triangle ABC, Fig. 335, which forms the basis from which to deduce a practical section. In *any* section, the c.g. G, of the material Ahs above any horizontal section hs, being projected vertically on to hs, determines the centre of pressure p on hs. In a triangular section dam, p will obviously always fall at one-third the horizontal breadth from h, or $hp = \tfrac{1}{3}hs$. For this position of the centre of pressure, it has already been shown that the intensity of

pressure at s is zero. Hence in a triangular section dam when the reservoir is empty, no compression exists at the outer or down-stream face, and a slight wind pressure acting on AC would be sufficient to set up a small tensile stress in the material near the outer face. In a practical section the following modifications of the theoretical triangular section become necessary. In Fig. 336, the dam must be carried a few feet above the water level L. The upper edge must be given a reasonable practical width by adding a mass of masonry A to the original triangular section. Most dams carry a roadway along the top, in which cases a width of several feet is needed. The weight of the additional masonry at A so affects the position of the centres of pressure on all horizontal sections below a certain horizontal level *hh*, as to entail the addition of a second mass of masonry, B, to the left of the original triangular section, in order to avoid creating tension on the outer face when the reservoir is empty. Lastly, in very high dams a third addition to the section is required at C to suitably increase the breadth of section, so that the intensity of pressure at the inner or outer face in the lower part of the dam may be kept within a safe limit, according as the reservoir is empty or full. The outline given to the outer face may be a series of straight batters, a continuous curve, or a combination of both. Whether one or another of these be adopted makes little difference in the practical economy of the section. A continuously curved outer face has a somewhat neater appearance, but is slightly more costly to build. In recent practice, the overflow is usually allowed to run down the face of the

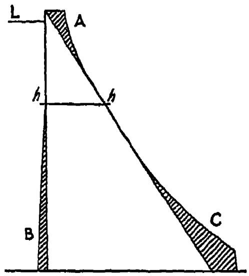


FIG. 336.

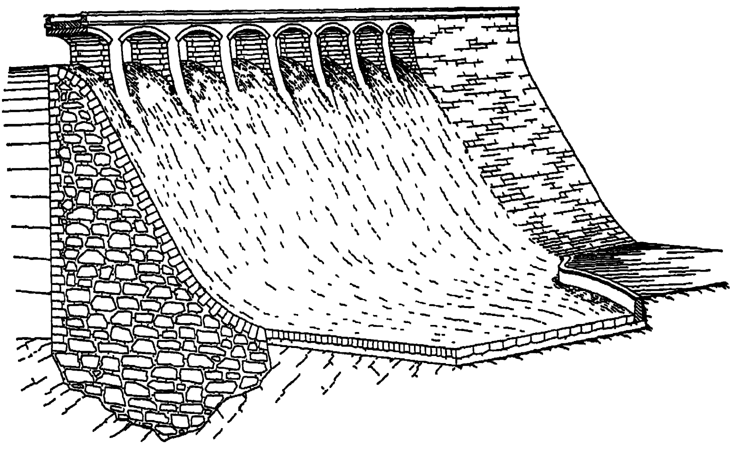


FIG. 337.

dam, after passing through a suitable number of arched spans beneath the roadway as in Fig. 337, such a dam being called an "overflow

dam." This arrangement obtains in the Vyrnwy, Burrator, Elan and Derwent Valley, and most of the modern dams, whilst in the older ones, the overflow is carried down a stepped fall beyond one end of the dam. The cross-section here shown is then necessary, the crest terminating in a blunt angle, whilst the toe is more gradually drawn out to conduct the water with reduced velocity into the overflow pool.

The effect of adding an additional mass of masonry A', Fig. 336, to a triangular section will now be considered. In Fig. 338, ABC

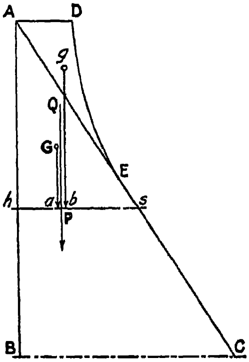


Fig. 338.

represents the original right-angled triangular section and ADE the addition necessary for carrying a roadway, or for providing a satisfactory top width AD for resisting the thrust of ice and effects of weathering. AD will generally vary between 5 ft. and 20 ft. G is the c.g. of the mass A*hs*, and *g* that of ADE. If any horizontal section *hs* be taken at a moderate depth below the top, and the points G and *g* be projected vertically on to it at *a* and *b*, G*a* and *gb* are the lines of action of the weight of the portions of masonry A*hs* and ADE respectively, and *a* and *b* their respective centres of pressure on *hs*. As before, *a* falls at $\frac{1}{3}hs$ from *h*, whilst *b* falls to the *right* of *a*. The line of action of the resultant weight of AD*sh* will therefore fall between G*a* and *gb* as at PQ, and the centre of pressure P due to the whole weight of masonry above *hs* will consequently also fall to the *right* of *a*, and therefore *within* the middle third of *hs*. Hence no tension will be developed at the horizontal section *hs*.

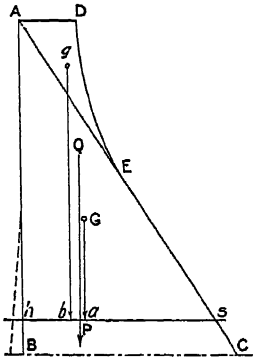


Fig. 339

If now, in Fig. 339, the same construction be repeated for a horizontal section *hs* taken considerably lower down, the point *b* is found to fall to the *left* of *a*. The centre of pressure P due to the whole weight above *hs*, now falls to the *left* of *a* and therefore *outside* the middle third of *hs*. In this case tensile stress will be developed in the material at *s* and for some distance in from the outer face. The width of the section *hs* must therefore be increased on the side towards *h*, and this is done by giving a slight batter to the inner face as shown by the dotted line. There will evidently be one particular horizontal section for which P falls neither within nor without the middle third of *hs*, the condition being that *g* and G must be situated on the same vertical line.

In Fig. 340 mark M the middle point of BC, and join AM. The centres of gravity of all triangles such as ABC lie on AM. Project *g* vertically to G on AM and make GH = $\frac{1}{2}$AG. Through H draw the horizontal *hs*. The resultant weight of masonry above *hs* now acts along *g*G, which being produced gives the centre of pressure at *p*, such

that $hp = \frac{1}{3}hs$. For all horizontal sections above hs the centre of pressure will fall *within* the middle third, whilst for those below hs, p will fall *outside* the middle third towards the inner face of the dam. Hence hs, which may be termed the "critical plane," marks the level at which the inside batter hF must be commenced in order to avoid creating tension at the outer face sC. Practically this batter will be commenced a little above the level hs, as it is undesirable for the compression to be reduced quite to zero at s, as would be the case in Fig. 340. The batter required to effect this adjustment is relatively slight and is readily ascertained after one or two trials.

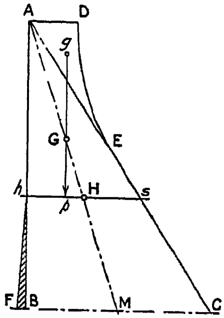


FIG. 340.

Stability of a Proposed Section.—Line of Resistance.—The following method of examination of the stability of a proposed section has been adopted in the design of most existing dams, and is often referred to as the *middle-third* or *trapezium* method. The procedure will be first outlined, after which the extent to which the results obtained agree with the probable actual stresses in the dam, will be noticed.

The section 0066 in Fig. 341 is assumed as that for a dam retaining 100 feet depth of water, and constructed of material weighing 150 lbs. per cubic foot. The section is divided into blocks or layers by the horizontal planes 1–1, 2–2, etc., 20 feet or other convenient distance apart, and the thickness of the section perpendicular to the plane of the paper is taken as one foot.

1. **Reservoir Empty.**—Considering first the reservoir empty, Fig. 341. Mark the positions a, b, c, d, e, f, of the centres of gravity of the masses 0011, 1122, etc. . . 5566. The centre of pressure on the plane 1–1, due to the weight of masonry above 1–1, is obtained by projecting a vertically to A on 1–1. The centre of pressure on any other plane follows similarly by projecting the c.g. of all the masonry above the plane vertically on to the plane. Thus, for the centre of pressure on plane 2–2, the c.g. of the mass 0022 is required. Its position may be most conveniently and accurately found from the known positions of a and b, and the weights of the first two masses 0011 and 1122 by taking moments about the vertical line oV.

$$\text{Weight of 0011 per ft. run} = \frac{10 \times 10 \times 150}{2240} = 6{\cdot}7 \text{ tons.}$$

$$\text{,, \quad 1122 \quad ,,} \quad = \frac{10 + 15{\cdot}5}{2} \times \frac{20 \times 150}{2240} = 17{\cdot}08 \text{ tons.}$$

$$\text{Moment of 0011 about } oV = 6{\cdot}7 \times 5' = 33{\cdot}50$$

$$\text{,, \quad 1122 \quad ,, } oV = 17{\cdot}08 \times 6\tfrac{2}{3}' = 113{\cdot}87$$

$$\text{Sum of moments} = 147{\cdot}37$$

$$\text{Sum of weights 0011, 1122} = 6{\cdot}7 + 17{\cdot}08 = 23{\cdot}78 \text{ tons.}$$

$$\text{Hence distance of c.g. of 0022 from } oV = \frac{147{\cdot}37}{23{\cdot}78} = 6{\cdot}2 \text{ ft.}$$

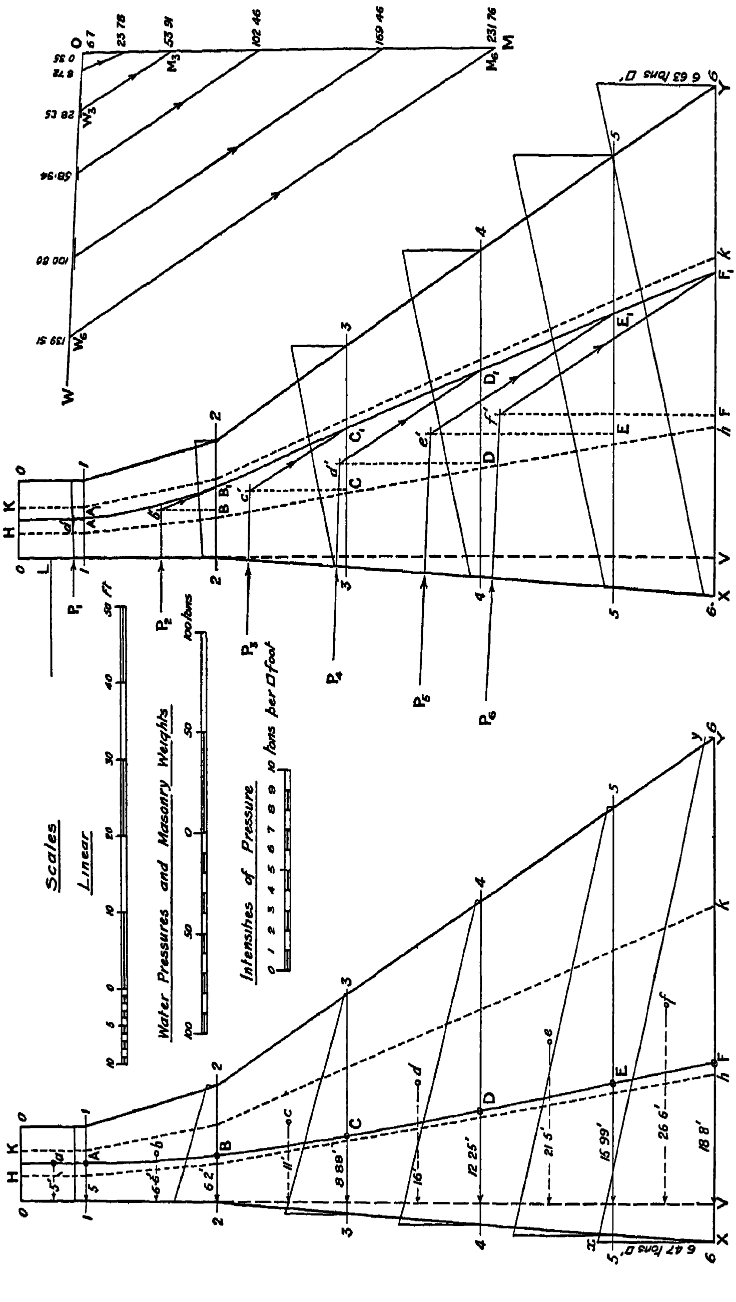


FIG. 342

FIG. 341.

It is unnecessary to determine the *height* of the c.g. of 0022, since being situated on a vertical 6·2 ft. from oV, it must, when projected on to 2–2, fall at B, also 6·2 ft. from oV. For the centre of pressure on plane 3–3,

$$\text{Weight of } 2233 = \frac{15{\cdot}5 + 29{\cdot}5}{2} \times \frac{20 \times 150}{2240} = 30{\cdot}13 \text{ tons.}$$

$$\text{Moment of 0022 about } oV \text{ (from above)} = 147{\cdot}37$$

$$\text{,, } \quad 2233 \quad \text{,, } \quad oV = 30{\cdot}13 \times 11' = 331{\cdot}43$$

$$\text{Sum of moments above 3–3} = \overline{478{\cdot}80}$$

$$\text{Total weight above 3–3} = 23{\cdot}78 + 30{\cdot}13 = 53{\cdot}91 \text{ tons.}$$

$$\text{Hence distance of c.g. of 0033 from } oV = \frac{478{\cdot}8}{53\ 91} = 8{\cdot}88 \text{ ft.}$$

This c.g. projected on to 3–3 falls at C, 8·88 feet from oV. In a similar manner, by adding the moment of 3344 to the moment of 0033 and dividing by the total weight of 0044 above plane 4–4, the position of D, the centre of pressure on plane 4–4 is obtained. The resulting distances of D, E, and F from oV are marked in Fig. 341. The curve connecting these centres of pressure A, B, C . . . F, is known as the **Line of Resistance or Line of Resultant Pressure** for the reservoir empty. It intersects any horizontal section of the dam at the centre of pressure due to the total weight of masonry above that section. From the positions of A, B, C, etc., the intensities of pressure perpendicular to the horizontal planes 1–1, 2–2, 3–3, etc., may be calculated.

For example, the centre of pressure F on the basal section is 18·8 ft. from oV, and therefore 23·8 ft. from the inside face at X. The base is 67 ft. wide, and the total weight of masonry above the base 231·76 tons. $m = \frac{23{\cdot}8}{67} = 0{\cdot}355$, and intensities of vertical pressure are

$$\text{at } X = \frac{2 \times 231{\cdot}76}{67}(2 - 3 \times 0{\cdot}355) = 6{\cdot}47 \text{ tons per square foot.}$$

$$\text{and at } Y = \frac{2 \times 231\ 76}{67}(3 \times 0{\cdot}355 - 1) = 0{\cdot}45 \quad \text{,,} \quad \text{,,}$$

These are plotted at Xx and Yy, the ordinates between XY and xy giving the intensities of vertical pressure on the base. The intensities on the other horizontal planes are plotted to the same scale in Fig. 341. The dotted lines Hh and Kk mark the limits of the middle third of the width of the dam at each horizontal section, and the centres of pressure A, B, C, etc., will be seen to fall just within these limits. With the reservoir empty, there is therefore no tension acting perpendicular to any horizontal section plane, whilst the maximum vertical compression is that of 6 47 tons per square foot at X.

2. **Reservoir Full**.—Fig. 342 indicates the method of finding the centres of pressure on the horizontal planes 1–1, 2–2, etc., when the reservoir is full and the water pressure acting against the inner face of the dam. Points A, B, C, etc., have been transferred from Fig. 341. Considering any horizontal plane as 3–3, the weight of the mass of

masonry 0033 above 3–3 acts along the vertical through C. P_3 represents the direction of the resultant water pressure acting on the inner face for the depth L3. Producing P_3 to meet the vertical through C at c', the resultant pressure on the plane 3–3 now acts in the inclined direction $c'C_1$. The centre of pressure is thus displaced from C to C_1, being nearer to the outer face than the inner, with the result that the maximum intensity of vertical pressure on any horizontal plane occurs at the outer face instead of at the inner, as was the case with the reservoir empty.

The directions $a'A_1$, $b'B_1$, etc., of the inclined resultant pressures are obtained from the right-hand diagram. The weights of masonry above the several horizontal planes 1–1, 2–2, etc., are set out to scale on the vertical line OM, and the corresponding water pressures along their respective directions drawn from O towards W. In this example the first two water pressures act horizontally, whilst the other four act at a slight inclination with the horizontal. These inclinations have been deduced in the manner referred to in Fig. 334, which is sufficiently accurate for the small batters prevailing on the inside faces of masonry dams. The water pressure

$$P_1 = \frac{62{\cdot}5 \times 5^2}{2 \times 2240} = 0{\cdot}35 \text{ ton acting horizontally at } \tfrac{2}{3}\text{L1 below L}$$

$$P_2 = \frac{62{\cdot}5 \times 25^2}{2 \times 2240} = 8{\cdot}72 \quad \text{,,} \qquad \text{,,} \qquad \tfrac{2}{3}\text{L2} \quad \text{,,} \quad \text{L}$$

$$P_3 = \frac{62{\cdot}5 \times 45 \times \text{L3}}{2 \times 2240} = 28{\cdot}25 \text{ tons acting perpendicular to L3 at } \tfrac{2}{3}\text{L3 below L.}$$

Similarly $P_4 = 58{\cdot}94$, $P_5 = 100\ 8$, and $P_6 = 139{\cdot}51$ tons. For plane 3–3, the masonry weight 0033 = 53·91 tons = OM_3 and water pressure $P_3 = 28{\cdot}25$ tons = OW_3. Join W_3M_3 which gives the direction and magnitude to scale of the resultant inclined pressure on plane 3–3. Hence, through c' draw $c'C_1$ parallel to W_3M_3, to intersect horizontal 3–3 in C_1. Repeating this construction for the five other horizontal planes, the centres of pressure A_1, B_1, C_1, . . . F_1 are obtained. The curve connecting these points is the Line of Resistance for the reservoir full. From the positions of A_1, B_1, C_1, etc., the altered intensities of pressure perpendicular to the horizontal planes may now be calculated.

On the basal section 6–6, the centre of pressure F_1 is 24 ft. from the outer toe Y, the breadth of section is 67 ft., and the resultant vertical pressure at F_1 = 240 tons, being the vertical component of W_6M_6. This vertical component is a little greater—about 8 tons—than the total masonry weight of 231·76 tons, since it includes the downward water pressure on the inside battered face from 2 to 6. The vertical components are obtained by projecting the inclined resultants on to the vertical load line. Hence m being $\frac{24}{67}$, the intensities of vertical pressure on the base are—

$$\text{at X} = \frac{2 \times 240}{67}\left(3 \times \frac{24}{67} - 1\right) = 0{\cdot}534 \text{ ton per square foot}$$

$$\text{and at Y} = \frac{2 \times 240}{67}\left(2 - 3 \times \frac{24}{67}\right) = 6{\cdot}63 \quad \text{,,} \qquad \text{,,}$$

The centres of pressure A_1, B_1, C_1, . . . F_1 again fall just within the line K*k* marking the outer limit of the middle third. With the reservoir full, there is therefore no tension acting perpendicular to any horizontal section plane, whilst the maximum vertical compression is that of 6·63 tons per square foot at Y. Further, since the lines of resistance ABC . . . F and A_1 B_1 C_1 . . . F_1 fall only just within the limits of the middle third, the section is "economical," since with a more liberal width the

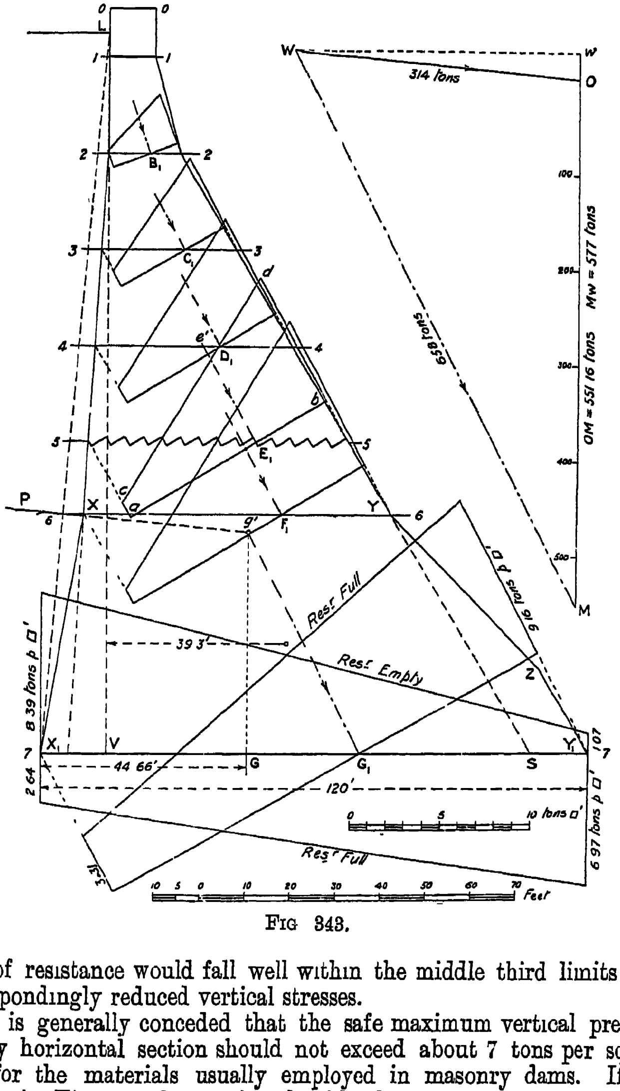


FIG 343.

lines of resistance would fall well within the middle third limits with correspondingly reduced vertical stresses.

It is generally conceded that the safe maximum vertical pressure on any horizontal section should not exceed about 7 tons per square foot for the materials usually employed in masonry dams. If the section in Fig. 341 be continued with the same inside and outside batters to a greater depth than 100 ft. below top water-level, then this

maximum safe pressure of 7 tons per square foot will be realized at a depth of about 110 ft., and for a dam of usual material to be continued for a still greater depth, this depth of about 110 ft. marks the level at which the width of the dam must be further increased in order to keep the maximum vertical pressure on horizontal planes in the lower portion of the dam within the above limit of 7 tons.

Dams not exceeding about 110 ft. in height are often alluded to as "low dams," and those above 110 ft. as "high dams." If, for example, the section in Fig. 341 be continued with the *same* batters to a depth of 150 ft. below water-level, the new width of base is 102 ft., and the centres of pressure, both for reservoir empty and full, still fall within the middle third of the base. The intensities of pressure are, however, 9·36 tons per square foot at inner face when empty, and 9·89 tons per square foot at outer face when filled. These pressures greatly exceed 7 tons, and demonstrate the need for increased width of section at lower levels of the dam.

Section of Dam for 150 ft. Depth of Water.—The method will now be applied to determining a suitable section for the dam down to a depth of 150 ft. In Fig. 343 the previous section is retained down to 100 ft. depth, but below this level the inside and outside batters are increased as shown to give a basal width of 120 ft. It should be noticed the increased outer batters from YZ to ZY_1 cause the middle third of the base X_1Y_1 to move considerably to the right, whilst the additional weight of masonry YSY_1Z has relatively little effect on the position of the c.g. of the total section, which moves very slightly towards the right, as compared with its position if the outer batter were continued from Y to S. The centre of pressure for the reservoir empty will therefore fall outside the middle third of the base unless the inside batter XX_1 be also increased. This causes the middle third to move back somewhat towards the left, and counteracts the effect of the increased outside batters YZ and ZY_1. The additional weight of masonry 6677 = 319·4 tons, and its c g. is 39·3 ft. from the vertical oV.

$$\begin{aligned} \text{Sum of moments of masonry about } oV \text{ down to } 6\text{–}6 &= 4{,}347 \\ \text{Additional moment of } 6677 \text{ about } oV = 319{\cdot}4 \times 39{\cdot}3 &= 12{,}552 \\ \hline \text{Sum of moments down to } 7\text{–}7 &= 16{,}899 \end{aligned}$$

Total masonry weight above 7–7 = 231·76 + 319·4 = 551·16 tons.

Distance of c.g. of 0077 from oV $= \dfrac{16{,}899}{551{\cdot}16} = 30{\cdot}66$ ft., and distance from inner face at $X_1 = 30{\cdot}66 + 14{\cdot}00 = 44{\cdot}66$ ft. Width of middle third $= \frac{120}{3} = 40$ ft. The centre of pressure G for reservoir empty therefore falls 4·66 ft. within the middle third of the base. For the intensities of vertical pressure on the base, $m = \dfrac{44\frac{2}{3}}{120}$ and

$$\text{intensity at } X_1 = \frac{2 \times 551{\cdot}16}{120}\left(2 - 3 \times \frac{44\frac{2}{3}}{120}\right) = 8{\cdot}39 \text{ tons per sq. ft.}$$

$$\text{and intensity at } Y_1 = \frac{2 \times 551{\cdot}16}{120}\left(3 \times \frac{44\frac{2}{3}}{120} - 1\right) = 1{\cdot}07 \quad \text{,,} \quad \text{,,}$$

Water pressure against inner face $= \dfrac{62{\cdot}5}{2} \times \dfrac{150 \times LX_1}{2240} = 314$ tons,

assumed acting at right angles to LX_1 at 50 ft. above the base. Combining OM = 551·16 tons, and OW = 314 tons, the inclined resultant is WM, having a vertical component $wM = 577$ tons. Drawing $g'G_1$ parallel to WM, the centre of pressure for reservoir full falls at G_1, 51 ft. from Y_1, or well within the middle third, $m = \frac{51}{120}$, and intensities of vertical pressure are

$$\text{at } X_1 = \frac{2 \times 577}{120}\left(3 \times \frac{51}{120} - 1\right) = 2{\cdot}64 \text{ tons per square foot}$$

$$\text{and at } Y_1 = \frac{2 \times 577}{120}\left(2 - 3 \times \frac{51}{120}\right) = 6{\cdot}97 \quad ,, \quad ,,$$

The following table gives the vertical pressures on horizontal planes at inner and outer faces as deduced by the above method.

RESERVOIR EMPTY.

Section plane	P	m	b	p_v at inner face	p_v at outer face
	tons		ft		
1–1	6 70	0 5	10	0·67	0·67
2–2	23·78	$\frac{6{\cdot}2}{15{\cdot}5}$	15·5	2·45	0 61
3–3	53·91	$\frac{10{\cdot}2}{29{\cdot}5}$	29·5	3·52	0·14
4–4	102·46	$\frac{15}{43}$	43	4·54	0·22
5–5	169 46	$\frac{20}{57}$	57	5 63	0 31
6–6	231·76	$\frac{23\ 8}{67}$	67	6·47	0 45
7–7	551 16	$\frac{44{\cdot}66}{120}$	120	8 39	1·07

RESERVOIR FULL.

Section plane	P_v	m	b	p_v at inner face	p_v at outer face
	tons		ft		
1–1	6·70	$\frac{4\ 8}{10}$	10	0 59	0 75
2–2	23 78	$\frac{6\ 5}{15\ 5}$	15·5	0·79	2 28
3–3	56 00	$\frac{11\ 5}{29{\cdot}5}$	29·5	0 65	3 15
4–4	106 00	$\frac{16}{43}$	43	0 57	4·36
5–5	176 00	$\frac{20{\cdot}5}{57}$	57	0 49	5·69
6–6	240·00	$\frac{24}{67}$	67	0 54	6 63
7–7	577·00	$\frac{51}{120}$	120	2 64	6·97

Actual Maximum Compression on the Material.—The intensities of vertical compression on horizontal planes as calculated above by the "trapezium rule" do not represent the actual maximum compression on the material of the dam. In Fig. 344 the curved lines represent generally the directions along which the maximum intensity of compression acts at any point in the dam. These lines, called "isostatic lines," or lines of principal stress, commence at right angles to the inner face, or in the same direction as the lines of action of the water pressure, and at the down-stream face are nearly parallel to the outer profile of the dam. The maximum pressure at any point P on the down-stream face will therefore act on some plane PQ perpendicular to the direction of the line of stress passing through P. If the directions of the isostatic lines were closely determinable, the positions of the planes on which the maximum pressure is exerted would be at once known. The directions of these lines of stress cannot, however, be exactly ascertained for any proposed section. That they follow generally the curves indicated in Fig. 344 is borne out both by theory and experiment.[1] The extent to which the dam may be supposed continuous with the rock foundation considerably modifies these directions, at and below the base of the dam.

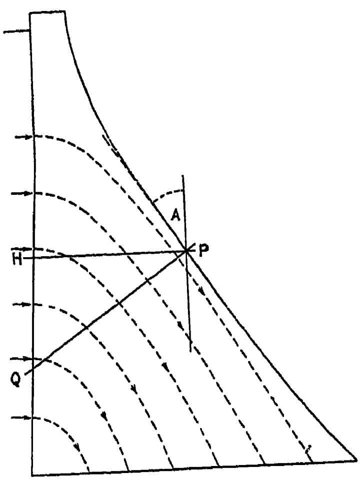


Fig. 344.

Theoretically, if A be the inclination of the down-stream face to the vertical, and p_v the intensity of vertical compression on the horizontal plane at P, as obtained by the preceding method, it may be proved that the maximum intensity of compression at P, acting normally on a plane PQ at right angles to the outer face $= \frac{p_v}{\text{Cos}^2 A}$. The maximum stresses in the dam are sometimes calculated by this formula, which, however, implies that the outermost lines of stress are parallel to the outer face of the dam, which, whilst sensibly true for the upper portion of the dam, is probably considerably at variance with the conditions existing near the toe. Here, wide variations exist in the outline given to the down-stream toe of existing dams, and if the rapidly changing inclination to the vertical be accepted as the governing factor in calculating the maximum pressure, such calculated pressures near the base must be in excess of the existing ones. Applying this in the case of the dam in Fig. 343, the resulting maximum pressures at the outer edges of the horizontal planes 2–2, 3–3, etc., for reservoir full, are as given in the third column of the following table.

[1] *Mins. Proceedings Inst. C. E.*, vol. clxxii. p. 107, and pl. 5.

Horizontal section	p_v tons per sq. ft by trapezium law.	$p_v - \cos^2 A$.	Maximum p, tons per sq ft. calculated from dentilated sections
2–2	2 28	2·44	2·72
3–3	3·15	4·50	4·00
4–4	4·36	6·03	5·61
5–5	5 69	7·87	7 37
6–6	6·63	9·17	8·61
7–7	6·97	9·64	9·16

The maximum intensities of compression at the outer face, especially towards the base, as given by this method, are probably appreciably greater than those actually existing in the dam.

From the results of experiments made with model dams under conditions approximating to those in actual dams,[1] the maximum intensity of pressure occurs at some distance in from the down-stream face and the actual variation of pressure intensity on horizontal sections, instead of following strictly the trapezium law, is indicated more exactly by the ordinates to a curve such as EFG, Fig. 345, instead

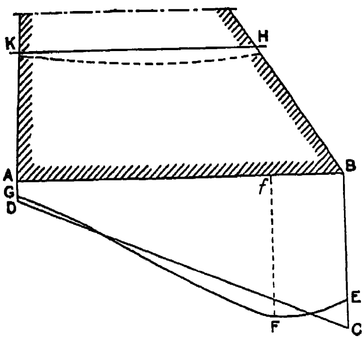


Fig. 345.

Fig 346.

of the straight line CD. This divergence from the variation of pressure intensity as indicated by CD is apparently due to the relative weakness of the acute-angled down-stream toe ABH, which is incapable of resisting a maximum intensity of vertical pressure at B, and yields accordingly, thus throwing the point of application f of maximum pressure intensity some distance in from the down-stream face This further shows the fallacy of the assumption that the material of a dam behaves according to the laws of bending for simple beams, since a plane section such as KH will not remain plane after stressing, but will take up some curved outline, as shown exaggeratedly by the dotted line. It would appear

[1] *Mins. Proceedings Inst. C. E.*, vol. clxxii pp 89, 107.

further that the inherent weakness of a very acute-angled toe might be corrected, and the material disposed with greater economy, by bending in the lower portion of the down-stream face, as in Fig. 346, which is a section of the Ban dam in the south of France. By so doing the toe is, as it were, better held up to its work, with the result that the point of application of the maximum pressure will approach more nearly the outer edge of the base. The original section proposed by Colonel Pennycuick for the Periyar dam in India also exhibits this reversed batter or hump on the down-stream profile, which, in the opinion of Colonel Pennycuick, is a preferable outline to the one actually adopted for the dam.[1]

Another method of estimating the maximum pressures is as follows. In Fig. 343 the inclined resultant pressures acting at points B_1, C_1, D_1, etc., have been transferred from Fig. 342. Any resultant pressure as $e'E_1$ is taken as acting on a dentilated section 5–5, the steps of which are parallel and normal to $e'E_1$. The total effective breadth of the normal faces of the steps is $ab = 49$ ft. The resultant pressure $e'E_1 = 201$ tons, and $bE_1 = 18$ feet, whence $m = \frac{18}{49}$, and the resulting intensities of pressure for reservoir full are 7·37 and 0·84 tons per square foot respectively at outer and inner faces at level 5–5. These are plotted at bd and ac, the intensities for the other sections being shown by the trapeziums plotted on the inclined base lines passing through B_1, C_1, D_1, etc. The maximum values at the down-stream face given by this method are inserted in the fourth column of the preceding table. Prof. Gaudard of Lausanne considers this method should give an excess of security.

The maximum stresses in the profile in Fig. 343 are well within the safe compression which may be put upon the materials of which modern dams are built, and keeping in view other causes of stress, such as variable temperature, partial penetration of water into the interior, lateral bending, etc., the extent of which cannot be approximately estimated, any closer estimate of the direct compression due to the water pressure and weight of masonry alone is of little importance.

Figs. 347 to 356 show the profiles of several of the more important earlier dams.

Vyrnwy Dam.—The Vyrnwy dam[2] (Fig. 347), built in 1881–90 for the water supply of Liverpool, has a length of 1172 feet and a maximum height of 144 feet from the deepest point of foundations to the crest. The maximum depth of water against the inner face of the dam is 84 feet. It is an overflow dam (see Fig. 337), straight in plan, and built of rubble concrete. Special care was taken to ensure the concrete having a high specific gravity as well as to render the masonry as watertight as possible. A system of drains formed along the beds of rock at the foundation level, collect the water from springs beneath the dam and conduct it into a small drainage tunnel formed within the dam, from which it is discharged on the down-stream face. This precaution was taken to avoid any possible upward pressure on the base of the dam. The maximum pressure on the material is approximately $7\frac{3}{4}$ tons per square foot.

[1] *Mins. Proceedings Inst. C. E.*, vol. cxv. p. 87. *Ibid.*, vol. clxxii. p. 146.
[2] *Ibid.*, vol. cxxvi.

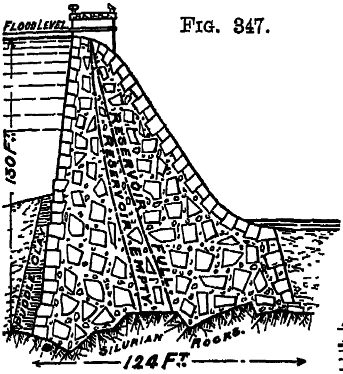


FIG. 347.

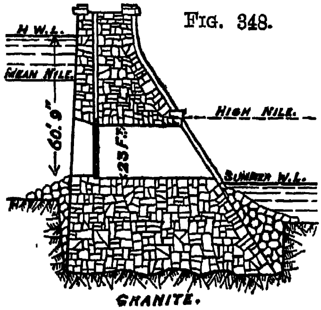


FIG. 348.

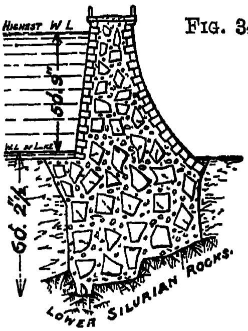


FIG. 349.

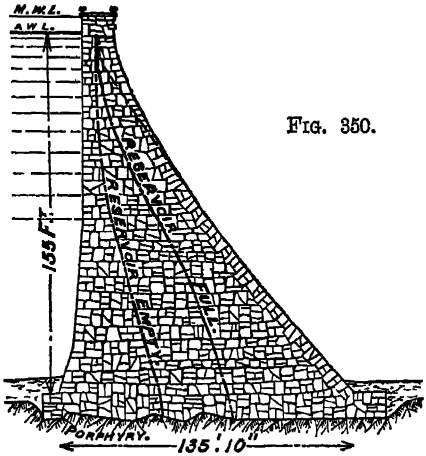


FIG. 350.

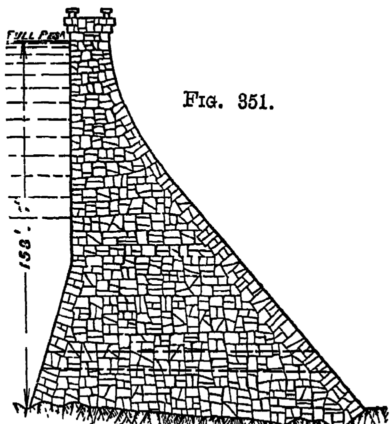


FIG. 351.

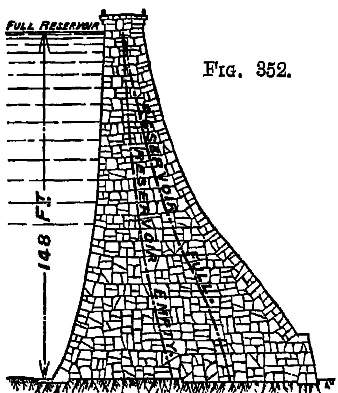


FIG. 352.

SCALE $\frac{1}{1000}$.

FT 50 0 50 100 150 FT.

Assuan Dam.[1]—The Assuan dam (Fig. 348) built across the river Nile is 6400 feet in length and contains 180 sluice-ways at different levels for discharging the impounded water. The sluices are opened in April, May, and June, the issuing water supplementing the flow of the Nile during the dry season, when in years of small discharge the natural flow is inadequate for the irrigation of the country below the site of the dam. The dam was raised during 1907–1912.

The dam is founded on granite rock and is constructed of rubble granite faced with coursed rubble masonry laid in cement mortar. The maximum height above foundation level is 127 feet, and the head of water above the bottom of the lowest sluice-ways is $60\frac{2}{3}$ feet. A maximum pressure of 5·8 tons per square foot on the masonry at the upstream face, with the reservoir empty, has been provided for, and in the event of the water rising to the level of the roadway, the maximum pressure on the down-stream face, with reservoir full, would be 4 tons per square foot.

Thirlmere Dam[2] (Fig. 349).—This dam has been constructed across the outlet of Thirlmere for raising the level of the lake a maximum height of 50 feet, the impounded water being used for the supply of Manchester. The dam consists of two portions 310 and 520 feet long, divided by a ridge of rock. The maximum depth is 110 feet. Fig. 349 shows the section at this maximum depth, and together with those of the Vyrnwy and New Croton dams indicates to what a great extent the foundations may add to the height and cost of dams when a considerable depth of pervious material has to be excavated in order to reach a sound rock foundation. The dam is of masonry-faced rubble concrete, with a roadway 16 feet wide along the top.

Chartrain Dam[3] (Fig. 350).—Built in 1888–92, the Chartrain dam impounds water for the supply of the town of Roanne in the Loire valley, in France, and also assists the controlling of floods in the district below the dam. It is curved to a radius of 1312 feet in plan and has a length of 720 feet, with a capacity of 990 million gallons It is founded on porphyry rock, has a maximum height of 177 feet, and retains a maximum head of water of 151 ft. 9 ins. The dam is constructed of rubble granite masonry in lime mortar, the inner face being rendered with 1 to 1 cement mortar for a thickness of 1·2 inches. The maximum pressure on the masonry at the inner toe, with the reservoir empty, is 9 tons, and at the outer toe, with reservoir full, 9·4 tons, per square foot.

Villar Dam, Spain[4] (Fig. 351).—Built in 1870–78 for the increased water supply of Madrid. The length is 546 feet, and in plan the dam is curved to a radius of 440 feet. The maximum height is 170 feet. For a length of 197 feet from one end the dam is utilized as a waste weir over which the roadway, 14 ft. 9 ins. wide, is carried by an iron bridge of twelve spans. The dam is constructed of rubble masonry, the maximum pressure on which is estimated at 6·5 tons per square foot.

[1] *Mins. Proceedings Inst. C. E.*, vol. clii. p. 78. *Ibid.*, vol. cxciv. p. 249.
[2] *Ibid.*, vol. cxxvi p. 3.
[3] *Les Réservoirs dans le Midi de la France*, Marius Bouvier, pp. 18 to 21. Congrès International de Navigation intérieure, Paris, 1892.
[4] *Mins. Proceedings Inst. C. E.*, vol. lxxi. p. 379.

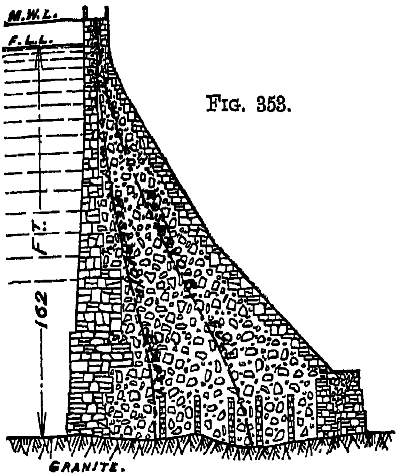


FIG. 353.

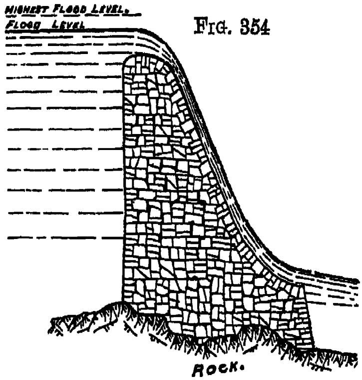


FIG. 354

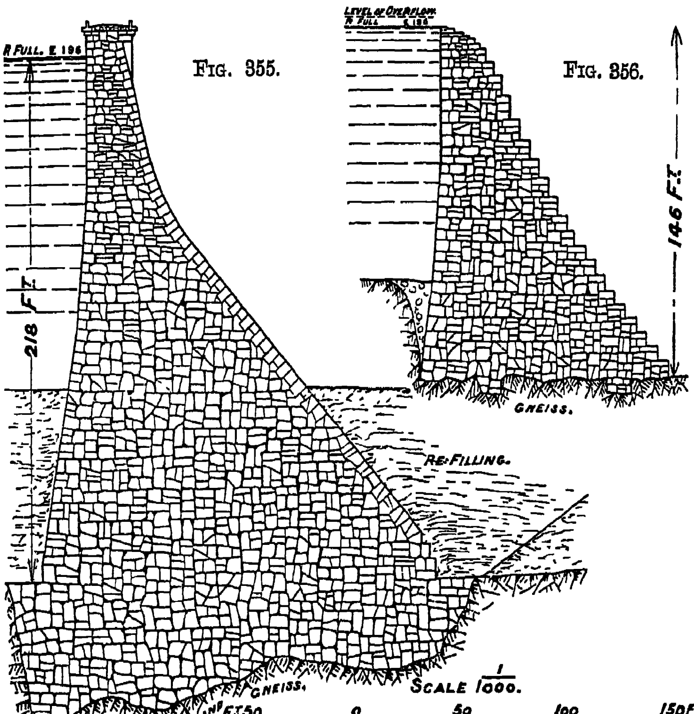


FIG. 355.

FIG. 356.

SCALE $\frac{1}{1000}$.

FT. 50 0 50 100 150 FT.

Ban Dam, France[1] (Fig. 352).—Erected across the River Ban in the lower Rhone valley in 1866–70. The length is 541 feet and the dam is built with a convex face up-stream to a radius of 1325·6 feet, in plan. It is built of rubble masonry founded on granite. The maximum head of water retained is 148 feet, with a capacity of 407 million gallons. The maximum pressure on the masonry is estimated to be 10 tons per square foot.

Periyar Dam, India[2] (Fig. 353).—This dam was erected in 1888–97, for impounding water in the valley of the Periyar, which water is diverted through a tunnel into the neighbouring valley of the Vaigai and there utilized for irrigation purposes. The dam is constructed of concrete faced with masonry and has a maximum height of 173 feet. The width at the base is 143 feet, with a slope of 1 to 1 from the top of the outer toe to a point about halfway up the outer face. The outer profile then becomes steeper, terminating with a top thickness of 12 feet carrying a roadway. The overflow level is 162 feet above the base, and the estimated flood level 11 feet higher.

La Grange Dam, California (Fig. 354).—This dam, built in 1891–94, has a length of 310 feet and is curved to a radius of 300 feet. The maximum height on the up-stream face is 125 feet, and width of base 90 feet. It is an overflow dam, and its outer profile was designed to coincide with the slope of the overflow water when flowing 5 feet deep over the crest of the dam. The cross section corresponds very closely with that of the waste-weir of the New Croton dam (Fig. 356) for a similar height, but the outer toe is made much stronger to withstand the shock of the falling flood water, which has already risen to a height of 12 feet above the crest of the dam. The La Grange Dam is built of uncoursed rubble masonry, and the water is employed for irrigation purposes.

New Croton Dam, New York[3] (Fig. 355).—The New Croton masonry dam constitutes a portion only of the dam across the Croton River for augmenting the water supply of New York. It is remarkable for its great height from foundation to crest. The maximum height above the deepest point of the foundation is 238 feet, although the maximum head of water impounded is only about 140 feet. The dam is of rubble masonry faced with ashlar above the level of the ground. The waste weir, Fig. 356, 1000 feet long, is built as a continuation of the main dam and curves round until at right angles with the axis of the dam, thus facing the side of the valley and diminishing in height from 150 feet to 13 feet. The great depth to which the foundations of the dam have been carried was necessitated by the existence of 80 feet of soft stratum overlying the rock, whilst a further maximum depth of about 57 feet of unsound rock had to be removed, in order to reach a sufficiently solid rock bed on which to found the dam.

Intensity and Distribution of Shear Stress.—During recent years considerable attention has been drawn to the question of shear stress in masonry dams. An accurate estimate of the shear stress would be possible only if the actual variation of the compressive stresses were

[1] *Annales des Ponts et Chaussées*, 1875, 1st Trimestre.
[2] *Mins. Proceedings Inst. C. E.*, vol. cxxviii p 140.
[3] *Transactions. Am. Soc. C. E.*, 1900, vol. xliii.

known. A brief examination, however, under an assumed distribution of vertical compressive stress probably worse than obtains in an actual dam will show that the shear stress does not exceed what may be safely resisted by the material. Adopting the section in Fig. 342, the intensities of vertical compression at inner and outer faces on plane 6–6 for reservoir full, are respectively 0·54 and 6·63 tons per square foot. Assuming uniform variation from X to Y, Fig. 357, the intensity at intermediate points will be given by the ordinates of the trapezium 66XY. Dividing the profile of the dam into vertical sections by planes *a–a*, *b–b*, etc., 10 ft. apart, the total vertical shear on *ah* = the upward pressure against *a*6, less the weight of the masonry *ah*6

$$= \frac{5{\cdot}72 + 6{\cdot}63}{2} \times 10' - 5{\cdot}39 = 56{\cdot}36 \text{ tons.}$$

Total shear on *b–b*

$$= \frac{4{\cdot}81 + 6{\cdot}63}{2} \times 20' - 4 \times 5{\cdot}39$$

$$= 92{\cdot}84 \text{ tons.}$$

Similarly total shear on	*c–c* =	109·44	tons
„ „	*d–d* =	106·16	„
„ „	*e–e* =	82·75	„
„ „	*f–f* =	32·00	„
„ „	*o*V =	8·65	„

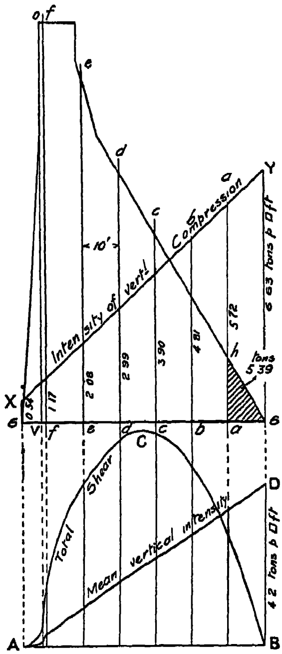


FIG. 357.

These values are plotted vertically to scale at ACB. Dividing each total shear by the vertical depth of the corresponding section, the mean shear on the vertical planes is obtained. Thus—

$$\text{Mean shear on } a\text{–}a = \frac{56{\cdot}36}{16{\cdot}1} = 3{\cdot}5 \text{ tons per square foot}$$

$$\text{,, ,, } b\text{–}b = \frac{92{\cdot}84}{32{\cdot}2} = 2{\cdot}88 \text{ ,, ,,}$$

$$\text{,, ,, } c\text{–}c = \frac{109{\cdot}44}{48{\cdot}3} = 2{\cdot}27 \text{ ,, ,,}$$

$$\text{,, ,, } d\text{–}d = \frac{106{\cdot}16}{64{\cdot}4} = 1{\cdot}65 \text{ ,, ,,}$$

$$\text{,, ,, } e\text{–}e = \frac{82{\cdot}75}{89{\cdot}0} = 0{\cdot}93 \text{ ,, ,,}$$

$$\text{,, ,, } f\text{–}f = \frac{32{\cdot}00}{105{\cdot}0} = 0{\cdot}30 \text{ ,, ,,}$$

$$\text{,, ,, } oV = \frac{8{\cdot}65}{75{\cdot}0} = 0{\cdot}12 \text{ ,, ,,}$$

These values are plotted vertically above AB, giving the line AD, and the intensity of shearing stress at any point being equal on horizontal and vertical planes, the curve AD indicates the variation of shear intensity along the horizontal plane 6–6. The intensity of shear on plane 6–6, *i.e.* on the 67 ft. base of the 100 ft. dam, is therefore zero at the inner face, and increases to a maximum of 4·2 tons per square foot, or practically twice the *mean* intensity, at the outer face.

Note that the *mean* intensity of shear on the base = horizontal water pressure above plane 6–6 ÷ 67 sq. ft. = 139 tons ÷ 67 = 2·08 tons per square foot. Since the vertical pressure intensity is not quite accurately represented by the trapezium 66XY (see Fig. 357), the maximum shear will be less than 4·2 tons per square foot. In an actual dam there is little doubt that the distribution of shear stress on horizontal planes in the upper three-fourths of the height closely agrees with that indicated by a curve such as AD. Near the base, however, the intensity and distribution of shear may be greatly modified, according to the extent to which the base of the dam is intimately bonded with the neighbouring rock foundation. In Fig. 358, if the dam be supposed firmly bonded to the mass of rock R on its up-stream face, the resistance of this mass to the horizontal displacing water pressure P, will set up considerable tension across the vertical plane AV. So long as this tension does not cause rupture in the neighbourhood of AV, proportionately large shearing stresses will be created on the basal plane at A, and for some distance to the right of A. For the dam acting independently of the rock mass R, the shear distribution on the base would be represented by a curve AB, similar to AD in Fig. 357. The effect of the shear set up at A by continuity between the dam and rock foundation R will be to tend to equalize the shear intensity on the base, so that it will be represented by some such curve as DE. If the rock on the up-stream face be much fissured or jointed, it will offer little or no resistance to tension at A, and the maximum shear BC on the basal section may approach twice the mean shear.

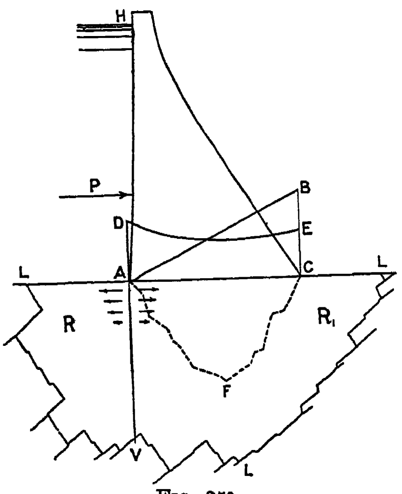


Fig 358.

If the dam be securely bonded with massive and unjointed rock, the shear on the base will approximate to the mean shear. It is doubtful if, on the majority of actual sites, much tensional resistance may be expected from the rock foundation for any considerable distance on the up-stream face, since the continuity must be interrupted sooner or later by some system of bedding and joint planes as LLL, even in the

soundest stratified rock. Although it has often been stated that no apparent cracks have developed at the inner toe in existing dams, it should be remembered that an exceedingly fine crack is sufficient to destroy the stress connection across the plane AV, and that such a crack would not be readily discernible by a diver, whilst the emptying of the reservoir for purposes of inspection would tend to close up such possibly existing cracks, by reason of the maximum intensity of compression reverting to the inner toe when the reservoir is empty.

Relatively few dams are founded at base level, the majority penetrating more or less deeply into the rock as at AFC. (See also Figs. 347 to 356.) The joint along AFC cannot but be regarded as a possible plane of weakness towards the up-stream face, and in the event of its opening for some distance from A towards F, the entry of water under the pressure due to the head in the reservoir would introduce a new and dangerous force beneath the dam. For this reason it appears desirable to continue the inner face *vertically* into the rock for some distance, since the penetration of water into a vertical crack is not of great moment since its horizontal pressure is resisted by the reaction of the rock mass R_1 on the down-stream face.

The total horizontal water pressure against the 150-feet dam in Fig. 343 is 314 tons, and the *mean* shear on the 120-feet base $= \frac{314}{120} = 2{\cdot}61$ tons per square foot. The maximum shear would probably amount to 4·5 or 4·6 tons per square foot under unfavourable conditions of bond with the surrounding rock. This, however, represents an outside figure, since recent research appears to indicate that the shear intensity is not uniform over the vertical planes *a–a*, *b–b*, etc., of Fig. 357, but is greater in the upper portion of the dam than near the base. Until more definite information is obtained on this point, however, the existence of shear intensities of the above magnitude must be regarded as possible.

The following appears to be the concensus of expert opinion on the subject of masonry dams at the present time (1923).

1. If a dam be designed according to the middle-third theory, so that the maximum vertical pressure on any horizontal plane does not exceed about 7 tons per square foot, corresponding with a possible maximum pressure on certain inclined planes of from 10·5 to 11 tons per square foot, and the lines of resistance for the reservoir empty and full be contained within the middle-third limits, the actual stresses in the dam will not exceed the above values.

2. The maximum shearing stresses accompanying these compressive stresses will be safely within the shearing resistance of the materials employed.

3. That tensile stress may occur in the region of the inner toe, the maximum intensity of which may exceed the average intensity of shearing stress on the base, but will be governed by the strength of the bond between the dam and foundation on the up-stream face. Such tension, when existent, will act generally in directions horizontal or slightly inclined with the horizontal. On other than a very sound foundation, it appears, however, impossible for such tension to exist.

The safety of the middle-third method of treatment is further substantiated by reference to the records of failures of actual dams. With the exception of those failures directly due to subsidence or sliding

on faulty foundations, practically all others have been directly traceable to the violation of one or other of the above-stated conditions, and no failure of a high dam has been recorded where those conditions have been satisfactorily complied with. It may be remarked that many gravity dams have been given a slight curvature in plan with the convex face up-stream, principally with a view to strengthening the dam against longitudinal bending. Such dams do not, however, come within the category of "arched dams," since they all possess "gravity" profiles.

The weight per cubic foot of the material employed in modern dams varies from 142 to 160 lbs. The Vyrnwy dam, built of heavy clay slate from the Lower Silurian formation, and in which special care was taken to obtain very dense concrete, has a specific gravity of 2·595. The Burrator dam, built of granite rubble concrete weighs 150 lbs. per cubic foot, the granite alone weighing 165 lbs. per cubic foot.

Within recent years, since about 1908, and especially within the last twelve years, many concrete dams have been built of greatly increased dimensions. The following table relates to some of the more important recent dams :—

Location of dam	Constructed.	Max. height above foundation
Periyar, India	1888 to 1897	173 feet.
New Croton, New York . . .	1892 „ 1906	238 „
Lake Cheeseman, Colorado . . .	1900 „ 1904	227 „
Wachusett, Massachusetts . . .	1900 „ 1906	207 „
Cataract Dam, N. S. Wales . .	1902 „ 1908	192 „
Shoshone, Wyoming	1903 „ 1910	328 „
Mauer, Germany	1904 „ 1912	203 „
Roosevelt, Arizona	1905 „ 1911	275 „
Burrinjuck, N. S Wales. . . .	1906 „ 1911	240 „
Olive Bridge, New York	1908 „ 1913	251 „
Elephant Butte, New Mexico . .	1910 „ 1916	275 „
Kensico, New York	1911 „ 1915	300 „
Arrowrock, Idaho	1912 „ —	351 „
Hetch Hetchy, San Francisco . .	1919 „ —	430 „
Hartebeestpoort, South Africa . .	1920 „ 1923	193 „

In several of the most recent of these dams, the method of construction has been considerably modified. The present American practice is to build the dam in independent sections of about fifty to ninety feet length, in order to allow of variations in length due to expansion and contraction and shrinkage of the concrete. By this means the vertical transverse cracks which have appeared in nearly all monolithic dams, are avoided. Special provision has to be made to prevent leakage across the transverse joints. The vertical end faces of the independent sections are carried up in vertical rectangular grooves G, as in Fig. 359, so that adjacent sections are interlocked the one with the other, thus preventing any tendency to relative transverse movement or unequal settlement. One end of each section is covered with a suitable plastic waterproof material or lubricated, to prevent adhesion with the adjacent

section. Leakage across the joint is prevented by a vertical copper strip S having its edges embedded in the concrete on opposite sides of the joint. Fig. 360 shows a horizontal section of this joint slightly opened

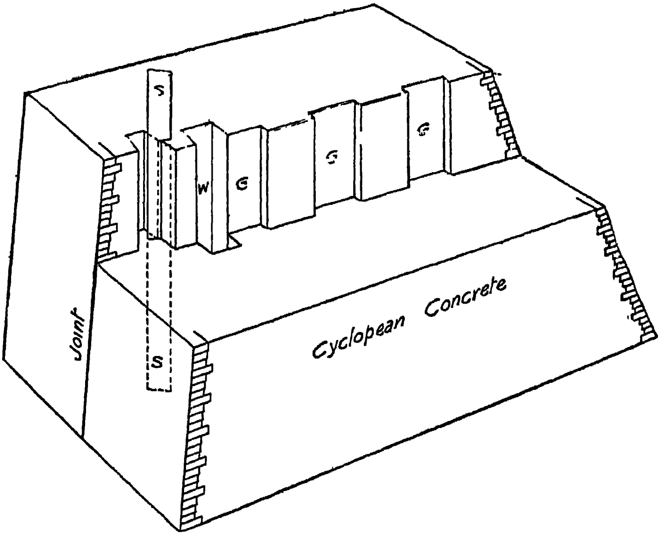


FIG. 359.

by contraction, a small width B, at the centre of the copper strip SS, being flexible enough to accommodate small movements of the adjacent sections in the directions of the arrows. Reinforcing rods R, R,

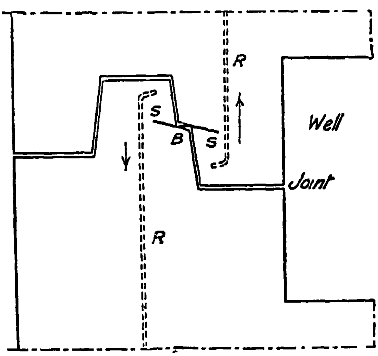


FIG. 360.

strengthen the interlocking tongues of the concrete. Immediately on the downstream side of the copper strip in each joint, a vertical or nearly vertical drainage well W is provided, large enough to permit access for inspection. These wells communicate at their lower ends

near the level of the bases of the expansion joints, with an inclined drainage and inspection gallery running longitudinally through the lower portion of the dam and communicating by a transverse tunnel D, Fig. 361, with the downstream face. Other wells about 18 inches in diameter, 12 feet apart and 15 to 20 feet from the upstream face of the dam, are also provided. The function of these vertical wells is to intercept any seepage penetrating the upstream face of the dam, Fig. 361. Such seepage may be considerable in the case of dams faced with

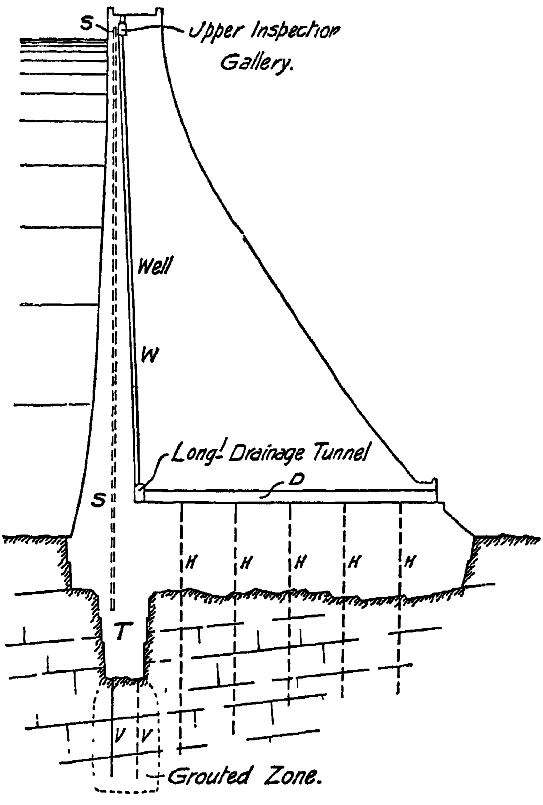


Fig 361.

concrete blockwork. These wells may advantageously be lined with lean concrete blocks to facilitate the collection of seepage. In the Elephant Butte dam across the Rio Grande in New Mexico, which retains 200 feet depth of water, two rows of vertical drainage holes, from 8 to 12 inches diameter and spaced from 4 to 6 feet apart, extend throughout the length of the dam from crest to foundation, a few feet behind the upstream face, and deliver into a longitudinal drainage tunnel near the base. This tunnel varies in cross-section from 5 ft. × 6 ft. 6 ins. near the ends to 6 ft. × 7 ft. under the centre.

Modern practice in regard to foundations is to sink a cut-off trench T, Fig. 361, into the rock 20 to 30 feet deep and 15 to 20 feet wide. This trench should preferably be channelled or quarried to avoid shaking the sides by heavy blasting. According as the exploratory borings will have indicated the extent to which the rock may be seamed and fissured, it may be necessary to drill two rows of staggered holes V, for a further depth of 30 to 40 feet and to grout these under air pressure up to 200 pounds per square inch. Such holes may be 3 to 6 feet apart and $2\frac{3}{8}$ inches diameter. The object of grouting under pressure is to fill the joints, fissures and seams intersected by and communicating with the boreholes. Good results may be expected by using grout consisting of one part cement and from three to five parts of water by volume. This treatment aims at producing a tight or impervious zone indicated by the dotted line, near the upstream face of the foundation. Any water which may subsequently pass beneath or through this grouted zone should be allowed to escape freely to the downstream side in order to avoid any accumulation of uplift pressure beneath the dam. The free escape of such water may be aided if necessary by drainage or relief holes H communicating with the main drain D. Several recent dams have been designed with a view to drilling such relief holes in the future, if considered necessary. In some cases it may be desirable to drill and grout the whole area of the foundation. In such cases care must be taken not to create a tighter and more impervious zone under the downstream face with consequent accumulation of hydrostatic pressure under the upstream portion of the foundation. This system of preparing a rock foundation carefully and intelligently carried out should adequately prevent the accumulation of any appreciable uplift due to water pressure beneath the dam.

Arrowrock Dam.– The Arrowrock dam is the highest completed dam at the present date (1923). Its maximum height above lowest level of foundation is 351 feet, the width at the lowest point being 238 feet. It dams the Boise River in Idaho, U S.A., and is employed for irrigation and power purposes. Its axis is curved in plan to a radius of 662 feet. The length is 1060 feet. It is built in eight sections with vertical contraction joints 150 feet apart, and a system of vertical drains. 6 inches wide spaced 10 feet apart, communicate with a longitudinal drainage tunnel from which 12-inch vertical drains 10 feet apart lead down into the rock beneath the foundation and so prevent the accumulation of pressure beneath the dam.

Hetch Hetchy Dam.—The Hetch Hetchy Water Supply Project is intended ultimately to supply San Francisco and the adjacent metropolitan area with 400 million gallons of water per day, and also to develop 200,000 hydro-electric horse-power.

The water is to be diverted from the Tuolumne River in the Sierra Nevada Mountains, about 150 miles from San Francisco. The Hetch Hetchy reservoir, the principal one of the project, will have a capacity of 113,500 million gallons, with its top water level 3800 feet above sea-level. The dam, Fig. 362, has a gravity section curved in plan to a radius of 700 feet at the upstream face.

Its maximum height above lowest point of foundation will be

ultimately 430 feet, with a profile as indicated by the dotted outline. It is of cyclopean concrete, 1 : 3 : 6 generally, with 1 : $2\frac{1}{2}$: 5 concrete in foundation, cut-off trench, upstream face and downstream face beneath the overflow section. The dam was commenced in 1919. It is being built in sections 97 ft. 6 ins. long with contraction joints sealed by bent sheet copper water stops S. The dam at present is to be carried up to a height of 344 feet as shown by the full-lined profile in Fig. 362, with a length on crest of 600 feet. In this stage its capacity will be 67,000 million gallons. Drainage wells D, 15 inches square,

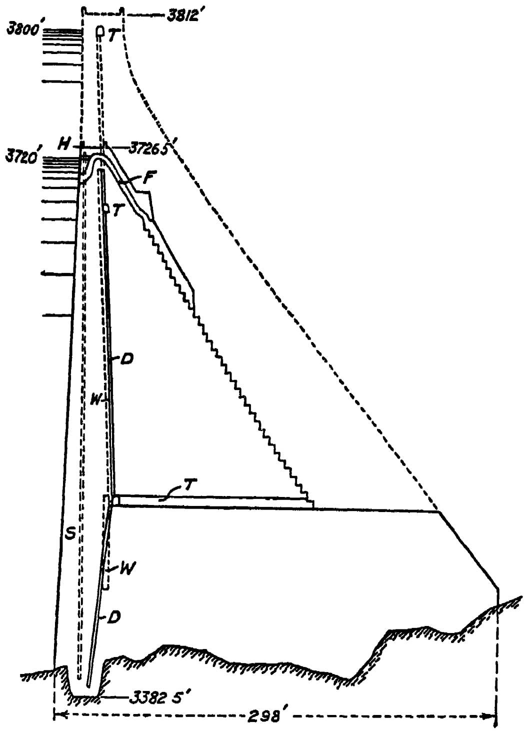


Fig. 362.

built with porous concrete blocks, and inspection wells W together with inspection and drainage tunnels T, are provided. The initial dam will have a siphon overflow F, in eighteen sections, having a total length of clear openings of 180 ft. 6 ins. Each section will be 8 feet high at the upstream face, tapering to 4 feet at the crest of the siphon, with two air vents each 12 by 24 inches. The outer face of the dam is stepped to facilitate bonding with the future addition. When the dam will be raised to its ultimate height of 430 feet, with crest length 900 feet, the siphon overflows will be stopped and replaced by an overflow weir at one end of the raised dam. The horizontal joint at H will also be

stopped by a sheet copper water stop. The following working pressures are being allowed in the design :—

Pressure normal to joint, upstream toe,

Reservoir empty, 25 tons per square foot.
Reservoir full, 16 „ „

Maximum pressure in section under worst combination of conditions and lasting for comparatively short periods of time, 27·5 tons per square foot.

A detailed and fully illustrated account by Mr. M. M. O'Shaughnessy, M.Am.Soc.C.E., of the progress of this scheme to date (1922), is published in the *Transactions* of the American Soc. C. E., Volume lxxxv., to which account the authors are indebted for the above particulars.

Masonry and Concrete Arches.

Arches of stone, brickwork or concrete are generally classed as masonry arches. They differ from steel arched structures in two important respects. 1. The dead load bears a greater ratio to the useful or live load and is usually less uniformly distributed. 2. Little or no tension may be permitted in the material. Arches of small span are seldom designed from first principles, since they are simply repetitions of types of well-established proportions, and the arch thickness may safely follow some empirical rule. Moreover, small span arches usually have a very large margin of safety. In the case of large span arches, more scope exists for the application of economical principles of design, and their dimensions are more carefully proportioned to the existing stresses.

Disposition of Load on Arches.—The spandril spaces of small span or very flat arches are usually filled up with earth, concrete or masonry, to the level of the road or railway to be carried. Fig. 363, shows the

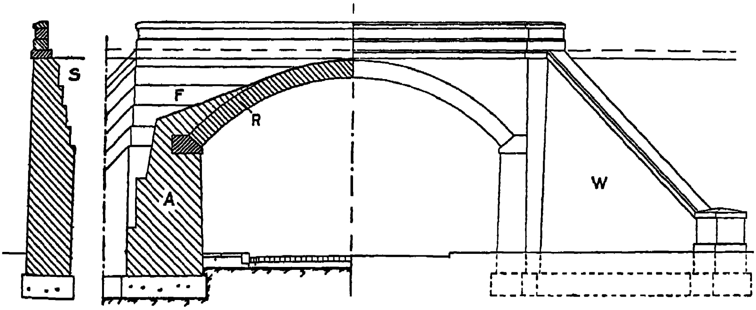


Fig. 363.

general construction of arched bridges of spans up to about 30 or 35 ft. The masonry or concrete of the abutment A is finished to a slope tangent to the back of the arch ring R, and the remainder of the spandril F filled with earth. S is a section of the wing-wall W. This

construction if adopted for large span arches would impose an excessive dead load upon the arch. The spandril space F is therefore left hollow, and the upper platform carried on jack arches J, turned between longitudinal bearing walls as in Fig. 364, or by a series of transverse arches

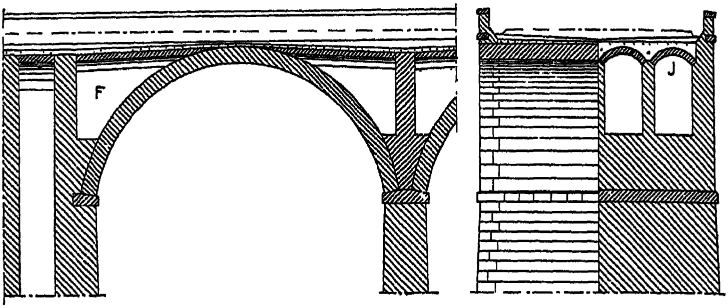


Fig. 364.

resting on piers standing on the back of the main rib, Fig. 365, which illustrates the 277-ft. arch at Luxembourg.[1]

The *Pont Adolphe* at Luxembourg, completed in July, 1903, has a span of 277 ft. 9 in., and a rise of 101 ft. 6 in. It consists of two separate arches in masonry, 18 ft. 6 in. wide, built side by side, having their axes 36 ft. 11 in. apart. The intervening space is bridged over by a reinforced concrete platform, the width between parapets being 52 ft. 6 in. The thickness of the arch at the crown is 4 ft. 8½ in., and at the joint where the greatest tendency to rupture occurs, the thickness is 7 ft. 10¼ ins. The masonry of the arch ring has a crushing strength

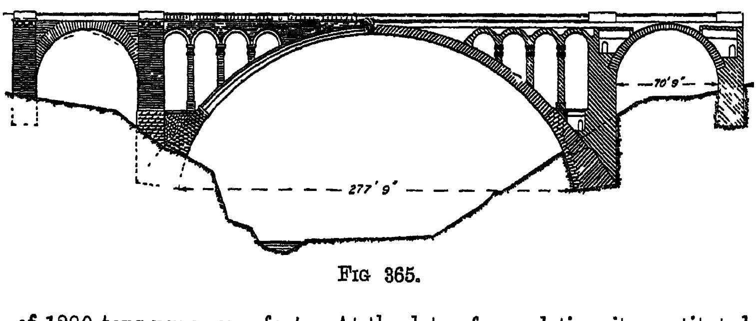


Fig 365.

of 1280 tons per square foot. At the date of completion, it constituted the largest existing masonry arch. Its span has, however, since been exceeded by that of the masonry arch at Plauen, in Saxony, of 295 ft. 2½ in. span and 59 ft. rise. The thickness at crown is 4 ft. 11 in. The width between parapets is 52 ft. 6 in., and the spandrils, which are hollow, contain a system of transverse and longitudinal vaulted spaces. The masonry has a crushing resistance of 1670 tons per square foot. This is at the present time (1923), the largest masonry arch in the

[1] *La Revue Technique*, May 25, 1904.

world. The Luxembourg arch was designed by M. Séjourné, and the Plauen arch by Herr Liebold. The largest masonry arch in England is the Grosvenor bridge over the river Dee at Chester, the span of which is 200 ft.

This type of construction is more generally followed on the continent, whilst in English practice the hollow spandril spaces are masked by continuous head or spandril walls. The magnitude and distribution of the load may be closely estimated for any proposed outline of arch when the arrangement of the superstructure has been decided. It remains, then, to ascertain whether such load may be safely carried by the outline of arched rib adopted, or whether the proposed design requires modification.

Reduction of Actual Load to Equivalent Load of Uniform Density.—It is convenient to convert the actual load consisting of varied materials into an equivalent load of uniform density equal to that of the material of the arch ring.

In Fig. 366 suppose the arch ring to be of masonry weighing 150 lbs. per cubic foot, the concrete backing B, 140 lbs., earth filling F,

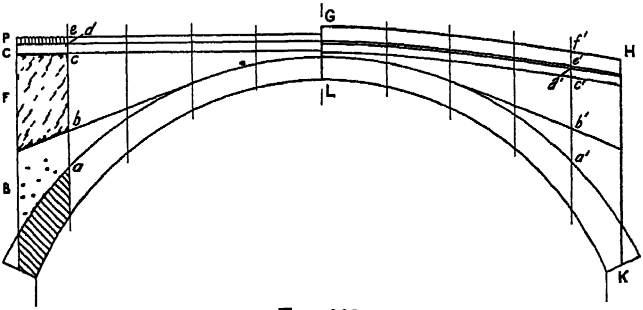


FIG 366.

100 lbs., concrete C, 140 lbs., and wood pavement P, 40 lbs. per cubic foot. Further, suppose that a live load of 200 lbs per square foot of roadway is to be provided for. Draw several verticals as *ae*, cutting the layers of materials in points *b*, *c*, *d*. To avoid confusion of lines the equivalent load area is drawn on the *right* of the centre line. $a'b' = \frac{140}{150}$ of *ab* gives the depth of material of 150 lbs. density required at A to equalize the depth *ab* of 140 lbs. density. Similarly for the other materials, $b'c' = \frac{100}{150}$ of *bc*, $c'd' = \frac{140}{150}$ of *cd*, and $d'e' = \frac{40}{150}$ of *de*. Repeating the construction for each vertical, the equivalent load area is obtained, each square foot of which represents 150 lbs. of load per foot width of the arch. The live load will be represented by an additional layer of vertical depth $e'f' = \frac{200}{150}$ or $1\frac{1}{3}$ ft. to scale. This equivalent load area GHKL may be conveniently subdivided to give the loading on short segments of the arch ring.

Line of Resultant Pressures.—Let *abcd*, Fig. 367, be the equivalent load area for one half of a proposed arch under symmetrical loading, deduced as above. Dividing *abcd* into a number of panels *b*–1, 1–2, 2–3, etc., by vertical lines 1, 2, 3, the centres of gravity of these panels, marked by the small circles, may be readily obtained, each panel

approximating closely to a trapezium in outline. If the right-hand half of the arch be supposed removed, the left-hand portion *cd* may be kept in equilibrium by the application of a horizontal thrust T applied at the crown. Taking moments round the springing point *d*, and calling *r* the rise of the arch—

$$T \times r = W_1 d_1 + W_2 d_2 + \ldots W_6 d_6,$$

from which T may be obtained. Set out the loads W_1, W_2, up to W_6 to any scale on a vertical line Ow_6 and the horizontal thrust T to the same scale at OP. Join P to $w_1, w_2 \ldots w_6$. The figure OPw_6 constitutes a *polar diagram*, having P as the *pole.* Produce T to cut the vertical through W_1 and continue thence a line parallel to Pw_1 to cut the vertical through W_2, a second line parallel to Pw_2 to cut vertical W_3 and so on, until the last line drawn parallel to Pw_6 emerges from the arch ring at *d*. The polygonal line from T to *d*, which would approximate more nearly to a curve if the number of panels were increased, is called the **Line of Resultant Pressures, Line of Resistance or Linear Arch** for the loads, span and rise here assumed.

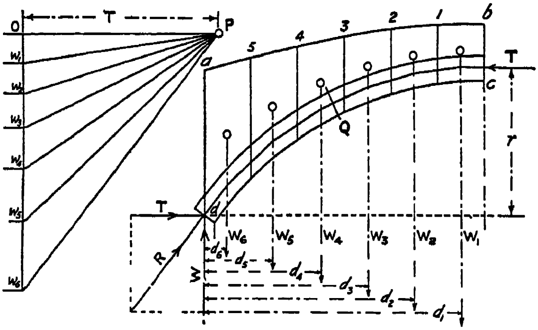


FIG. 367.

Its direction indicates the direction of the resultant pressure on any section of the arch, and the *magnitude* of the pressure on any section Q is obtained from the length of the corresponding parallel ray Pw_3 of the polar diagram, measured to the same scale as the loads and thrust T. The emergent line at *d* gives the direction of the inclined thrust against the abutment, the magnitude being Pw_6. The radial lines Pw_1, Pw_2, etc. of the polar diagram, which determine the thrust at various points along the arch, all have the same horizontal component OP = T, so that the *horizontal* thrust at any point in the arch is constant, and the reaction R at the abutment is compounded of a horizontal thrust T and a vertical reaction W equal to the sum of the loads $W_1, W_2, \ldots W_6$.

It is necessary for the design of an arch to determine first the line of resultant pressure due to the proposed load, span, and rise. It is obvious that for a symmetrical load the right- and left-hand halves of the complete line of resistance will be similar, and that only one-half need be drawn, as in Fig 367. If the loading be unsymmetrical, as, for instance, when one half of the span carries the dead load only, and

the other half both dead and live load, the line of pressures will be also unsymmetrical, and the thrust at the crown will no longer act horizontally. As this condition of loading is the one which, practically speaking, most severely stresses the arch, it is customary to examine any proposed design, (1) under this condition of loading, and (2) when the arch carries both dead and live load over the whole span. As the live load on masonry arches bears a relatively small ratio to the dead load, the former condition of loading is usually found to create the maximum stresses in the material.

A line of resultant pressure passing through three fixed points A, B, and C (Fig. 368), for any system of loads, may be drawn as follows.

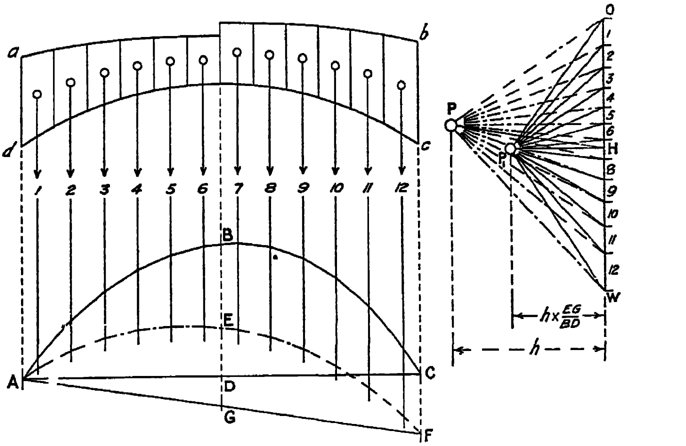


FIG 368.

Let *abcd* be the equivalent load area for a proposed arch carrying live load on the right-hand half span, together with dead load over the whole span. Divide up the load area as before, and draw the lines of action 1, 2, 3 . . . 12 of the panel loads. Set out the loads to scale on a vertical line OW, and select *any* point P as a pole. Join P1, P2, . . . P12, and draw the corresponding link or funicular polygon AEF, having its sides parallel to the rays P1, P2, . . . P12 of the polar diagram. AEF would be the line of resultant pressure for an arch springing from A and F, having a central rise EG, and carrying the proposed loads. The linear arch passing through A, B, and C has a rise BD *greater* than EG, and will consequently have a *less* horizontal thrust than the arch AEF, in the ratio $\frac{EG}{BD}$, since the horizontal thrust varies inversely as the rise. Join AF, and draw PH parallel to AF. Draw HP_1 parallel to AC and equal to h (the original polar distance of P) $\times \frac{EG}{BD}$. P_1 is then the correct pole position from which to draw a new system of rays whose directions will give the required linear arch passing through ABC. P_1H to the scale of the loads is the constant horizontal thrust acting in this arch, and OP_1 and P_1W the

2 F

reactions at A and C respectively. It will be noticed that ABC is also a bending moment diagram for the system of loads 1, 2, 3, . . . 12 on the span AC.

In order to draw the line of pressures for a proposed arch, the positions of points A, B, and C must be known beforehand. These positions, however, may only be *accurately* fixed by providing the arch with hinged or pin joints at the crown and springing points. Many

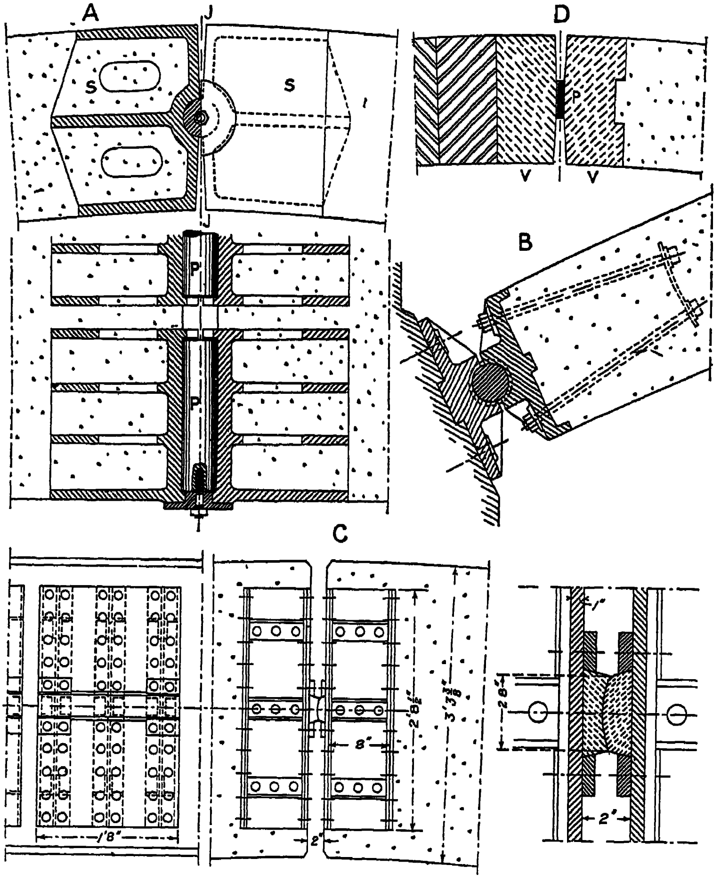


FIG. 369.

arches have been so erected, and it is only in such arches that an *exact* estimate of the stresses is possible. Fig. 369 shows some of the methods of applying hinges or articulations to masonry and concrete arches. At A a number of cast-steel shoes are embedded in the ends of each semi-arch and abut on steel pins P, a narrow joint J–J being left at crown and springing to allow of slight movement under extremes of temperature. This arrangement permits of giving the arch rib an appearance

of continuity if desired, the joints being scarcely noticeable, whilst they are sometimes masked by decoration. The concrete filling immediately behind the casting should be of smaller aggregate to ensure thorough ramming into the pockets of the castings. At B is a similar arrangement, the bearings being secured by long tie-rods taken back into the concrete. The hinges are more exposed, and become a noticeable feature in this design. C shows the detail of the built-up articulations employed for a concrete arched bridge of 164 ft. span erected over the Danube at Munderkingen in 1893. For masonry ribs the arrangement at B may be adopted, the castings being attached to the terminal voussoirs by rag-bolts, or the ends of the ribs adjacent to the hinges may be of concrete, or if of stone, the terminal stones are carefully cut to fit suitable pockets in the castings. Frequently the arrangement at D has been employed. Lead plates P, from $\frac{1}{5}$ to $\frac{1}{3}$ the depth of the arch rib are inserted, the abutting voussoirs V, V, being preferably of granite, basalt, or hard sandstone. The thickness of the lead is from $\frac{3}{4}$ in. to 1 in. for spans up to 150 ft, and the maximum pressure on it may be 1500 lbs. per square inch. Lead begins to yield slightly under a pressure of about 1000 lbs. per square inch, so that in the event of the line of pressure approaching the edge of the plate, the increased intensity of compression causes the lead to yield slightly, with consequent increase of bearing surface and automatic reduction of pressure per square inch.

In addition to ensuring more accurate location of the line of pressure, the provision of hinges has the important effect of annulling the stresses due to change of temperature. Expansion and contraction, which in straight girders is provided for by roller bearings, creates in rigid arches a considerable amount of bending stress which is incapable of exact determination. In a three-hinged arch the two semi-ribs rise and fall slightly after the manner of a toggle joint, and consequently suffer no stress under change of temperature. The stresses induced by expansion and contraction are, however, relatively slight in a rigid masonry arch as compared with those in structural steel arched ribs, since heat does not penetrate a mass of masonry to the same extent as built-up steel members composed of thin plates and section bars.

Line of Pressure in a Rigid Masonry Arch.—In an arch provided with three hinged joints, the centres of the pins fix the position of the line of pressure. In a rigid arch the abutting surfaces at crown and springing have considerable depth, and it is possible to draw several lines of pressure in the same arch, the outlines of which will vary with the positions assumed for the points A, B, and C, in Fig. 368. Of any two possible lines of pressure *abc* and ABC. Fig. 370, due to the same loads over a horizontal span L, the rise *bd* is greater than the rise BD. Since the horizontal thrust is inversely proportional to the rise, the line of pressure *abc* will possess a less horizontal thrust than the line ABC. Hence the horizontal component H_a of the inclined thrust R_a at the abutment will be less than the horizontal component H_A of the inclined thrust R_A. Many other pressure curves might be drawn by varying the positions of A, B, and C. Of these, one will possess a less horizontal thrust than all the others, and by the principle of least resistance, that particular pressure curve will come into operation which entails the least resistance on the part of the abutments. In

other words, the reactions at the abutments must be the least possible consistent with satisfying certain other conditions relating to stress intensity. The vertical reaction V, at either abutment (Fig. 370), is fixed by the load distribution; consequently, by combining the least possible value of H with V, the least value of the inclined thrust R follows. Hence the particular pressure curve sought in a rigid arch is that one which emerges from the arch ring at the greatest inclination with the horizontal, and which satisfies the further following condition.

In order to avoid creating compressive or tensile stresses exceeding the safe resistance of the material of the arch ring, the curve of pressure must not cut any section of the arch outside certain pre-determined limits. If tension is to be *entirely* avoided, the pressure curve must fall within the middle third of the arch thickness. It does not, however, follow that the arch will be unsafe if tension exist at certain points, provided the maximum intensity of compression is still within the safe limit. In masonry arches with good cement joints, a tension of 3 tons per square foot should not endanger the joints, and even if some of these should slightly open, the stability of the arch is not impaired, provided the compression at the opposite edge of the joint be not excessive. In continuous concrete ribs also a similar amount of tension may be safely allowed.

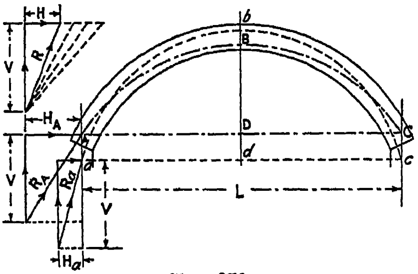


FIG. 370.

If 32 tons per square foot be taken as the maximum safe compression at the edge of a joint nearest to the centre of pressure, and the pressure curve be allowed to approach within three-tenths of the breadth of the joint from that edge, the tension at the opposite edge of the joint would amount to 2·91 tons per square foot, or 45 lbs. per square inch, which in *good* work is quite permissible. It will therefore be assumed that for a maximum pressure of 32 tons per square foot, the pressure curve of least resistance must be contained within the central two-fifths of the arch thickness. In the case of ashlar masonry arches, where the maximum compression per square foot may be limited to 20 to 25 tons, the corresponding tensile stresses for the centre of pressure at three-tenths the breadth, would be 1·82 and 2·28 tons per square foot respectively.

Method of Drawing the Curve of Least Resistance for a Rigid Masonry Arch.—The curve of least resistance may be obtained by a series of trials by varying the positions of points A, B, and C, Fig. 370, within the prescribed limits. This is, however, very tedious, and the following procedure, originally due to Prof. Fuller, gives a direct determination.

In Fig. 371 let DEFG be the equivalent load area, and verticals 1, 2, 3, etc., the lines of action of the weights of the various segments (ten) into which DEFG is divided. Taking AC, the line joining the centres of the springing beds, as the effective span, assume any point B and draw the line of resistance ABC for the loads 1, 2, 3, etc., and span

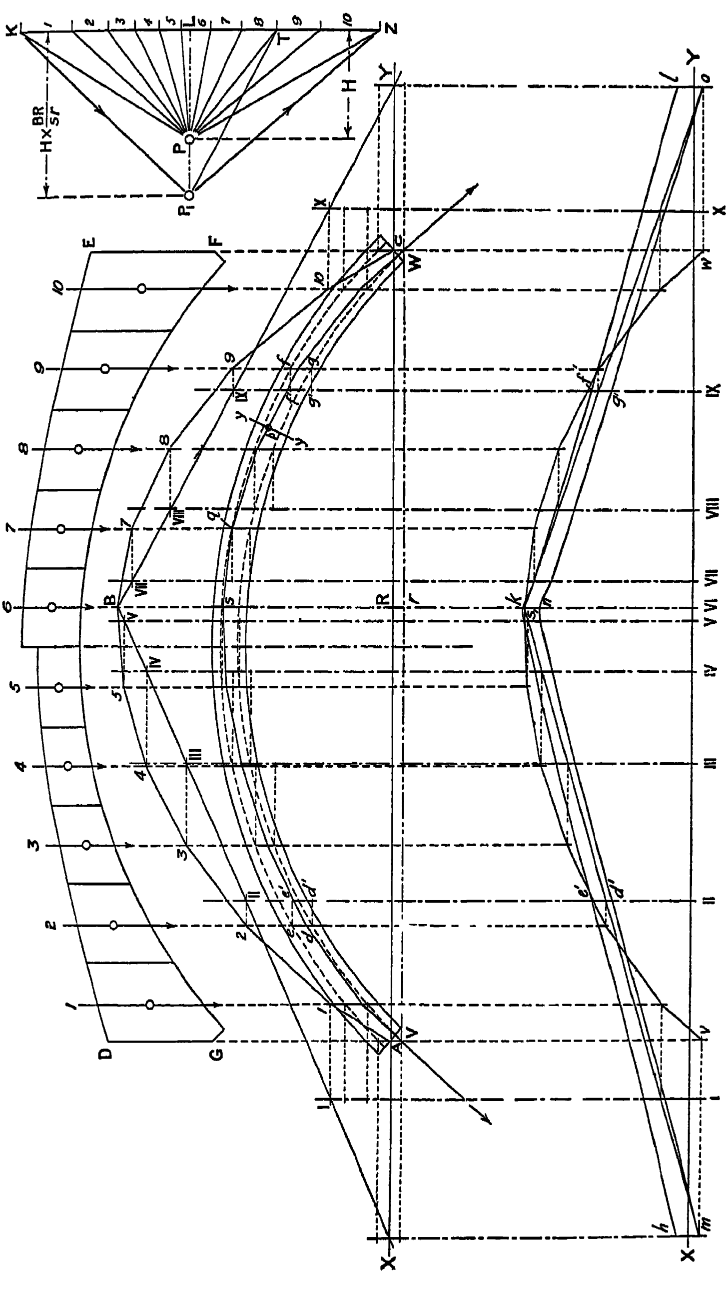


FIG. 371.

AC. The method of drawing ABC has already been given in Fig. 368. The polar diagram from which ABC has been drawn is shown at PKZ, Fig. 371. On the trial outline of the arch, mark the limits between which it is desired to confine the line of least resistance. These are indicated by the dotted curves, and have here been taken to include the middle *half* of the arch thickness. On AC produced, select *any* two points X and Y, and join BX and BY. Project points A, 1, 2, 3, etc., horizontally on to the lines BX and BY, so obtaining points X, I, II, III, etc. Through these latter draw a new series of vertical lines, which, for the sake of clearness, are carried through to the lower figure. Where any original vertical as 2–2 cuts the middle half limits of the arch in d and e, project these points horizontally to d' and e' on vertical II–II. Similarly f and g on 9–9 will project to f' and g' on IX–IX. Transfer the heights above XY of all the points so obtained to the lower figure, and connect them by the irregular boundaries *hkl* and *mno*. These boundaries form a distorted outline of the middle half limits of the arch, having the same degree of horizontal distortion as was given to the line of resistance A1234 . . . BC, by projecting it on to the straight lines BX and BY. Since the line of resistance sought will have ordinates in the same ratio as those of ABC, it will be represented on the lower distorted diagram by two *straight* lines included between the irregular boundaries *hkl* and *mno*. Further, the line of *least* resistance will be represented by the two lines most steeply inclined to the horizontal, which may be drawn between *hkl* and *mno*. These are shown by s_1m and s_1o. If two lines such as s_1m and s_1o cannot be drawn within *hkl* and *mno*, the arch thickness must be increased.

It remains, then, to re-proportion these lines horizontally by projecting their points of intersection with verticals I, II, III, etc., back on to verticals 1, 2, 3, etc. The polygonal system of lines vs_1w connecting these points is the required curve of least resistance. This line of resistance vs_1w is transferred to the upper figure, taking care to place it in the same relative position to the horizontal line XY as in the lower figure, when it will be found to lie within the middle half limits of the arch.

To obtain the reactions due to this curve of pressure, join VW intersecting BR in r. The rise of the original line of resistance ABC = BR The corresponding rise of VsW = sr. The horizontal thrust being inversely proportional to the rise, the horizontal thrust for the linear arch VsW $= \text{PL} \times \dfrac{\text{BR}}{sr}$, PL being the horizontal thrust scaled from the polar diagram PKZ, originally employed for drawing the linear arch ABC. From L draw LP_1 parallel to VW, and mark P_1 at a *horizontal* distance from the load line KZ $= \text{PL} \times \dfrac{\text{BR}}{sr}$. Join P_1K and P_1Z. P_1 is the correct pole position for the line of resistance VsW, and the reactions at V and W due to this line of resistance are respectively P_1K and P_1Z, measured to the scale of the vertical loads. These reactions are equal and opposite to the inclined thrusts at V and W, which are required in designing the abutments for the arch. If P_1 be joined to the points 1, 2, 3, etc., on the load line KZ, the rays of the new polar diagram so formed will be parallel to the segments of the line of resistance VsW.

The intersection of the line of least resistance VsW with any section as yy, determines the centre of pressure p on yy, whilst the length of the corresponding ray P_1T of the polar diagram gives the magnitude of the thrust on the section yy. From these data the intensities of pressure or tension (if any) at opposite faces of the section yy may be calculated in the usual manner. Generally the most heavily stressed sections will be those at which the line of least resistance approaches most nearly to the outer or inner faces of the arch ring. In the figure these sections occur at q, V, and W. These sections are often referred to as the *joints* or *planes of rupture*, since at these sections the tendency to failure of the arch ring is greatest.

Design for Three-hinged Concrete Arched Bridge.—*Span* 150 *ft. Rise* 15 *ft. To carry an equivalent distributed live load of* 150 *lbs. per square foot. Clear width between parapets* 30 *ft.*

The general arrangement is shown in the half longitudinal and transverse sections in Fig. 372. The spandrils are hollow, the platform being carried on jack arches of 4 ft. 6 in. span turned between longitudinal spandril walls L, L, stiffened by two transverse walls T, T. A 3-in. asphalte roadway is laid over the upper surface of the concrete backing of the jack arches. Between vertical section No. 2 and the crown the filling is solid.

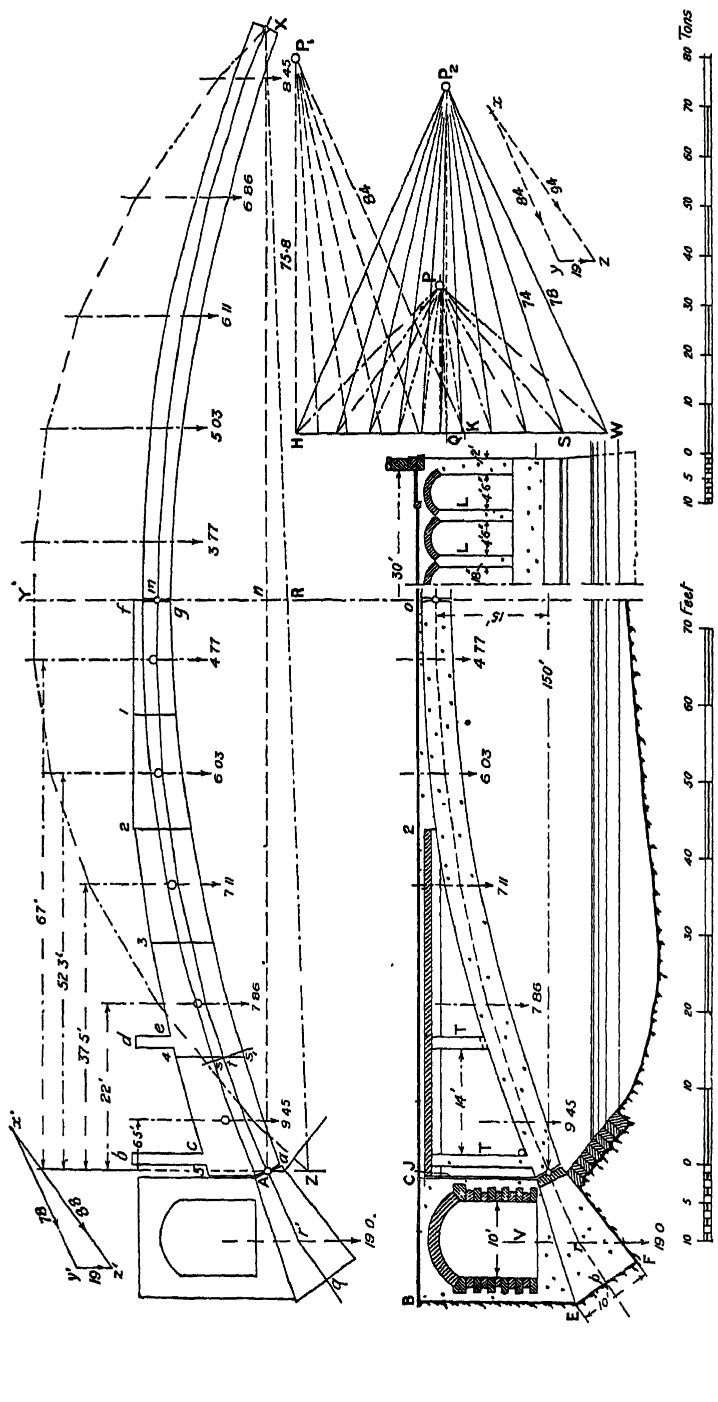
In the upper figure the equivalent load area for one half of the span, inclusive of the live load, is shown by $a5bcd \ldots 2fg$. The weight of the longitudinal walls, platform and load is considered as evenly distributed across the width of the arch sheet, the two vertical projections at b and d representing the weight of the intermediate portions of the transverse walls T, T, equalized for the full breadth of the arch. The arch thickness is taken as 3 ft. 6 in. at crown and springing, and 4 ft. 6 in. at the centre of each semi-rib. Considering first the live load over the whole span, the load area and line of resistance will be similar for each semi-span, and only one half of the line of resistance need be drawn. The load area is here divided into five equal panels of 15 ft. width by verticals 1, 2, 3, 4, and 5. Preferably about ten panels should be taken, so that the resulting line of resistance may approximate more closely to a curve, and the figure be drawn to a much larger scale than is here permissible. The weights of the panels 0–1, 1–2, 2–3, 3–4, and 4–5 per foot width of the arch, estimated at 135 lbs. per cubic foot, are respectively 4·77, 6·03, 7·11, 7·86, and 9·45 tons, and their centres of gravity are indicated by the small circles.

The moments of these loads about the springing point A

$$\begin{aligned} = 4{\cdot}77 \times 67' &= 319{\cdot}6 \\ 6{\cdot}03 \times 52{\cdot}3' &= 315{\cdot}4 \\ 7{\cdot}11 \times 37{\cdot}5' &= 266{\cdot}7 \\ 7{\cdot}86 \times 22' &= 173{\cdot}0 \\ 9{\cdot}45 \times 6{\cdot}5' &= 61{\cdot}5 \\ \hline \text{Sum of moments} &= 1136{\cdot}2 \text{ ft.-tons.} \end{aligned}$$

Dividing by the rise, 15 ft., horizontal thrust at crown $= \dfrac{1136{\cdot}2}{15}$

$= 75{\cdot}8$ tons.



X
P1
P2
P
H
Q
K
S
W
Y'
R
A
Z
B
CJ
E
F
V
L
T
67'
52 3'
37 5'
22'
30'
14'
10'
150'
8 45
6 86
6 11
5 03
3 77
4 77
6 03
7 11
7 86
9 45
19 0
75 8
84
94
74
78
88
19
Tons
Feet

FIG. 372

On the polar diagram $HP_1 = 75{\cdot}8$ tons to scale is drawn horizontally, and the panel loads set off in order to the same scale, between H and K. Lines drawn parallel to the polar rays of this diagram P_1HK, determine the line of resistance (indicated by similar dotted lines) on the semi-arch in the lower sectional elevation. The direction and magnitude of the thrust at the hinge A is given by P_1K, which scales off 84 tons. It should be noted that if a greater number of panels were taken the line of resistance would sensibly approximate to a curve tangent to the polygonal lines in the figure.

Over the abutment is provided an arched passage 10 ft. wide for accommodation of traffic, towage, etc., along the river-bank. The weight of masonry in this arch, together with that of the trapezoidal abutment and live load from B to C equals 19 tons per foot width, and the common centre of gravity is situated on the vertical Vr. Drawing xy parallel to P_1K and equal to 84 tons, and yz vertical $= 19$ tons, $xz = 94$ tons gives the direction and magnitude of the resultant pressure on EF. (A smaller scale has been employed for the triangle xyz than for the polar diagrams.) Drawing rp parallel to xz, the centre of pressure p on EF falls practically at the centre of EF, whence the intensity of pressure on $EF = \frac{94}{10} = 9{\cdot}4$ tons per square foot. In designing an abutment of this type, the position and inclination of EF may, after a few trials, readily be adjusted so that the resultant thrust acts normally to EF and sensibly through its middle point.

Considering next the left-hand half span as carrying dead and live load whilst the right-hand half span carries dead load only, the loads per foot width of the arch are as indicated in the upper figure. The right-hand loads are obtained by deducting the live load per panel $= \frac{15 \times 150}{2240} =$ say, 1 ton, from the left-hand loads. These loads are set off to scale from H to W, commencing with $8{\cdot}45$ tons and following in order from right to left. Selecting *any* pole P the polar diagram PHW is drawn, the rays being indicated by chain-dotted lines. Commencing at X, the corresponding linear arch XYZ is drawn having its segments parallel to the polar rays of PHW. Join ZX and draw PQ parallel to XZ. The correct pole position P_2 for the linear arch passing through X, m and A will be situated horizontally opposite to Q. To calculate the horizontal thrust QP_2, the rise $YR = 35$ ft., $mn = 15$ ft., and horizontal thrust in the polar diagram $PHW = 30$ tons. Hence horizontal thrust or polar distance—

$$QP_2 = 30 \times \frac{YR}{mn} = 30 \times \tfrac{35}{15} = 70 \text{ tons.}$$

Join P_2 to the load intervals on HW and draw in the line of resistance XmA having its segments parallel to the rays of the polar diagram P_2HW. These are indicated by full lines. The actual *curved* line of resistance will be sensibly tangent to the polygonal line XmA. This line of resistance due to the *unsymmetrical* loading will be found to be slightly raised on the left-hand or more heavily loaded semi-span, and slightly depressed on the right-hand semi-span, as compared with the line of resistance in the lower figure for a state of symmetrical loading. The line of resistance approaches most nearly to the outer face

of the arch at the section ss_1. At this section $ss_1 = 50$ in., $st = 18$ in., and the pressure on the section per foot width $= P_2S = 74$ tons.

Hence intensities of pressure at ss_1 are

$$\text{at extrados} = \frac{2 \times 74}{4\frac{1}{6}}\left(2 - \frac{3 \times 18}{50}\right) = 32 \cdot 7 \text{ tons per square foot}$$

$$\text{and at intrados} = \frac{2 \times 74}{4\frac{1}{6}}\left(3 \times \frac{18}{50} - 1\right) = 2 \cdot 9 \quad \text{,,} \quad \text{,,}$$

This maximum pressure of 32·7 tons per square foot, or 509 lbs. per square inch, represents the usual limiting compressive stress allowed on concrete arches. In Prussia and the United States the official allowance is 500 lbs. per square inch, whilst this has been occasionally exceeded in large span arches.

The reactions at A and X are respectively equal to P_2W and P_2H. P_2W scales off 78 tons, and is combined with the vertical abutment weight of 19 tons at $x'y'z'$, $x'z' = 88$ tons, giving the resultant pressure on the base EF of the abutment. $r'q$ parallel to $x'z'$ gives the centre of pressure q, which again falls very nearly at the centre of EF, giving for this disposition of load an intensity of pressure on EF slightly greater than $\frac{88}{10} = 8 \cdot 8$ tons per square foot.

Note.—The directions of the final thrusts on EF for the two cases of loading considered are very slightly different, since the live load bears only a small ratio to the dead load. A much larger scale drawing should be made to enable these directions to be accurately ascertained.

In order to annul stresses due to changes of temperature, the ends of the longitudinal spandril walls L, L, must be discontinuous and just out of contact with the face of the abutment to allow of freedom of movement under the slight rise and fall of the semi-ribs. In an arch of these dimensions an interval of 1 in. to $1\frac{1}{2}$ in. is ample, the magnitude of the movement at J being very small, as will be seen from the following calculation. The length $AmX = 155$ ft. The increase in length for a variation in temperature of 100° F. would be $155 \times 12 \times 0 \cdot 0000055 \times 100 = 1 \cdot 02$ inches, assuming the temperature of the ribs to rise 100° F. throughout, which is most unlikely. The increase in *rise* corresponding with this increase in length is therefore only a small fraction of an inch, whilst the horizontal closure at J will be still less (about $\frac{1}{5}$) by reason of the shorter leverage about the hinge A. The relatively thin continuous road bed of concrete above J is sufficiently flexible to accommodate itself to this slight movement. In laying the concrete at J and over the crown, thin sheets should be placed over the clearance spaces to prevent mortar or stones from falling into and blocking them up.

The hinged joints may be of one of the types shown in Fig. 369. The total breadth of arch at springing = 32 ft. 6 in. Allowing 4 ft. 6 in. for intervals between pins, 14 steel pins 2 ft. long, and 3 in. diameter, would give a projected bearing area of $28 \times 12 \times 3 = 1008$ square inches.

The maximum total thrust at springing $= 84$ tons $\times 32 \cdot 5 = 2730$ tons, and intensity of compression on hinges $= \frac{2730}{1008} = 2 \cdot 7$ tons per square inch.

Skew or Oblique Arches.—When one road or railway crosses another obliquely by means of an arched bridge, it becomes necessary to build a skew arch if the angle of obliquity of crossing is more than a few degrees. In a square arch the bed joints of the masonry are parallel to the springing line and the pressure in each foot width of the arch acts normally to the joints. In a skew arch, Fig. 373, with joints *j*, *j*, parallel to the springing *s*, *s*, the pressure P on any foot width of the arch may be resolved into a normal pressure N, and a tangential or horizontal component T, which tends to cause lateral sliding of the arch courses. This component T obviously increases with the angle of skew or obliquity of crossing, and in order to prevent dislocation of the masonry the bed joints are arranged in directions sensibly perpendicular to that of the oblique pressure P. They therefore wind across the surface of the arch in helical, or, as they are commonly but erroneously called "spiral" curves. The ultimate thrust at the springing is delivered on to checked masonry *skewbacks* *s*, *s*, Fig. 374, and the

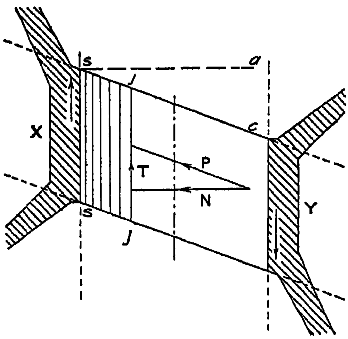


FIG 373.

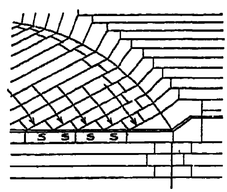


FIG. 374

lateral displacing effect of the horizontal component T has eventually to be resisted by the abutments X and Y, Fig. 373.

Skew arches constructed of masonry throughout are very expensive, since each stone, being of considerable size, has an appreciable amount of twist on both bed and end joints, and requires its faces dressed to twisted or helicoidal surfaces. For these reasons bricks are almost universally employed, excepting where architectural effect is desired. The amount of twist on a surface the size of the face of a brick is small enough to be readily compensated for in the thickness of the mortar joint, and the twist on a course of brickwork consequently augments by frequent small steps instead of constantly as in the case of accurately worked large stones. In the best structural skew arches the sheeting is of brick, with ring stones or voussoirs at the faces bonded with the brickwork, each ring stone being 9 in., 12 in., 15 in., etc., in breadth on the soffit in order to bond with 3, 4, 5, etc., bricks, the size adopted being in pleasing proportion to the dimensions of the arch.

The *angle of skew acs*, Fig. 373, is fixed by the exigencies of the crossing. The angle of inclination of the checks or steps on the skew-backs or springers then requires to be accurately ascertained, so that

the bed joints or "heading spirals" may traverse the arch as nearly as possible at right angles to the pressure. Upon this angle, known as the *angle of skew back*, depends the shape of the springers and ring stones. The checks on the springers being cut to the correct inclination, the courses of brickwork, commencing from the skewback checks, automatically follow the correct curves as they are laid on the centering.

Fig. 375 shows the method of drawing the development of the soffit or intrados of a skew arch, from which the angle of skewback and shapes of springers may be determined. 0123 . . . 12 represents the elevation of the arch looking along the axis PQ. The square span is 40 ft., and angle of skew 66° 30′. The outline of the arch is a circular segment having a rise of 15 ft. DEFG is the plan of the soffit. The segment 0123, etc., is divided into any number (12) of equal parts by points 1, 2, 3, etc. These being projected on to the plan, represent horizontal lines running along the inner surface of the arch. If the curved surface be supposed opened out or developed into a flat sheet by rotating it about the line EF, the joints and outlines of ring stones and springers drawn upon such development, will show their true shapes and inclinations. The square length 0–12′ of the development will be the real length of the arc 0123 . . . 12. This may be stepped off in short portions, but is preferably calculated. The angle 12 C 0 = 147° 28′, and radius 12C = 20 ft. 10 in. Hence, length of arc 0123 . . . 12

$$= \frac{147^\circ\ 28'}{360^\circ} \times 2 \times 3{\cdot}1416 \times 20\tfrac{5}{6} = 53'\ 7\tfrac{3}{4}''\ \text{bare.}$$

This is set out at 0–12′ and divided into twelve equal parts by points 1′, 2′, 3′, etc. Vertical lines through these points determine the developed positions of the lines 1_1, 2_1, 3_1, etc., on the plan DEFG. Projecting the points 1_1, 2_1, 3_1, where the dotted verticals cut DE and FG, horizontally on to verticals 1′, 2′, 3′, etc., points on the development of the opposite edges of the soffit are obtained, which being joined give the curved boundaries E*d* and F*g* of the development.

Accurate Method of Drawing the Joints.—The correct directions for the bed joints so that the lines of pressure may everywhere cut them at right angles, are shown on the right-hand half *pqgd* of the development. The method of drawing them (sometimes called the French method) is as follows. Several curves *aa*, *bb*, *cc*, shown dotted, similar to *pd* and *qg* are marked on the development. These represent developed lines of oblique pressure, and the joints are then carefully drawn in so that they intersect these curves, and the face curves *pd* and *qg*, perpendicularly. As this results in a system of tapered courses, and necessitates several breaks of bond in order to preserve the same width of voussoirs on opposite faces of the arch, the construction is tedious and expensive in stonework, and inapplicable for brickwork arches.

Approximate Method.—A system of parallel courses of uniform width may be substituted for the exact courses without seriously affecting the condition of perpendicularity of pressure. This method, known as the English method, although in very general use, is illustrated on the left-hand half EF *qp* of the development. F*g* is joined and divided

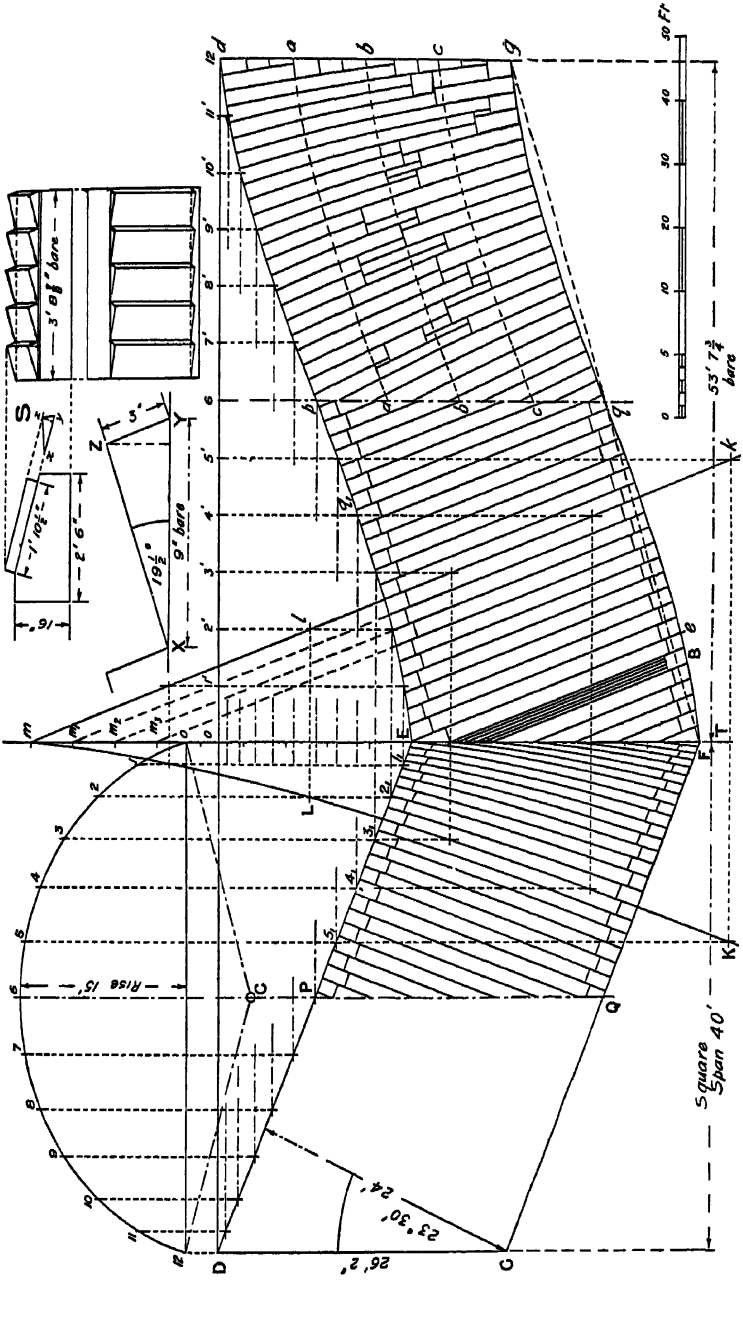

Square Span 40'
Rise 15'
53' 7¾" bare
23° 30'
24'
26' 2"
9" bare
19½°
3'
3' 8⅝" bare
2' 6"
50 Ft

FIG. 375.

into a suitable number of voussoir widths. In this example, F*g* measures 56 ft. 3 in., and represents practically the real length measured round the *oblique* edge FG of the soffit. Adopting a width of voussoir of 15 in., so that each ring stone will bond with five courses of brickwork, $\frac{56' \, 3''}{1' \, 3''} = 45$, gives the number of voussoirs. This should be an *odd* number if the appearance of a key-stone be desired on the face of the arch, although it will be noticed there is no real key-stone in a skew arch in the same sense as in a square arch, the central course at q on one face running over to q_1 some distance from the crown on the opposite face. Dividing F*g* into 45 equal parts, E is joined to that division *e* which locates E*e* as nearly as possible perpendicular to F*g*. All the other joints are then ruled in parallel to E*e*. A comparison of the two halves of the development shows that the directions of the joints so obtained are most in error at the springing, whilst at the crown they almost coincide with the ideal joints. The angle FE*e* = $19\frac{1}{2}°$ is the angle of skewback to which the checks on the springers are to be cut.

If it be desired to insert the joints on the plan of the soffit DEFG, although these curves are not of much practical value, they may be drawn most accurately as follows. Produce any joint as *kl* both ways to intersect the springing line at *m* and centre line *pq* of the development. This point falls off the figure. Where *klm* intersects any vertical as 2′ on the development, in *l*, project *l* horizontally to L on the corresponding vertical 2 of the plan. The curve *m*LK continued through to the centre line PQ, determines the outline in plan of a complete bed joint from springing to crown. Every joint on the plan contained between DE and FG, will be a portion of this curve, and by making a template in stiff paper having the outline *m*KT, and sliding it vertically along *m*T by successive equal intervals mm_1, m_1m_2, m_2m_3, the joints are rapidly and neatly drawn in. For the other halves, here omitted, the template is reversed, and slided along DG. The points m_1, m_2, m_3, etc., are the intersections of the bed joints produced, with the springing line *m*T. In the example, seven of these intervals occur between E and F. EF = 26 ft. 2 in., and mm_1, mm_2, etc , therefore equal $26' \, 2'' \div 7$. Five of the intermediate brickwork courses are indicated at B on the development. The springers may be cut with separate checks for each brick course, or be dressed to a butt heading joint with the whole five bricks, the latter requiring much larger blocks of stone in the rough. One of the intermediate springers is shown at S, the arch being assumed as $22\frac{1}{2}$ in. thick. There being seven springers in the length 26 ft. 2 in., the length of each stone will be $26' \, 2'' \div 7 = 3' \, 8\frac{7}{8}''$ bare, or a little less to allow for joints. The length of the base XY of each check $= 3' \, 8\frac{7}{8}'' \div 5 = 9''$ bare, and drawing XZ at $19\frac{1}{2}°$ slope, and YZ perpendicular to XZ, the correct template for one check is obtained.

The end springers at E and F are respectively acute and obtuse at the outer corners. They are shown in plan and elevation in Fig. 376, to the same scale as S in Fig. 375. In practice, only the exposed faces, beds and joints are worked smooth, the non-fitting portions of the stones being left rough. The end springers are usually not cut with

separate checks, and the springer E, Fig. 376, is dressed square as at *abc*, or obtuse, if the angle *dbc*, which depends on the angle of skew, is very acute.

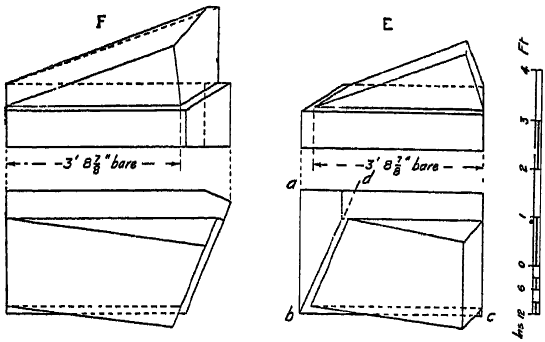


FIG. 376

TALL CHIMNEYS.

The stability of tall chimney stacks depends upon the weight of the brickwork of the shaft and the pressure of the wind upon the outer surface. In calm weather, the centre of pressure at any horizontal section coincides with the centre of gravity of the section, and uniform intensity of compression exists over the whole section. The application of lateral wind pressure causes displacement of the centre of pressure at every horizontal section, and this displacement should be restricted within such limits as to prevent the occurrence of tensile stress in the mortar joints. The *effective* wind pressure on the face of a tall chimney varies considerably with the section of the stack. Colonel Moore[1] gives the following table of coefficients to be applied in calculating the effective wind pressure on the faces of various solids.

TABLE 31.—COEFFICIENTS OF EFFECTIVE WIND PRESSURE ON SOLIDS.

Solid	K	Wind acting
Sphere	0·31	——
Cube	0 81	Normal to face.
"	0 66	Parallel to diagonal of face.
Cylinder (height = diameter)	0·47	Normal to axis.
Cone (height = diameter of base)	0·38	Parallel to base

K = Ratio of the effective pressure on a body to the pressure on a thin plate of area equal to the projected area of the body on a plane normal to the direction of the wind pressure.

It appears reasonable, therefore, to assume the effective pressure on a circular chimney stack as about one-half that on a thin plate of area

[1] *Sanitary Engineering*, Col. E. C. S. Moore, R.E., p. 752.

equal to the projected area of the chimney, that is, the area seen in elevation, and the following effective horizontal wind pressures may be employed for chimneys of various sections :—

On square chimneys,	50	lbs. per square foot,	
„ octagonal „	32	„	„
„ circular „	25	„	„

Let W = Weight of brickwork in tons above any horizontal section.
A = Area of horizontal section in square feet.
I = Moment of inertia of horizontal section in feet units.
R = Outside radius and r = inside radius of horizontal section in feet.
P = Total wind pressure in tons against surface of chimney above the horizontal section considered.
h = Height in feet of centre of pressure of wind above the horizontal section considered.

Then, direct compression on section due to dead weight of brickwork $= \frac{W}{A}$ tons per square foot.

Bending moment due to wind pressure $= P \times h$ foot-tons. Stress due to bending $= \pm \frac{P \times h \times R}{I}$ tons per square foot, the *plus* sign denoting compression at the leeward face and the *minus* sign tension at the windward face of the section. The tensile stress due to the bending action of the wind pressure should not exceed the direct compression due to the dead weight above any horizontal section. It is desirable that the stress be not reduced quite to zero on the windward face, and a residual compression of 100 to 200 lbs. per square foot should remain after deducting the tensile bending stress from the permanent direct compression.

The moment of inertia of a hollow circular section $= 0{\cdot}7854 (R^4 - r^4)$, and of a hollow square section $= \frac{1}{12}(D^4 - d^4)$, D and d being the breadths of outer and inner faces respectively

EXAMPLE 37.—Fig. 377 shows the results of the application of the above rules to the design of a circular chimney stack 160 feet high above ground-level. The dimensions and other data for the calculations are given in the various columns opposite the respective horizontal sections to which they refer. The weight of brickwork is taken as 112 lbs. per cubic foot. This is a little on the safe side as regards the stability, since brickwork will usually weigh from 115 to 118 lbs per cubic foot. As the calculations for the intensities of compression at any horizontal section are similar, only those for one section need be stated.

Considering the horizontal section at 60 feet below the top, weight of brickwork above the section $= 41{\cdot}9 + 64{\cdot}5 = 106{\cdot}4$ tons. Outer diameter = 11 ft. $4\frac{1}{2}$ in., inner diameter = 8 ft. $4\frac{1}{2}$ in., and sectional area = 46·54 sq. ft.

$$\text{Hence direct compression} = \frac{106{\cdot}4}{46{\cdot}54} = 2{\cdot}29 \text{ tons per square foot.}$$

Projected area of the chimney above the section

$$=282 + 318 = 600 \text{ sq. ft.}$$

Height of centre of pressure above the section = 28·5 feet. The positions of the various centres of pressure are indicated on the figure by small circles.

Area in Sq Ft	Weight in Tons	Diameter, Outside	Diameter, Inside	Sectional Elevation	I, in Foot Units	Projected Wind Area	Compression in Tons p Sq Ft, Windward	Compression in Tons p Sq Ft, Leeward
		8′ 3″	6′ 0″					
	41 9					282		
30 62		9′ 9¾″	7′ 6¾″ / 6′ 9¾″		294		0 60	2 12
	64 5					318		
46 54		11′ 4½″	8′ 4½″ / 7′ 7½″		582		0 42	4 16
	90 6					365		
65 18		12′ 11¼″	9′ 2¼″ / 8′ 5¼″		1121		0 41	5·63
	166 0					559		
89 91		15′ 0″	10′ 6″ / 9′ 9″		1885		0 10	7 96
	237 0					473		
153 0		15′ 9″	9′ 9″		4375		0 94	6 90
	187 0							
576 0					27648		0 66	2 28
	490 0							
784 0					51221		1 21	2 39

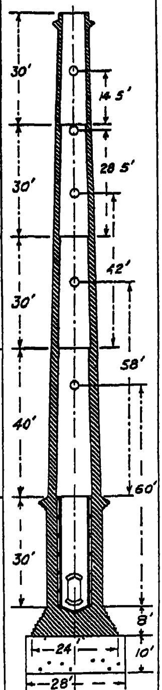


FIG. 377.

$$\text{Bending moment due to wind pressure} = \frac{600 \times 25 \times 28{\cdot}5}{2240}$$
$$= 190{\cdot}84 \text{ ft.-tons.}$$

I, for the section = 582 foot units, outer radius = 5·7 ft., and stress due to bending $= \frac{190{\cdot}84 \times 5{\cdot}7}{582} = \pm\ 1{\cdot}87$ tons per square foot.

Hence compression on leeward face = 2·29 + 1·87 = 4·16, and

compression on windward face = 2·29 − 1·87 = 0·42 ton per square foot.

At the ground-level the section of the pedestal, which is assumed as square for 30 feet above the ground, is a hollow square of 15 ft. 9 in. and 9 ft. 9 in. outer and inner faces. Its moment of inertia = 4375, and total wind pressure against the shaft = 61,750 lbs. This total pressure consists of the pressure on the circular portion of the shaft above the pedestal, taken at 25 lbs. per square foot, and the pressure on the face of the pedestal taken at 50 lbs. per square foot.

The point of application of the resultant of these two pressures is at 60 feet above ground-level.

$$\text{Hence bending moment} = \frac{61750 \times 60}{2240} = 1654 \text{ ft.-tons}$$

$$\text{and stress due to bending} = \frac{1654 \times 7\frac{7'}{8}}{4375} = \pm\ 2{\cdot}98 \text{ tons per square foot.}$$

The total weight of chimney above ground-level (excluding the inner fire-brick lining of the pedestal which is not bonded with the pedestal and consequently rests on the footings) equals 600 tons, and sectional area is 153 sq. ft.

Therefore direct compression at ground-level = $\frac{600}{153}$ = 3·92 tons per square foot, giving 3·92 + 2·98 = 6·9 tons per square foot at leeward face and 3·92 − 2·98 = 0·94 ton per square foot at windward face.

At the base of the footings the total vertical load is the weight of stack, pedestal, fire-brick lining and footings = 787 tons, to which must be added the weight of earth resting on footings, which, at 100 lbs. per cubic foot, amounts to 61 tons. The area of base of footings = 24′ × 24′ = 576 sq. ft.

$$\text{Direct pressure} = \frac{787 + 61}{576} = 1{\cdot}47 \text{ tons per square foot.}$$

$$\text{Bending moment due to wind at this level} = \frac{61750 \times 68}{2240} \text{ foot-tons,}$$

and $I = \frac{1}{12} \times 24^4 = 27{,}648$.

$$\text{Stress due to bending} = \frac{61750 \times 68 \times 12}{2240 \times 27648} = \pm\ 0{\cdot}81 \text{ tons per sq ft.}$$

Compression at leeward face = 1·47 + 0·81 = 2·28 ,, ,,
,, at windward face = 1 47 − 0·81 = 0 66 ,, ,,

A similar calculation at the foundation level gives the intensities of pressure on the soil as 2·39 and 1 21 tons per square foot at leeward and windward edges respectively. The total vertical load at foundation level equals 787 tons due to brickwork in stack, lining and footings, + 490 tons in concrete base (at 140 lbs. per cubic foot) + 136 tons due to weight of earth on footings and upper ledges of concrete block = 1413 tons. The resultant wind pressure acts at a leverage of 78 feet and I for the 28 ft. square base = $\frac{1}{12} \times 28^4 = 51{,}221$.

$$\text{Direct pressure} = \frac{1413}{784} = 1{\cdot}8 \text{ tons per square foot,}$$

and stress due to bending $= \frac{27 \cdot 6 \text{ tons} \times 78' \times 14}{51221} = \pm 0 \cdot 59$ ton per square foot.

Compression at leeward edge of foundation $= 1 \cdot 8 + 0 \cdot 59 = 2 \cdot 39$ tons per square foot, and at windward edge $= 1 \cdot 8 - 0 \cdot 59 = 1 \cdot 21$ tons per square foot. The side of the pedestal at which the flue enters should be suitably thickened to compensate for the weakening of the horizontal section at this level.

London County Council Regulations require that the height of square chimney stacks shall not exceed *ten* times the width of basal section, measured immediately above the footings. The height of circular chimney stacks must not exceed *twelve* times the diameter clear of the footings, and octagonal shafts *eleven* times the diameter. The shaft is to batter not less than 2½ inches in every 10 feet, and no projection is to extend beyond the face of the shaft to a greater distance than the thickness of the wall at that level. The minimum thickness of the uppermost section to be 9 inches and the thickness to increase by 4½ inches at least every 20 feet from the top downwards. The footings are to project beyond the basal section, on all sides, to a minimum distance equal to the thickness of the wall at the base.

The circular chimney realizes the greatest stability with the minimum quantity of brickwork and most efficient section of flue, and is equally resistant to the wind pressure applied in any direction. Strongly projecting copings, caps, or string courses are undesirable, since they offer increased resistance to the wind, the former at the maximum leverage above the base. Dangerous oscillations of stacks with heavily projecting caps have occasionally been considerably reduced by removal of the cap. Inverted arches beneath the centre of the shaft are probably more dangerous than advantageous, since they may contribute to unequal settlement at the base with resulting fracture.

The fire-brick lining must be free to expand and contract independently of the main stack. Its height will depend on the particular purpose of the chimney. It may be as low as 20 ft. or 30 ft., but is commonly 50 ft. to 80 ft. high in lofty chimneys—one-fifth to one-fourth the height of the shaft is a common rule. In the St. Rollox chimney the inner shaft rises to a height of 243 ft., the total height of the chimney being 455 ft. 6 ins.

An efficient air space should be provided between the fire-brick lining and the outer shaft, suitably covered or protected by corbelled courses of brickwork projecting from the inside of the outer shaft, and leaving sufficient clearance for maximum expansion of the lining. This latter may be estimated at one inch per 35 to 40 feet of height.

The following are particulars of the great St. Rollox chimney at Glasgow. The chimney is of circular section and rises to a height of 455 ft. 6 ins. above foundation level. The thickness of the outer shell of brickwork at ground level is 3 ft., spreading to footings 6 ft. wide at 15 ft. depth. These rest on 5 ft. thickness of concrete having a basal width of 50 ft. The chimney is founded on Old Red Sandstone rock.

St. Rollox Chimney.

Level	Thickness.	Outer Diameter
Ground to 54 ft. 6 ins.	33 ins.	40 ft. 0 ins. to 35 ft. 0 ins.
54 ft. 6 ins. to 116 „ 6 „	28 „	35 „ 0 „ „ 30 „ 6 „
116 „ 6 „ „ 209 „ 6 „	$22\frac{1}{2}$ „	30 „ 6 „ „ 24 „ 0 „
209 „ 6 „ „ 349 „ 6 „	18 „	24 „ 0 „ „ 16 „ 9 „
349 „ 6 „ „ 435 „ 6 „	14 „	16 „ 9 „ „ 13 „ 6 „

The section of minimum stability is stated to be at the 209 ft. 6 ins. level, where the chimney is calculated to withstand with safety a wind pressure of 55 pounds per square foot.

The internal shaft is externally cylindrical of 10 ft. 6 ins. internal diameter at bottom and 12 ft. at the top. Its height above ground level is 243 ft., and its thickness $22\frac{1}{2}$ ins. for the lower 62 ft. of its height. It is carried up in three sections of diminishing thickness and stiffened by buttresses on the outside.

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